Two-loop matching of the chromo-magnetic dipole operator with the gradient flow

Janosch Borgulat,[a] Robert Harlander,[a,*] Matthew D. Rizik[b] and Andrea Shindler[b]

[a] *TTK, RWTH Aachen University, D-52056 Aachen, Germany*

[b] *FRIB & Physics Department, Michigan State University, East Lansing, MI 48824, USA*

E-mail: janosch.borgulat@rwth-aachen.de, harlander@physik.rwth-aachen.de, rizik@nscl.msu.edu, shindler@frib.msu.edu

The chromo-magnetic dipole operator is expressed in terms of operators at finite flow time in the gradient-flow formalism. The matching coefficients are evaluated through next-to-next-to-leading order QCD.

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*Speaker

1. Introduction

Beyond the Standard Model (BSM) effects on observables measured at hadronic scales of a few GeV are encoded in higher dimensional ($D > 4$) operators. The relevant degrees of freedom are the same as the ones of QCD, quarks and gluons, and QED effects should also be included if the precision requires it. In most cases, the calculation of hadronic matrix elements using lattice QCD (LQCD) is challenging because of the complicated renormalization pattern.

A long-standing unresolved question in modern physics is the measured matter-antimatter asymmetry in the Universe [1], that is orders of magnitudes larger than the Standard Model (SM) prediction [2, 3]. Sakharov conditions [4], among other constraints, dictate that any interaction responsible for the baryon asymmetry should violate CP-symmetry. The amount of CP-violation in the SM (from the CKM matrix) is wildly insufficient to explain the measured baryon asymmetry, thus providing a very strong argument to search for new sources of CP-violation.

The electric dipole moment (EDM) of the neutron provides a system sensitive to CP-violating BSM sources and free of any background for many order of magnitude. Many different higher dimensional operators contribute to the neutron EDM and in particular the so-called $D = 5$ quark-chromo EDM (qCEDM) provides an important contribution (see Ref. [5] for a review on LQCD calculations on the neutron EDM). The qCEDM has a complicated renormalization pattern on the lattice [6], which has prevented the calculation of the qCEDM contribution to the neutron EDM.

In a set of publications [7–12], it was proposed to use the gradient flow [13–15], to solve the renormalization of the qCEDM as well as all the other higher dimensional operators contributing to the neutron EDM. The method proposed is based on the short flow-time expansion (SFTX) of the higher dimensional operators and it needs non-perturbative LQCD computations combined with calculations in perturbative QCD. Perturbative QCD is needed to match the LQCD results obtained at finite flow time t, with the renormalized matrix elements at $t = 0$. In Refs. [11, 12], a 1-loop calculation of the matching coefficients of the qCEDM and the corresponding CP-even operator, the chromo-magnetic (CM) operator was presented.

Using the methods and tools of Ref. [16], it is possible to extend this calculation to the 2-loop level, as it has been done in similar applications including the energy-momentum tensor [17], the hadronic vacuum polarization [18], and the effective weak Hamiltonian [19] (see also Ref. [20]). In these proceedings we present this extension to two loops for the chromo-magnetic dipole operator in the case of massless quark. This avoids nuisances with the definition of γ_5 in a generic D dimension. We emphasize that the calculation of the matching coefficients on the CM operator is a phenomenologically relevant calculation on its own. For example for strangeness changing $\Delta S = 1$ CM operators contribute to rare kaon decays (CP-even and CP-odd), to the $K^0 - \overline{K}^0$ oscillation and to the ϵ'/ϵ ratio parametrizing direct CP-violation [21, 22].

2. Operator basis

Throughout this paper, we work in single-flavor massless QCD. The general case will be deferred to a forthcoming publication. Working in $D = 4 - 2\epsilon$ space-time dimensions in order to regularize UV and IR divergences, the quark chromomagnetic dipole operator is

$$O_{CM} = \mu^{2\epsilon} g_{\mathrm{B}} \bar{\psi} t^a \sigma_{\mu\nu} \psi F^a_{\mu\nu} \,, \tag{1}$$

where we factored out one power of the bare strong coupling $g_{\rm B}$, and introduced the 't Hooft mass μ in order to compensate for the non-integer mass dimension of the operator. ψ is the bare quark field, t^a the generators of SU(3) in the fundamental representation, and the field strength tensor is given by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_B f^{abc} A^b_\mu A^c_\nu\,, \tag{2}$$

with the bare gluon field A_μ and the SU(3) structure constants f^{abc}. The goal of this paper is to express this operator in terms of its flowed counter part, defined through

$$\tilde{O}_{CM}(t) = g\bar{\chi}(t)t^a\sigma_{\mu\nu}\chi(t)G^a_{\mu\nu}(t)\,, \tag{3}$$

with g the $\overline{\rm MS}$-renormalized strong coupling. The flowed quark and gluon fields are defined through the flow equations

$$\partial_t\chi(x,t) = \Delta\chi(x,t)\,, \qquad \partial_t B_\mu(x,t) = D_\nu G_{\nu\mu}(x,t)\,, \tag{4}$$

where $\Delta = D_\mu D_\mu$ contains derivative acting on the fundamental representation, while the derivative acting on the field tensor reads $D_\mu = \partial_\mu + \left[B_\mu,\cdot\right]$. The boundary conditions satisfied by the fields are

$$\chi(t=0) = \psi\,, \qquad B^a_\mu(t=0) = A^a_\mu\,. \tag{5}$$

Taking the limit $t \to 0$ of $\tilde{O}_{CM}(t)$ leads to an operator-product expansion in terms of regular-QCD operators with t-dependent coefficients.

Up to leading order in t, it involves O_{CM}, the dimension-3 operator

$$O_S = i\mu^{2\epsilon}\bar{\psi}\psi\,, \tag{6}$$

plus operators that vanish in physical matrix elements due to gauge invariance or equations of motion. Even though the latter can be neglected in the final result, they will play a role in our calculation as outlined in the next section.

In order to express O_{CM} in terms of flowed operators, we therefore need to introduce also the flowed counter part $\tilde{O}_S(t)$ of O_S, which allows us to establish a one-to-one relation between regular and flowed operators, given by the bare matching matrix $\zeta^{\rm B}(t)$:

$$\begin{pmatrix}\tilde{O}_{CM}(t)\\ \tilde{O}_S(t)\end{pmatrix} = \zeta^{\rm B}(t)\begin{pmatrix}O_{CM}\\ O_S\end{pmatrix} + \cdots\,, \tag{7}$$

where

$$\tilde{O}_S(t) = i\bar{\chi}(t)\chi(t)\,, \tag{8}$$

and the ellipsis denotes terms that vanish as $t \to 0$. Since the flowed operators lead to UV finite Green's functions, while those of the regular operators are divergent in general, the $\zeta^{\rm B}(t)$ will contain UV singularities in general. We can, however, define a renormalized matching matrix as

$$\zeta(t) = \zeta^{\rm B}(t)Z^{-1} = \begin{pmatrix}c_{CM}(t) & c_S(t)/t\\ 0+\cdots & s_S(t)\end{pmatrix}\,, \tag{9}$$

where again the ellipsis denotes terms that vanish as $t \to 0$. The operator renormalization matrix Z is diagonal in our case, because operators of different mass dimension do not mix in the $\overline{\text{MS}}$ scheme:

$$Z = \begin{pmatrix} Z_{CM} & 0 \\ 0 & Z_m \end{pmatrix} . \tag{10}$$

Z_m is the quark mass renormalization constant, and Z_{CM} can be extracted through N^{3}LO from Ref. [23].

3. Determination of the matching matrix

The elements of the matching matrix ζ^{B} can be determined with the help of the method of projectors [24]. The projections consist of certain derivatives of suitable Green's functions w.r.t. external momenta, collectively denoted by p in the following, such that[1]

$$P_i[O_j] \equiv \mathcal{P}_i(\partial_p)\langle i|\Gamma_i O_j|0\rangle\Big|_{p=0} = \delta_{ij} , \tag{11}$$

where Γ_i is a matrix in Dirac space, $\mathcal{P}_i(x)$ is polynomial in x, and $\langle i|$ is a state in (adjoint) Fock space. It is important that the derivatives ∂_p and the nullification of p are understood to be taken before any loop integration is carried out. This means that it is sufficient to satisfy Eq. (11) at tree-level, because all loop contributions are scaleless and vanish in dimensional regularization.

Using these projections, one thus directly obtains

$$\zeta^{\text{B}}_{ij}(t) = P_j[\tilde{O}_i(t)] . \tag{12}$$

Since the Green's functions considered in Eq. (11) do not represent physical matrix elements, one needs to take into account operators which vanish by equations-of-motion in the determination of the projectors.

For example, it is suggestive to define the following projector for O_{CM}:

$$P'_{CM}[X] = -i\frac{t^a_{ij}}{16\, g_{\text{B}} D(D-1)}\frac{\partial}{\partial q_\rho}\langle ija\mu, p, q|\sigma^{\mu\rho}X|0\rangle\Big|_{q=p=0} , \tag{13}$$

where $\langle ija\mu, p, q|$ contains a quark and an antiquark of color i and j and momenta p and $-(p+q)$, as well as a gluon of color a, Lorentz index μ, and momentum q. Indeed, this gives $P'_{CM}[O_{CM}] = 1$ and $P'_{CM}[O_S] = 0$. However, one easily shows that $P'_{CM}[O_{D^2}] = -i/2$, where

$$O_{D^2} = \mu^{2\epsilon}\bar{\psi}\not{D}^2\psi , \tag{14}$$

In order to subtract this contribution in the proper definition of the projector onto O_{CM}, we define a projector onto O_{D^2} as

$$P_{D^2}[X] = i\frac{t^a_{ij}}{2D g_{\text{B}}}\frac{\partial}{\partial q_\mu}\langle ija\mu, p, q|X|0\rangle\Big|_{q=p=0} , \tag{15}$$

[1]The general case also includes derivatives w.r.t. masses.

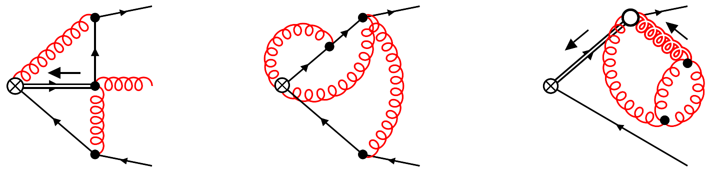

Figure 1: Examplary (non-vanishing) diagrams for the calculation of $c^{\mathrm{B}}_{CM}(t)$, $c^{\mathrm{B}}_{S}(t)$, and $s^{\mathrm{B}}_{S}(t)$, (a)–(c). Double lines denote "flow-lines", the direction of the flow is indicated by the arrow next to the line. The vertex on the left is due to the flowed operator. Vertices with white filling are at finite flow time. See Ref. [16] for more details. Diagrams produced using `FeynGame` [25].

which leads to the actual projector onto O_{CM}:

$$P_{CM} = P'_{CM} + \frac{i}{2} P_{D^2} \,. \tag{16}$$

The projector onto O_S is simply

$$P_S[X] = -i\frac{\delta_{ij}}{12}\langle ij, p|X|0\rangle\Big|_{p=0} \,, \tag{17}$$

where $\langle ij, p|$ contains a quark-antiquark pair of color ij and momenta p and $-p$. In total, this amounts to 3375 Feynman diagrams for c^{B}_{CM} at 2-loop level (45 at 1-loop level), 226 (5) for c^{B}_{S}, and 383 (10) for s^{B}_{S}. One examplary 2-loop diagram for each of these coefficients is shown in Fig. 1. We compute them using the setup described in Ref. [16]. After performing the Dirac traces and setting the external momenta to zero, they lead to integrals of the form

$$I(\mathbf{c}, \mathbf{a}, \mathbf{b}) = \int_0^1 d\mathbf{u}\, \mathbf{u}^{\mathbf{c}} \int \frac{d^D\mathbf{p}}{(2\pi)^{lD}} \frac{\exp\left[-t\mathbf{a}(\mathbf{u})\cdot\mathbf{D}(\mathbf{p})\right]}{\mathbf{D}^{\mathbf{b}}(\mathbf{p})} \tag{18}$$

where the $\mathbf{D} = (D_1, \ldots, D_k)$ are quadratic polynomials of the $\mathbf{p} = (p_1, \ldots, p_l)$, and the $\mathbf{a} = (a_1, \ldots, a_k)$ are polynomials of the (dimensionless) flow-time variables $\mathbf{u} = (u_1, \ldots, u_f)$. Furthermore, $\mathbf{u}^{\mathbf{c}} = u_1^{c_1}\cdots u_f^{c_f}$ and $\mathbf{D}^{\mathbf{b}} = D_1^{b_1}\cdots D_k^{b_k}$, with integer c_i and b_k. At one-loop level, the parameters take the values $l = 1$, $0 \le f \le 2$, and $k = 1$, while at two-loop level, it is $l = 2$, $0 \le f \le 4$, and $k = 3$.

Using integration-by-parts [26, 27], including its extension to flow-time integrals [16], it is possible to reduce all one-loop integrals to the single master integral

$$I(\{\}, \{2\}, \{0\}) = \int \frac{d^Dp}{(2\pi)^D}\, e^{-2tp^2} = \frac{1}{(8\pi t)^{D/2}} \,. \tag{19}$$

At two-loop level, we find four master integrals:

$$
\begin{aligned}
I(\{\},\{2,2,0\},\{0,0,0\}) &= \frac{1}{(8\pi t)^D}\,,\\
I(\{\},\{2,0,0\},\{1,1,0\}) &= \frac{t^2}{(8\pi t)^D}\left(\frac{4}{\epsilon}+4+O(\epsilon)\right),\\
I(\{0\},\{2-u_1,u_1,u_1\},\{0,0,0\}) &= \frac{1}{(8\pi t)^D}\left(\frac{1}{2\epsilon}-\frac{7}{6}+\ln 2-\frac{1}{2}\ln 3+O(\epsilon)\right),\\
I(\{0\},\{1-u_1,1+u_1,1+u_1\},\{0,0,0\}) &= \frac{1}{(8\pi t)^D}\left(\frac{2}{3}+\frac{1}{2}\ln 3+O(\epsilon)\right).
\end{aligned}
\tag{20}
$$

4. Result

After renormalization according to Eq. (9), the dimensionless coefficients of the matching matrix $\zeta(t)$ through NNLO QCD read, in single-flavor QCD,

$$
\begin{aligned}
c_{CM} &= 1+\frac{\alpha_s}{\pi}\left(-4.0228+0.16667\,l_{\mu t}\right)+\left(\frac{\alpha_s}{\pi}\right)^2\left(-11.61-10.15\,l_{\mu t}+0.2292\,l_{\mu t}^2\right),\\
c_S &= -2\frac{\alpha_s}{\pi}+\left(\frac{\alpha_s}{\pi}\right)^2\left(-6.136-3.167\,l_{\mu t}\right),\\
s_S &= 1+\frac{\alpha_s}{\pi}\left(-2.6895-l_{\mu t}\right)+\left(\frac{\alpha_s}{\pi}\right)^2\left(-4.546-8.328\,l_{\mu t}-0.7917\,l_{\mu t}^2\right),
\end{aligned}
\tag{21}
$$

where $\alpha_s = g^2/(4\pi)$, and $l_{\mu t} = \log(2\mu^2 t)+\gamma_E$, with $\gamma_E = 0.577216\ldots$ the Euler-Mascheroni constant. For the sake of brevity, we have inserted SU(3) color factors in this expression, and expressed all coefficients as floating point numbers. Through NLO, the coefficients of c_{CM} and c_S agree with Ref. [12],[2] all other results are new.

5. Summary

The renormalization of the chromo-magnetic dipole operator on the lattice is particularly challenging because of the mixing with operators of the same and lower dimensions. The gradient flow provides a tool to resolve this challenge, because it allows a matching of the calculation of matrix elements of flowed operators with the renormalized physical ones at vanishing flow time. While the matching with lower dimensional operators will eventually have to be performed non-perturbatively, a perturbative calculation provides a strong guidance when analyzing lattice QCD data. We have presented a 2-loop calculation of the matching coefficients for massless quark. The extension to include operators with non-vanishing mass is in progress. Beside a direct application to kaon physics, these results pave the way to determination of renormalized CP-odd quark-chromo electric dipole moment operator, a very important contribution from beyond-the-standard-model physics to the neutron electric dipole moment.

[2]Note the different normalization of the operators and matching coefficients though.

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