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A lattice QCD study of the $B \to \pi\pi\ell\bar{\nu}$ transition

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V_{ub} is the smallest and least known of all CKM matrix elements; the community currently determines its magnitude primarily through the exclusive process $B \to \pi\ell\bar{\nu}$. Here we present our progress toward a lattice QCD determination of the V_{ub} matrix element from a novel transition – the $B \to \pi\pi\ell\bar{\nu}$ process, where the $\pi\pi$ system is in a P wave and scattering features the $\rho(770)$ resonance as an enhancement. We perform our calculation on $N_f = 2 + 1$ isotropic clover fermions on a lattice of $L \approx 3.6$ fm and a pion mass of ≈ 320 MeV; for the b-quark we use the anisotropic clover action. After a brief overview of the theoretical framework, we will discuss some preliminary results.

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1. Introduction

V_{ub} is the smallest and least known of all Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The community currently determines its magnitude primarily through the exclusive process $B \to \pi\ell\bar{\nu}$, although the purely leptonic $B \to \tau\nu$ and fully inclusive $B \to X_u\ell\bar{\nu}$ semileptonic decay also contribute [1]. There is, however, a puzzle regarding the determination of V_{ub} - the determinations of $|V_{ub}|$ from $B \to X_u\ell\bar{\nu}$ are in tension[1] with those using the exclusive $B \to \pi\ell\bar{\nu}$ decay rate. To better understand the tension, an additional exclusive channel to determine V_{ub} would be beneficial. Such a channel is $B \to \rho(\to \pi\pi)\ell\bar{\nu}$ where the $\rho(770)$ resonance is present, which in addition to opening a new determination of V_{ub}, also provides complementary constraints on right-handed $b \to u$ currents for beyond the standard model physics [2]. Experimental data for this channel are available from Babar, Belle, and Belle II [3–5]; however, the relevant hadronic matrix elements from theory are not yet known to sufficient precision.

Previous lattice calculations of the $B \to \rho(\to \pi\pi)\ell\bar{\nu}$ process were done in the quenched approximation and assumed the ρ resonance to be stable under the strong interaction [6, 7]. Here we present our preliminary results for the $B \to \rho(\to \pi\pi)\ell\bar{\nu}$ transition from lattice QCD; we perform our calculation at a pion mass where the ρ appears as a resonance. To take care of the finite-volume normalization, we use Lellouch-Lüscher factors in our analysis, enabling us to determine the transition amplitude in a range of $\pi\pi$ invariant masses, $E^\star$, and momentum transfers q^2. This work is the extension of our previous $\pi\gamma \to \pi\pi$ study [8].

2. Gauge Ensemble

We present preliminary results on a single gauge field ensemble with $N_f = 2+1$ clover Wilson fermions whose quark masses correspond to $m_\pi \approx 320$ MeV. The lattice spacing is approximately $a = 0.114$ fm, and the lattice volume is $N_L^3 \times N_t = 32^3 \times 96$. The pion dispersion relation is shown in Fig. 1 of Ref. [9]. For the b-quark, we use an anisotropic action [10, 11], in which we tune the quark mass and anisotropy parameters to match the B_s meson rest and kinetic mass. This gives the B-meson dispersion shown in Fig. 1.

3. The 3-point correlation functions

The 3-point correlation functions definition is

$$C_3^i = \langle\Omega|O_i(\Delta t; \vec{P}, \Lambda, r)J(t_J; \vec{q}, \mu)B^\dagger(0; \vec{p}_B)|\Omega\rangle, \tag{1}$$

where the source is at timeslice 0, the current $J(t_J; \vec{q}, \mu)$ with momentum $\vec{q}$ at timeslice t_J and the sink at timeslice Δt. The source part of the 3-point correlation functions consists of a single B-meson interpolator, $B(0; \vec{p}_B) = \sum_{\vec{x}} e^{i\vec{p}_B\cdot\vec{x}}\bar{q}\gamma_5 b(\vec{x})$ with momentum $\vec{p}_B$. For the sink part, we use three or four interpolators $O_i(\Delta t; \vec{P}, \Lambda, r)$ built from either one- or two-hadron operators in the irreducible representation Λ and row r. Here $\vec{P}$ is the total momentum of the two-hadron system,

[1]Recent more conservative error estimates for the inclusive determination have reduced the tension [1].

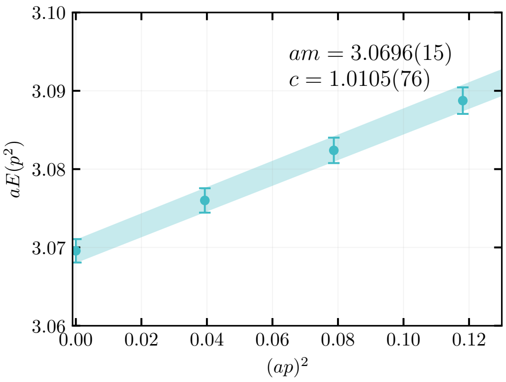

Figure 1: The B-meson dispersion relation.

projected to the irreducible representation Λ of the Little Group defined by $\vec{P}$. The current insertion, $J(t_J;\vec{q},\mu)$, is $O(a)$ improved through:

$$J(t_J;\vec{q},\mu) = \sqrt{Z^u Z^b}(\bar{u}\Gamma b + d^{(b)}\bar{u}\Gamma\gamma^i\nabla_i b), \tag{2}$$

where Γ is either γ^μ or $\gamma^\mu\gamma_5$, $d^{(b)}$ is the improvement coefficient, and Z^f are renormalization coefficients of the flavor-conserving temporal vector current for quark flavor f. To determine the

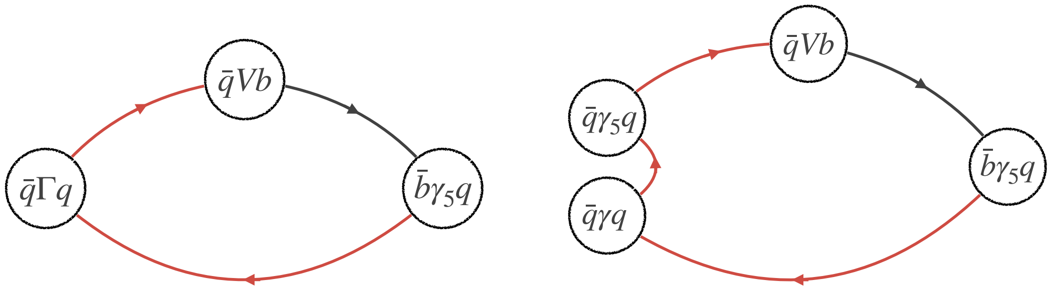

Figure 2: The Wick contractions relevant to the $B \to \pi\pi\ell\bar{\nu}$ transition. Left is the Wick contraction for the single-hadron sink operator, while the right is the Wick contraction for the two-hadron sink operator. We project both Wick contractions to $I = 1$, $I_z = 0$ in the sink.

3-point correlation functions, we evaluate the Wick contractions shown in Fig. 2; Fig. 3 shows an example of the 3-point correlation functions in the B_3 representation of $\vec{P} = \frac{2\pi}{L}[0,1,1]$. The first two panels employ one-hadron, $\bar{q}\gamma_i q$ and $\bar{q}\gamma_t\gamma_i q$ sink operators, while the last two panels show 3-point correlation functions with two-hadron operators at the sink. To construct 3-point functions with dominant overlap to a single finite-volume state, we construct a linear combination C_3^n of the four 3-point correlation functions C_3^i with coefficients u_i^n taken as the n-th state generalized eigenvector of the GEVP analysis [9], $C_3^n = u_i^n C_3^i$. The projection leads to an optimized correlation function that dominantly overlaps with a single finite-volume state:

$$C_3^n = \langle n,\Lambda,\vec{P}|J|B,\vec{p}_B\rangle\langle B,\vec{p}_B|O_B|\Omega\rangle \frac{e^{-E_n(\Delta t-t_J)}e^{-E_0^B(t_J-0)}}{2E_n 2E_0^B} + \text{excited state cont.}, \tag{3}$$

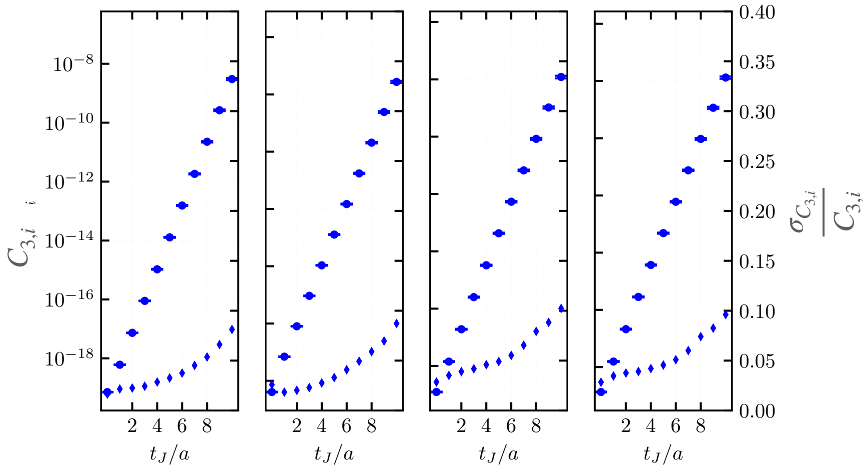

Figure 3: Examples of 3-point correlation functions are shown with filled circles; the left-most panel shows the $O_{i=1} = \bar{q}\gamma_i q$ operator, the second-to-left-most shows the $O_{i=2} = \bar{q}\gamma_t\gamma_i q$ operator, the second-to-right-most shows $O_{i=3} = \pi(\vec{p}_1)\pi(\vec{p}_2)$ with $|\vec{p}_1| = 0$, $|\vec{p}_2| = \frac{2\pi}{L}\sqrt{2}$, and the right-most shows $O_{i=4} = \pi(\vec{p}_1)\pi(\vec{p}_2)$ with $|\vec{p}_1| = \frac{2\pi}{L}$, $|\vec{p}_2| = \frac{2\pi}{L}\sqrt{3}$. The relative uncertainties, $\frac{\sigma_{C_{3,i}}}{C_{3,i}}$, are shown as filled diamonds.

where $\langle n, \Lambda, \vec{P}|J|B, \vec{p}_B\rangle$ is the sought-for matrix element, and the "excited state cont." are similar products with matrix elements involving excited states of the source and sink irreducible representations. These differ from the desired matrix element in their size and temporal dependence. If we multiply the leading-order time dependence out of C_3^n, we can fit the matrix elements with models where we can consider the source, sink, or both sides of excited state contaminations. We show an example of such a matrix element in Fig. 4, where the dots with uncertainties represent the lattice data, the light-shaded region the fit of the full model, including excited state contamination, and the dark-shaded region the matrix element value. To determine the matrix elements, we vary the fit models and fit windows, which yields a total of 64 matrix elements spread across different q^2 and $\sqrt{s}$.

4. Fitting the Matrix Elements

The significant interactions in the finite-volume state (i.e., those that lead to the ρ resonance) affect the normalization of the matrix elements, an effect taken into account by the Lellouch-Lüscher factor [12, 13]. In these proceedings, we follow the approach of Briceño, Hansen, and Walker-Loud [14] and use the particular implementation discussed in Ref. [15] to map the infinite-volume amplitudes onto the finite-volume matrix elements. For full generality, we thus do not map each finite-volume matrix element to its infinite-volume counterpart (even though this is possible in the $\pi\pi$ channel) but rather fit the finite-volume matrix elements directly. The general form of the transition amplitude $\mathcal{H}^{\mu}_{1,m_\ell}$ can be written as

$$\mathcal{H}^{\mu}_{1,m_\ell}(q^2, E^{\star 2}) = \mathcal{A}^{\mu}_{1,m_\ell}(q^2, E^{\star 2})\frac{T(E^{\star 2})}{k}, \tag{4}$$

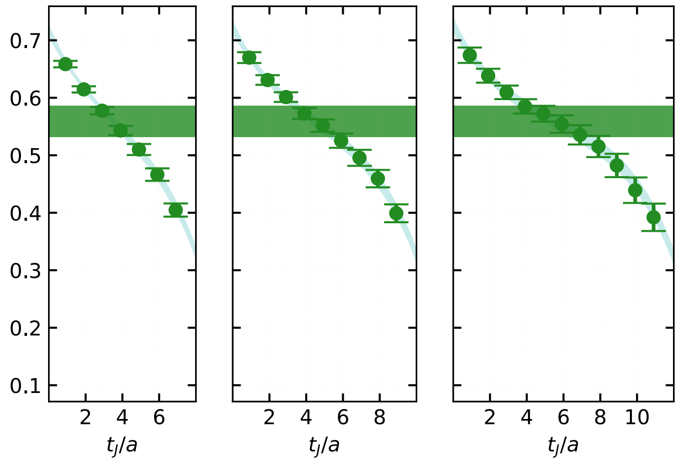

Figure 4: An example of the state-projected 3-point correlation function without the Lorentz symmetry factor. We factor out the leading-order temporal dependence to demonstrate the matrix element and excited state contributions. Shown is the ground state of the irreducible representation B_3 of $\vec{P} = \frac{2\pi}{L}[0, 1, 1]$ with $\vec{p}_B = \frac{2\pi}{L}[0, 1, 1]$. The discrete data points are the lattice-determined matrix element, the light-shaded region is the full model, which includes source and sink excited-state contamination, and the dark-shaded region is the determined matrix element.

where, for the case of the vector current, $\mathcal{A}^{\mu}_{1,m_\ell}$ will have the following Lorentz decomposition

$$\mathcal{A}^{\mu}_{1,m_\ell} = \frac{iV}{m_B + 2m_\pi} \varepsilon^{\mu\nu\alpha\beta} \epsilon^{\nu *}(P, m_\ell) P_\alpha (p_B)_\beta. \tag{5}$$

Here, V is the transition form factor, P is the four-momentum of the two-hadron state, p_B is the four-momentum of the initial B-meson, and ϵ is the polarization vector of the two-hadron state with $J = 1$ and third component m_ℓ. The invariant $P \cdot P = E^{\star 2}$ denotes the $\pi\pi$ invariant mass, and the invariant q^2, where $q = P - p_B$, is the momentum transfer.

To determine the infinite-volume transition amplitude, we first pick a parameterization for V, set its parameters to initial values, and then obtain the finite-volume matrix elements through

$$\langle n, \Lambda, \vec{P} | J | B, \vec{p}_B \rangle = \frac{1}{\sqrt{2E^B_0}\sqrt{2E_n}} \sqrt{\frac{2E^\star_n}{-\mu^{\star\prime}_0}}\, V, \tag{6}$$

where $\mu^\star_0$ is the non-zero eigenvalue of the residue matrix R,

$$R = 2E_n \lim_{E \to E_n} \frac{E - E_n}{F^{-1} + T}, \tag{7}$$

and E_n is the finite-volume energy corresponding to the state $\langle n, \Lambda, \vec{P}|$. In Eq. (6), $\mu^{\star\prime}_0$ is the derivative of $\mu^\star_0$ with respect to $E^\star$.

From the model matrix element and the lattice data, we construct a χ^2 function and then minimize the χ^2 function for the model parameters. In these proceedings, we consider a single

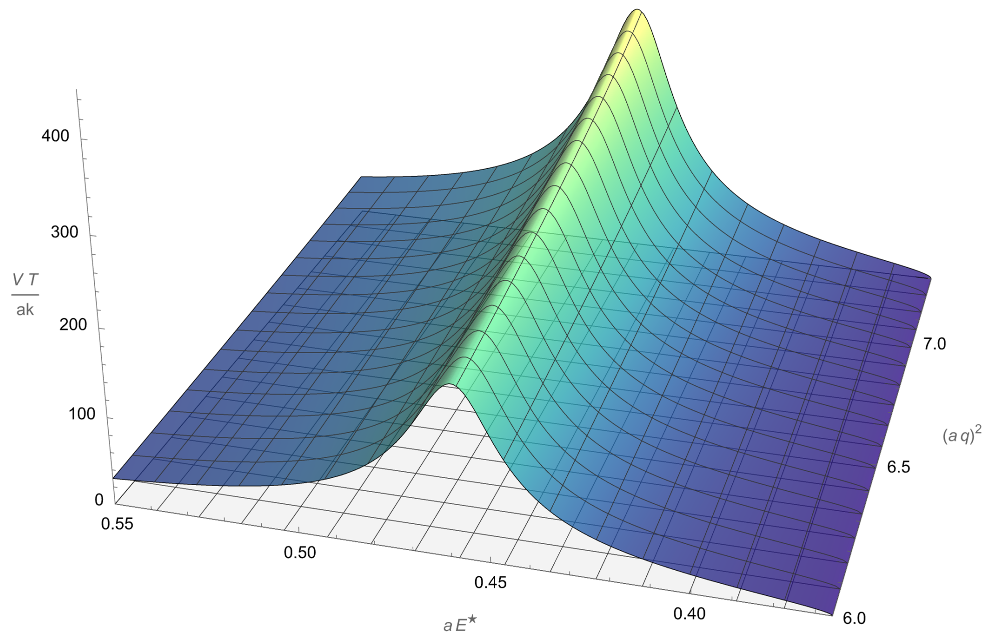

Figure 5: The transition amplitude $\mathcal{A}(q^2, E^\star) = \frac{VT}{ak}$ of the vector current of the $B \to \pi\pi\ell\bar{\nu}$ transition in the isospin basis. Shown is the region of q^2 and $E^\star$, where lattice data is available.

model of the transition amplitude, where the $\pi\pi$ scattering amplitude T is the Breit-Wigner I amplitude used in Ref. [9]; it fully describes the pole structure in the $E^\star$ variable. As such, V is a smooth function of $E^\star$, but still has singularities in the momentum transfer, q^2, variable, above the semileptonic region. We parametrize V using a generalization of the z-expansion [16, 17]

$$V = \frac{1}{1 - \frac{q^2}{m_{B^\star}}} \sum_{n=0,m=0}^{n_{\max},m_{\max}} a_{n,m} z^n \mathcal{S}^m, \tag{8}$$

where $\mathcal{S} = \frac{E^{\star 2} - (2m_\pi)^2}{(2m_\pi)^2}$ and

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}. \tag{9}$$

Here, t_+ corresponds to the $B\pi$ threshold, and we use $t_0 = 6.0$ in lattice units. The B^*-meson pole is included explicitly as a prefactor in Eq. (8). Our preliminary fit does not include $\mathcal{S}$ dependence, we used $n_{\max} = 1$, $m_{\max} = 0$. We show the central value of the resulting transition amplitude in Fig. 5; as our matrix elements are in the isospin basis of the final $\pi\pi$ state, so is the transition amplitude. The fit presented used 51 points at various $E^\star$ and q^2 and yields a $\chi^2/\mathrm{dof} = 1.4$. The corresponding parameters are $a_0 = 0.2405(45)$ and $a_1 = -0.09(11)$.

5. Summary

We have presented our preliminary results for the vector form factor of the $B \to \pi\pi\ell\bar{\nu}$ transition; we plan to determine the axial-vector form factors as well. Here we presented the state-projected 3-point correlation functions and their fits used to determine the matrix elements. Taking the Lellouch-Lüscher factors into account, we normalize the finite-volume matrix elements that enter the global analysis of the transition amplitude. In this manner, we have determined the $B \to \pi\pi\ell\bar{\nu}$ transition amplitude in the region of large q^2 and $\pi\pi$ invariant mass near the $\rho(770)$ resonance. We have shown an initial fit to a subset of all our data and demonstrated the approach's viability.

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