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Topological susceptibility, scale setting and universality from $Sp(N_c)$ gauge theories

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In this contribution, we report on our study of the properties of the Wilson flow and on the calculation of the topological susceptibility of $Sp(N_c)$ gauge theories for $N_c = 2, 4, 6, 8$. The Wilson flow is shown to scale according to the quadratic Casimir operator of the gauge group, as was already observed for $SU(N_c)$, and the commonly used scales t_0 and w_0 are obtained for a large interval of the inverse coupling for each probed value of N_c. The continuum limit of the topological susceptibility is computed and we conjecture that it scales with the dimension of the group. The lattice measurements performed in the $SU(N_c)$ Yang-Mills theories by several independent collaborations allow us to test this conjecture and to obtain a universal large-N_c limit of the rescaled topological susceptibility.

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PoS(LATTICE2022)400

1. Introduction

Theories based on $Sp(N_c)$ gauge symmetry have been under scrutiny in the past few years from different points of view, prominently for their role in realizing UV-complete composite Higgs models (CHMs) in which to implement top partial compositeness [1, 2]. In addition, they provide an alternative routes, besides the families of groups $SU(N_c)$ and $SO(N_c)$, in the study of the universal features of the large-N_c limit of gauge theories [3, 4]. Several contributions relevant to this programme have been presented at this conference [5, 6], and a review of recent results can be found in Ref. [7].

This contribution reports on a lattice study of the topological features of the vacuum of $Sp(N_c)$ pure gauge theories for $N_c = 2$, 4, 6, and 8. The scale of the lattice theory is set using the Wilson Flow (WF) and the continuum and large-N_c limits of topological susceptibility are obtained. In QCD, the latter enters crucially in the solution to the $U(1)_A$ problem [8, 9]. Moreover, it appears as the coefficient of the $O(\theta^2)$ term in the expansion of the the free energy in powers of a complex-valued θ. Thus, it encodes properties of gauge theories that might play a role in understanding the physics of the strong-CP problem (and the properties of the axion) and in the determination of the electric dipole moment of hadrons. Several studies have been devoted to this quantity for $SU(N_c)$ gauge theories [10–14]. The comparison between these results and the ones obtained for $Sp(N_c)$ suggest to propose a conjecture on the universal properties of the topological susceptibility at large-N_c. For additional details, see Refs. [15–18].

2. Lattice setup

The lattice theory is defined on a four dimensional euclidean hypercubic lattice, with L/a sites in each direction. Sites are labelled by x and links by (x, μ). The elementary degrees of freedom, defined on links, are $Sp(N_c)$-valued and denoted by $U_\mu(x)$. The action is defined as

$$S_W[U] = \beta \sum_{x,\mu>\nu} \left(1 - \frac{1}{N_c} \mathrm{ReTr}\, U_{\mu\nu}(x)\right) , \qquad \text{with} \quad \beta = \frac{2N_c}{g_0^2} , \tag{1}$$

where g_0 is the bare coupling and $U_{\mu\nu}(x) = U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)$ is the so-called *plaquette variable*. Expectation values of gauge invariant operators can be defined as finite integrals in the measure $\mathcal{D}U_\mu = \prod_{x,\mu} dU_\mu(x)$,

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}U_\mu O(U_\mu) e^{-S_W[U]} \tag{2}$$

where $Z = \int \mathcal{D}U_\mu e^{-S_W[U]}$, and computed numerically as *ensemble averages*. The ensembles are sets of configurations of the gauge field generated with Monte Carlo sampling and stored for later analysis. Successive configurations are separated by a number N_{sw} of *full lattice sweeps*, i.e. successive updates of all the links on the lattices with the traditional $1 - 4$ combination of heat bath (HB) and over-relaxation (OR) updates. Ensembles of ~ 4000 configurations were obtained for various values of the input parameters β, N_c, and L. The specific values of the latter were chosen to reduce finite size effects. Details can be found in Ref. [15].

The difficulties in calculating the topological charge Q and the topological susceptibility χ from their lattice discretizations are well known [19]. As a consequence of the discretization of the theory, the lattice topological charge Q_L is not integer valued. Moreover, the lattice topological suspceptibility χ_L must be additively renormalized and is dominated by UV fluctuations as $a \to 0$. In order to overcome these difficulties, and to set the scale of the lattice at each value of β, the properties of the WF were employed [20, 21]. The flowed field $V_\mu(x,t)$ is defined by

$$\frac{\partial V_\mu(t,\, x)}{\partial t} = -g_0^2 \left\{\partial_{x,\mu} S_{\mathrm{W}}[V_\mu]\right\} V_\mu(t,\, x), \qquad V_\mu(0,\, x) = U_\mu(x)\,, \tag{3}$$

where t is known as flow time. At large t, the WF drives the theory to its classical limit and, at leading order in the coupling, $V_\mu(x,t)$ can be shown to be a gaussian smoothening of $U_\mu(x)$ with mean radius $1/\sqrt{8t}$. As a consequence of the suppression of UV fluctuations, the lattice topological charge takes on quasi-integer values and the lattice topological susceptibility has an improved behaviour towards the continuum.

Several definitions of the lattice topological charge density are possible, that differ from one another by terms of order $O(a^4)$ as $a \to 0$. In this study, the clover-leaf expression was used

$$q_L(x,\, t) \equiv \frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\, C_{\mu\nu}(x,\, t)C_{\rho\sigma}(x,\, t)\,, \tag{4}$$

which is computed on $V_\mu(x,\, t)$. We refer to Ref. [15] for the full expression of $C_{\mu\nu}(x,\, t)$ as a function of $V_\mu(x,\, t)$. The lattice topological charge can then be obtained as $Q_L(t) = \sum_x q_L(x,\, t)$ and the effects of discretization can be further reduced by employing the α-rounding procedure [12], that yelds integer values for Q_L.

Sampling different topological sectors at small lattice spacing with a local update algorithm is a notoriously difficult task, especially at larger values of N_c. As $a \to 0$, the integrated autocorrelation time τ_Q of the Q_L is known to diverge exponentially [11], with an exponent that increases with N_c. Several remedies have been designed to restore ergodicity [22–25] as $a \to 0$ at large-N_c. In this study, we tuned the value of N_{sw} so that, for each ensemble, $\tau_Q \sim 1$. That the distribution of the sampled topolofical charges was indeed gaussian, as expected, was verified for each ensemble separately by performing a gaussian fit to the frequency histogram of Q_L. Finally, the topological susceptibility was evaluated as

$$\chi_L(t)a^{-4} = \frac{\langle Q_L^2(t)\rangle}{L^4}\,. \tag{5}$$

for each N_c, β and L.

The scale of the lattice can be set by defining the renormalized coupling

$$\alpha(\mu) \equiv k_\alpha t^2\langle E(t)\rangle \equiv k_\alpha \mathcal{E}(t)\,, \quad E(t) = \sum_x \tfrac{1}{2}\mathrm{Tr} V_{\mu\nu}(x,\, t)V_{\mu\nu}(x,\, t) \tag{6}$$

at scale $\mu = 1/\sqrt{8t}$, where k_α is a constant that can be calculated perturbatively, and $V_{\mu\nu}(x,\, t)$ is the plaquette variable calculated from $V_\mu(x,\, t)$. As an alternative way of setting the scale [26], the quantity

$$\mathcal{W}(t) = t\frac{d}{dt}\{\mathcal{E}(t)\} \tag{7}$$

can be used instead of $\mathcal{E}(t)$. As $\mathcal{W}(t)$ is expected to be affected by milder discretization effects than $\mathcal{E}(t)$, their comparison also provides an approximate way to quantify the magnitude of discretization effects. The scales t_0 and w_0 can then be defined implicitly by $\mathcal{E}(t_0) = \mathcal{E}_0$ and $\mathcal{W}(t_0) = \mathcal{W}_0$, where $\mathcal{E}_0$ and $\mathcal{W}_0$ can be chosen arbitrarily. A way to perform the large-N_c limit at fixed 't Hooft coupling, $\lambda = 4\pi N_c \alpha$, can be understood from the perturbative expression

$$\mathcal{E}(t) = \frac{3\lambda}{64\pi^2} C_2(F) , \tag{8}$$

where $C_2(F)$ is the quadratic Casimir operator of the gauge group. For the fundamental representation of $Sp(N_c)$ one has $C_2(F) = (N_c + 1)/4$. The above scaling law was tested in the context of $SU(N_c)$ gauge theories and good agreement was found with data, see Ref. [27]. A constant 't Hooft coupling is obtained at different N_c by choosing the reference values $\mathcal{E}_0$ and $\mathcal{W}_0$ according to,

$$\mathcal{E}_0 = c_e C_2(F), \qquad \mathcal{W}_0 = c_w C_2(F) . \tag{9}$$

where c_e and c_w are constants. While these tests are based on perturbative arguments, we adopt the scaling in Eq. (9) throughout, to compare different theories.

3. Numerical results

Each configuration in an ensemble was used as an initial condition for the numerical integration of the WF equations, Eq. (3). The quantity $\mathcal{E}(t) = t^2 E(t)$ was calculated on the entire interval of t, using both the the plaquette (pl.) and the clover (cl.) expression for $V_{\mu\nu}$. The discretisation effects could be estimated and were only found to be non-negligible for a small neighbourhood of $t = 0$. The value of t_0 and w_0 were determined at each N_c from the reference values $\mathcal{E}_0$ and $\mathcal{W}_0$, obtained from Eq. (9), by setting $c_e = c_w = 0.225$. Alternative scales were obtained by the choice $c_e = c_w = 0.5$ and used for comparison. The behaviours of $\mathcal{E}(t)$ and $\mathcal{W}(t)$, normalized to $C_2(F)$, are displayed in Fig. 1. They are expressed as functions of t/t_0 and t/w_0^2, and are shown to overlap with each other over a wide range of values of t. The overlap is more evident for ensembles that have a similar value of the convenently defined coupling $\tilde{\lambda} = \lambda d_G/\langle P\rangle$, where $\langle P\rangle$ is the expectation value of the average action per plaquette; hence, the scaling law Eq. (8) seems to capture some scaling property that holds also non-perturbatively.

The α-rounded lattice topological charge was compouted at $t = t_0$, where t_0 is obtained from Eq. (9) with both the choices $c_e = c_w = 0.225$ and $c_e = c_w = 0.5$, for comparison purposes. The values of the lattice topological susceptibility obtained from Eq. (5) are displayed in Fig. 2, as a function of a^2/t_0 and a^2/w_0^2. The extrapolations to $a = 0$ are performed according to a best fit of

$$\chi_L(a) t_0^2 = \chi_L(a = 0) t_0^2 + c_1 \frac{a^2}{t_0} , \tag{10}$$

to the data, using c_1 and $\chi_L(a = 0)$ as fit parameters and are visible as dashed lines. A similar analysis was carried out in units of w_0. The best-fit values are reported in Table 1.

Both the topological susceptibility and the string tension are assumed to tend to a finite limit as $N_c \to \infty$. As explained in Ref. [16], the gauge-group dependence of χ is inherited from the free energy of the theory. In particular, each of the d_G gauge fields is expected to contribute equally to

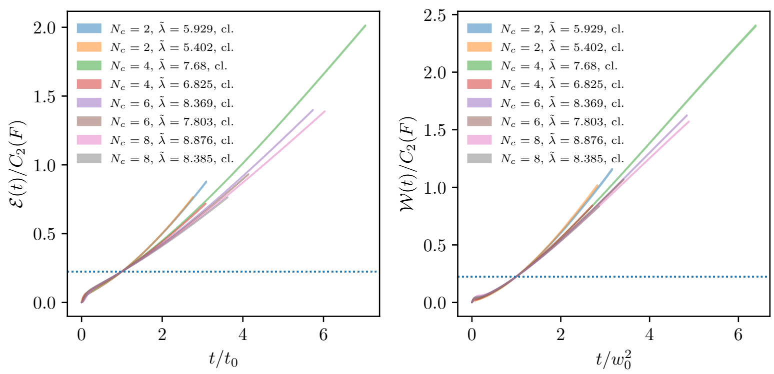

Figure 1: The quantities $\mathcal{E}(t)/C_2(F)$ (left panel) and $\mathcal{W}(t)/C_2(F)$ (right), computed with the clover-leaf discretisation of $E(t)$ on the available ensembles corresponding to the finest and coarsest available lattices, for each N_c, with $C_2(F) = (N_c + 1)/4$, displayed as a function of the rescaled flow times t/t_0 and t/w_0^2. The value of $\tilde{\lambda}$ is also reported in each case. The figure adopts the choice $c_e = c_w = 0.225$ (horizontal dashed line).

the latter. Moreover, assuming that the string tension σ scales with the quadratic Casimir operator of the fundamental representation, we expect the following ratio to capture universal features:

$$\eta_\chi \equiv \frac{\chi C_2(F)^2}{\sigma^2 d_G} = \frac{\chi}{\sigma^2} \cdot \begin{cases} \frac{N_c^2-1}{4N_c^2} & \text{for } SU(N_c) \\ \frac{N_c+1}{8N_c} & \text{for } Sp(N_c) \end{cases} . \tag{11}$$

In the limit $N_c \longrightarrow \infty$, we expect that $\eta_\chi \longrightarrow \eta_\chi(\infty)$, where $\eta_\chi(\infty)$ is finite and universal. The latter was estimated from the values of the topological susceptibility in $Sp(N_c)$ and $SU(N_c)$ at finite N_c, displayed in Fig. 3 (left) as a function of $1/N_c$. The values obtained are displayed in Fig. 3 (right) as a function of $1/d_G$. The result of a 2-parameter fit of $\eta_\chi(\infty) + b/d_G$ to the data, using $\eta_\chi(\infty)$ and b as fitting parameters, is displayed as a dotted line. The estimate obtained for $\eta_\chi(\infty)$ is

$$\eta_\chi(\infty) = (48.42 \pm 0.77 \pm 3.31) \times 10^{-4} , \tag{12}$$

where the first error is a statistical error produced by the fit, and the second is a systematic error related to the choice of fitting function. The latter is estimated by the magnitude of the variation in the value of $\eta_\chi(\infty)$ if a term proportional to $1/d_G^2$ is added to the fitting function, and a 3-parameters fit is performed.

4. Conclusion

This study focuses on the scaling properties of the WF and on the topological susceptibility in $Sp(N_c)$ gauge theories. Ensembles were collected at $N_c = 2$, 4, 6 and 8 over a range of values of the bare parameter β and lattice volume L, chosen to avoid finite size effects. An integer-valued lattice topological charge was obtained from each WF smoothed configuration by α-rounding. For each value of N_c, the topological susceptibility in units of t_0 and w_0 was then extrapolated to the continuum limit. The scales t_0 and w_0 were obtained at approximately constant 't Hooft coupling.

PoS(LATTICE2022)400

Table 1: The continuum limits of the topological susceptibility for $Sp(N_c)$ gauge theories with N_c = 2, 4, 6, 8, obtained from the best fit of Eq. (10). The results are reported in units of t_0 (top section of the table) and in units of w_0 (bottom), obtained from the reference values $c_e = 0.225 = c_w$ (left section of the table) or $c_e = 0.5 = c_w$ (right). The value of the reduced chi-square for each extrapolation is labelled as $\tilde{X}^2$ and is reported in the last column. The individual measurements and the extrapolations are displayed in Fig. 2, where each color corresponds to a different value of N_c.

N_c	c_e	$\chi_L t_0^2(a=0)$	$\tilde{X}^2$	c_e	$\chi_L t_0^2(a=0)$	$\tilde{X}^2$
2	0.225	0.000600(22)	1.47	0.5	0.002353(93)	1.29
4	0.225	0.000452(17)	2.18	0.5	0.002305(90)	1.89
6	0.225	0.000315(23)	1.08	0.5	0.00185(11)	1.02
8	0.225	0.000303(23)	2.32	0.5	0.00194(15)	1.39
N_c	c_w	$\chi_L w_0^4(a=0)$	$\tilde{X}^2$	c_w	$\chi_L w_0^4(a=0)$	$\tilde{X}^2$
2	0.225	0.000572(24)	1.16	0.5	0.001698(69)	1.50
4	0.225	0.000584(22)	1.63	0.5	0.001991(69)	2.14
6	0.225	0.000503(32)	1.39	0.5	0.00180(12)	1.13
8	0.225	0.000535(39)	1.41	0.5	0.00189(13)	1.80

The topological susceptibility for $Sp(N_c)$ and for $SU(N_c)$ gauge theories was then compared at $N_c = \infty$, and their values used to test a universality conjecture. The latter was found to be in agreement with the available data.

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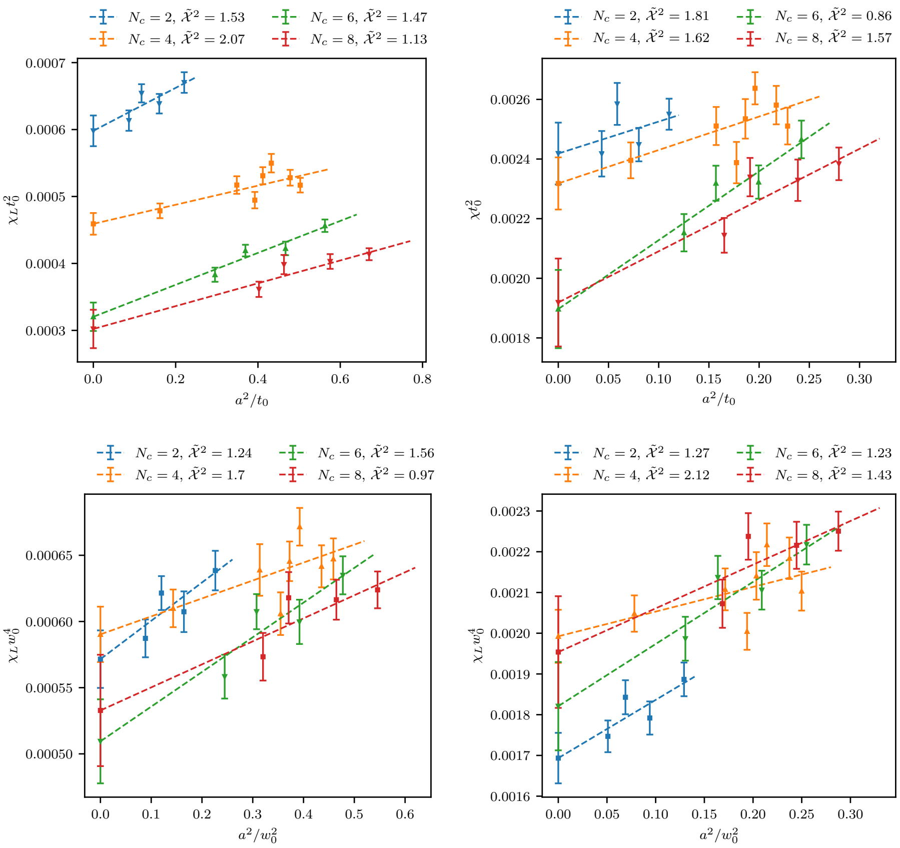

Figure 2: Topological susceptibility per unit volume $\chi_L t_0^2$ as a function of a^2/t_0 (top panels) and $\chi_L w_0^4$ as a function of a^2/w_0^2 (bottom), in $Sp(N_c)$ Yang-Mills theories with $N_c = 2, 4, 6, 8$. We adopt reference values $c_e = c_w = 0.225$ (left panels) and $c_e = c_w = 0.5$ (right). Our continuum extrapolations are represented as dashed lines. The results are reported in Table 1.

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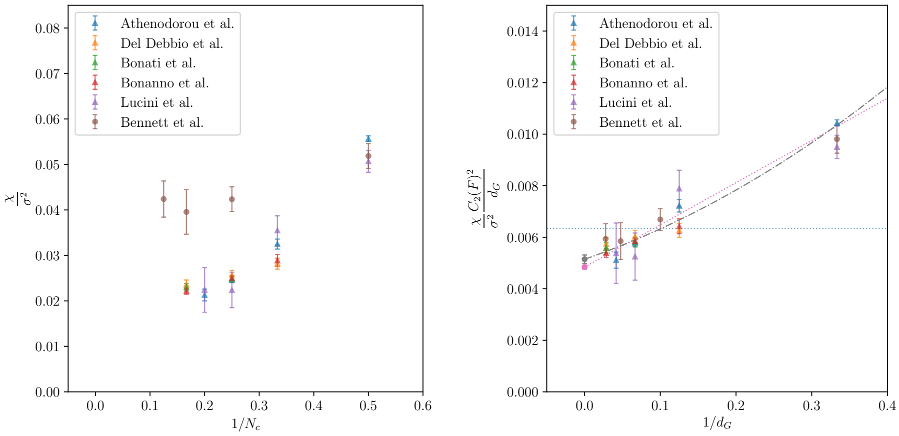

Figure 3: Topological susceptibility χ, in units of the string tension σ, in the continuum limit, for various groups $SU(N_c)$ and $Sp(N_c)$, as a function of $1/N_c$(left panel) and of $1/d_G$(right). The measurements reported here are labelled by the collaboration that published them. Figures are taken from Ref. [16]

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