PROCEEDINGS OF SCIENCE

CPT and unitarity constraints for higher-order CP asymmetries at finite temperature

Tomáš Blažek,[a] **Peter Maták**[a,*] **and Viktor Zaujec**[a]

[a] *Comenius University in Bratislava,*
Mlynská dolina F1, 84248 Bratislava, Slovak Republic
E-mail: tomas.blazek@fmph.uniba.sk, peter.matak@fmph.uniba.sk, viktor.zaujec@fmph.uniba.sk

We use an unconventional diagrammatic approach to formulate CPT and unitarity constraints for higher-order CP asymmetries entering the source term in the Boltzmann equation. Usually, the reaction rate asymmetries in these constraints are computed within the classical kinetic theory, using zero-temperature quantum field theory to describe particles' interactions. We approximate the rates, otherwise obtained within the closed-time-path formalism, in terms of diagrams drawn on a cylindrical surface and their holomorphic cuts. The resulting equilibrium asymmetry constraints incorporate thermal-mass effects and allow tracking the cancellations of reaction rate asymmetries computed with quantum statistics. We use the top Yukawa corrections to the asymmetries in the seesaw type-I leptogenesis as an example. The contribution is primarily based on Ref. [1].

8th Symposium on Prospects in the Physics of Discrete Symmetries (DISCRETE 2022)
7-11 November, 2022
Baden-Baden, Germany

*Speaker

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1. Introduction

The CPT and unitarity constraints form a set of relations between reaction rate CP asymmetries entering the Boltzmann equation describing particle asymmetry evolution in the early universe [2, 3]. In leading-order zero-temperature calculations, these constraints come with a natural diagrammatic representation [4]. However, higher-order corrections [5] or, as discussed in Refs. [6, 7], the inclusion of thermal effects may cause substantial difficulties. In this proceedings, we briefly discuss the diagrammatic representation of the asymmetry cancellations as introduced in Ref. [1] when thermal corrections are considered.

2. CP violation at zero and finite temperature

The violation of CP requires the presence of irreducible complex phases in couplings. Furthermore, on-shell intermediate states must be present in the amplitude to produce its imaginary part represented by Cutkosky cuts over the respective Feynman diagrams. At higher orders, more on-shell cuts can be made simultaneously, and thus, for $\mathrm{i}T_{ij} = S_{ij} - \delta_{ij}$ the complex conjugation in the right-hand side of the unitarity condition [3]

$$\Delta|T_{ij}|^2 = |T_{ij}|^2 - |T_{ji}|^2 = -2\Im\left[\left(TT^\dagger\right)_{ij} T^*_{ji}\right] + \left|\left(TT^\dagger\right)_{ij}\right|^2 \tag{1}$$

cannot be omitted, spoiling the diagrammatic approach introduced in Ref. [4]. This obstacle can be overcome using the holomorphic cutting rules [8–10] based on an expansion of the S-matrix unitarity condition into a geometric series

$$(1+\mathrm{i}T)^\dagger = (1+\mathrm{i}T)^{-1} \quad \rightarrow \quad \mathrm{i}T^\dagger = \mathrm{i}T - (\mathrm{i}T)^2 + (\mathrm{i}T)^3 + \dots \tag{2}$$

Then, for the CP asymmetry, we obtain [5]

$$\begin{aligned}\Delta|T_{fi}|^2 &= \sum_n (\mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fn}\mathrm{i}T_{ni}) \\ &\quad - \sum_{n,m}(\mathrm{i}T_{in}\mathrm{i}T_{nm}\mathrm{i}T_{mf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fm}\mathrm{i}T_{mn}\mathrm{i}T_{ni}) + \dots\end{aligned} \tag{3}$$

where the summation over the final state $|f\rangle$ leads to a vanishing sum of the asymmetries for a given initial state – the CPT and unitarity constraints known in the literature [2, 3]. The first term in each row in Eq. (3) can be viewed as a forward-scattering diagram cut into several pieces. The first piece corresponds to $\mathrm{i}T_{fi}$ and defines the process for which the asymmetry is calculated.

In thermal equilibrium, analogous constraints hold for asymmetries of reaction rates γ_{fi} counting the number of $i \rightarrow f$ processes occurring per unit of volume per unit of time, which is essential to the fulfilment of Sakharov's conditions [11]. We distinguish two ways how these rates can be calculated. As a first approximation, we can use Maxwell-Boltzmann particle densities to describe the multiparticle state and zero-temperature Feynman rules for describing the interactions. In this case, we denote the rates by a small circle

$$\mathring{\gamma}_{fi} = \frac{1}{V_4}\int\prod_{\forall i}[d\mathbf{p}_i]\mathring{f}_i(p_i)\int\prod_{\forall f}[d\mathbf{p}_f]\left(-\mathrm{i}T_{if}\mathrm{i}T_{fi} + \sum_n \mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} + \dots\right) \tag{4}$$

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where $[d\mathbf{p}_i] = d^3\mathbf{p}_i/((2\pi)^3 2E_i)$. Using $\mathring{f}_i(p_i) = \exp\{-E_i/T\}$, the detailed balance condition ensures

$$\sum_f \Delta\mathring{\gamma}_{f\,i} = 0 \tag{5}$$

inheriting the pairwise cancellations of Eq. (3).

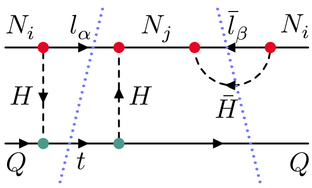
Alternatively, we may wish to include the Bose-Einstein or Fermi-Dirac quantum phase densities and final-state statistical factors leading to uncircled rates $\gamma_{f\,i}$. When computing the rate asymmetries, it is impossible without modifying the cutting rules, as statistical factors also alter the on-shell parts of propagators. The correct form of the asymmetry source term can be obtained by considering the evolution of the multiparticle density matrix, as it is in the closed-time-path formalism [12, 13]. In the Markovian approximation of such evolution, the resulting rates can be expressed as an infinite series of circled rates obtained from cuttings of forward scattering diagrams drawn on a cylindrical surface [7].

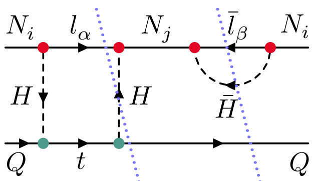
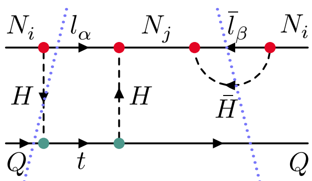
3. Unitarity and Higgs thermal mass in right-handed neutrino decay asymmetry

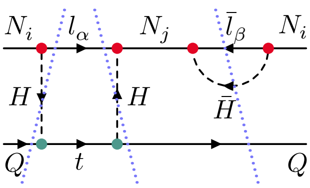
To illustrate the principle within a simple example, we consider a single forward-scattering diagram contributing to the top Yukawa corrections to the asymmetries in leptogenesis. Let us begin with the following lagrangian density

$$\mathcal{L} \supset -\frac{1}{2}M_i\bar{N}_iN_i - (Y_{\alpha i}\bar{N}_iP_Ll_\alpha H + Y_t\bar{t}P_LQH + \text{H.c.}) \tag{6}$$

where N_i corresponds to the heavy right-handed neutrino field, while l, H, Q, and t stand for standard-model leptons, Higgs doublet, and left- and right-handed top quarks, respectively. At the $\mathcal{O}(Y^4Y_t^2)$ order, we consider a specific forward-scattering diagram cut according to Eq. (3), contributing to the $NQ \to lt$ and $N_iQ \to lHQ$ circled rate asymmetries as

$\Delta\mathring{\gamma}_{NQ\to lt} \quad \leftarrow$  $-$ m.t. (7)

$\Delta\mathring{\gamma}_{NQ\to lHQ} \quad \leftarrow$  +  (8)

$-$  $-$ m.t.

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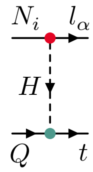
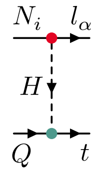
where the phase-space integration is implicitly performed as in Eq. (5). The abbreviation m.t. stands for the *mirrored terms* with the intermediate states arranged in a reversed order [5]. While the calculation of the first asymmetry in Eq. (7) is rather straightforward, the second is slightly more subtle. When putting one of the t-channel Higgs propagators on its mass shell by cutting it, the second will become on-shell, too, leading to an ill-defined expression [14]. To resolve this issue, we must carefully treat the intermediate states in our calculations. Using the distributional identity

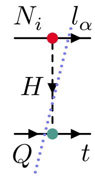
$$\frac{1}{k^2+\mathrm{i}\epsilon} = \mathrm{P.V.}\frac{1}{k^2} - \mathrm{i}\pi\delta(k^2) \quad \rightarrow$$



$= \mathrm{P.V.}$



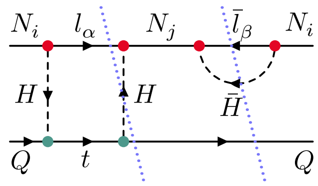
$+\frac{1}{2}$



(9)

we can rewrite Eq. (8) as

$$\Delta\mathring{\gamma}_{NQ\to lHQ} \quad \leftarrow \quad 2\mathrm{P.V.}$$



$-$ m.t. (10)

and apply [14, 15]

$$2\delta_+(k^2)\mathrm{P.V.}\frac{1}{k^2} = -\frac{1}{(k^0+|\mathbf{k}|)^2}\frac{\partial\delta(k^0-|\mathbf{k}|)}{\partial k^0} \tag{11}$$

with k labelling the four-momentum of the t-channel Higgses. The derivative with respect to its zeroth component can be turned into the derivative with respect to the Higgs mass through [16]

$$\frac{\partial}{\partial k^0}\bigg|_{k^0=|\mathbf{k}|}\frac{\mathcal{F}(k^0,\mathbf{k})}{(k^0+|\mathbf{k}|)^2} = \frac{\partial}{\partial m_H^2}\bigg|_{m_H^2=0}\frac{\mathcal{F}(E_\mathbf{k},\mathbf{k})}{2E_\mathbf{k}} \quad \text{for} \quad E_\mathbf{k} = \sqrt{m_H^2+\mathbf{k}^2} \tag{12}$$

such that

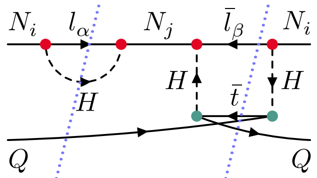
$$\Delta\mathring{\gamma}_{N_iQ\to lHQ} = \Delta\mathring{\gamma}_{N_i(Q)\to lH(Q)} + \frac{1}{4}\mathring{m}^2_{H,Y_t}(T)\frac{\partial}{\partial m_H^2}\bigg|_{m_H^2=0}\Delta\mathring{\gamma}_{N_i\to lH} \tag{13}$$

where the Higgs thermal mass

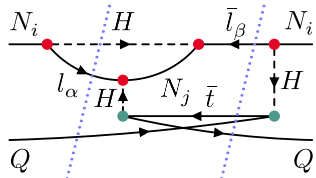
$$\mathring{m}^2_{H,Y_t}(T) = 12Y_t^2\int[d\mathbf{p}_Q]\mathring{f}_Q \tag{14}$$

is obtained from classical or circled phase-space density. In Eq. (13) we denote

$$\Delta\mathring{\gamma}_{N_i(Q)\to lH(Q)} =$$



$+$



$-$ m.t. (15)

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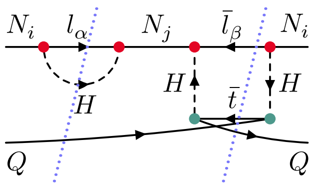
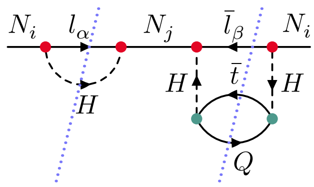
approximating the Pauli blocking factor of Q in the asymmetry-generating loop contributing to the $N_i \to lH$ asymmetry. This can be observed in



$$= -\mathring{f}_Q \times$$



(16)

before the initial-state momentum integration is carried out. Finally, considering the full list of forward-scattering diagrams of the same perturbative order allows us to write the CPT and unitarity constraints [1, 5]

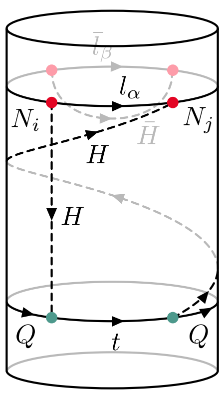
$$\Delta\mathring{\gamma}_{N_i Q\to lt} + \Delta\mathring{\gamma}_{N_i(Q)\to lH(Q)} + \Delta\mathring{\gamma}_{N_i(Q)\to \bar{l}\bar{H}(Q)} + \Delta\mathring{\gamma}_{N_i Q\to \bar{l}QQ\bar{t}} = 0 \tag{17}$$

and a separately vanishing mass-derivative of the right-handed neutrino decay asymmetries

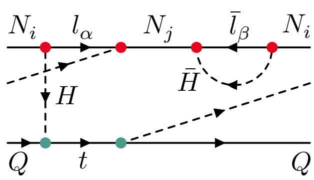
$$\frac{1}{4}\mathring{m}^2_{H,Y_t}(T)\frac{\partial}{\partial m_H^2}\bigg|_{m_H^2=0}\left(\Delta\mathring{\gamma}_{N_i\to lH} + \Delta\mathring{\gamma}_{N_i\to \bar{l}\bar{H}}\right) = 0. \tag{18}$$

It is remarkable that even though starting with classical kinetic theory and zero-temperature Feynman rules, an approximation of thermal mass effect in the decay asymmetry kinematics inconspicuously entered the computation. We emphasize it is a natural consequence of unitarity and the so-called anomalous thresholds [10, 17–19]. In our calculation, they entered Eq. (8) and were represented by cuts dividing the amplitude into connected and disconnected diagrams.

We may wish to generalize the relations in Eqs. (17) and (18) to include quantum densities as well as uncircled thermal mass. For that purpose, we consider a diagram similar to that in Eq. (7) drawn on a cylindrical surface [1]



$$\longrightarrow$$



(19)

where the winding number of the Higgs internal line has been increased to one. Then, the resulting forward-scattering diagram is cut in all possible ways according to Eq. (3) and a set of relations analogous to Eqs. (17) and (18) is obtained. Summation of resulting circled asymmetries obtained from cylindrical diagrams with all possible winding numbers of all lines leads to a simple replacement

$$\mathring{f}_{N_i}\mathring{f}_Q \quad\to\quad f_{N_i}f_Q(1+f_H)(1-f_l)(1+f_{\bar{H}})(1-f_{\bar{l}}) \tag{20}$$

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to be made in Eq. (7), leading to uncircled asymmetry of the $NQ \to lt$ reaction [1]. Finally, applying the same procedure to all diagrams contributing to the asymmetry at the given perturbative order, we may erase the circles in Eqs. (17) and (18), and thus, the CPT and unitarity constraints for the CP asymmetries including thermal corrections have been obtained.

4. Conclusions

In this contribution, a new diagrammatic method [7] has been applied to the example of the $O(Y^4 Y_t^2)$ asymmetries of the seesaw type-I leptogenesis with right-handed neutrino and left-handed third-generation quark in the initial state. The systematic procedure leading to the CPT and unitarity constraints for such asymmetries, in which thermal effects are included, has been described briefly. The details, including explicit expressions for the CP asymmetries, can be found in Ref. [1].

Acknowledgements

The authors have been supported by the Slovak Ministry of Education Contract No. 0466/2022 and by the Slovak Grant Agency VEGA, project No. 1/0719/23. Viktor Zaujec received funding from Comenius University in Bratislava, project No. UK/317/2022: *Unitarita, CP asymetrie a diagramatická reprezentácia tepelných efektov v leptogenéze*. We thank our colleague Fedor Šimkovic for his long-term support.

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