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$B^0 - \bar{B}^0$ entanglement for the direct CP violation $\phi_3 = \gamma$ phase

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$B^0 - \bar{B}^0$ entanglement offers a conceptual alternative to the single charged B-decay asymmetry for the measurement of the direct CP-violating ϕ_3/γ phase. With $f = J/\psi K_L, J/\psi K_S$ and $g = (\pi\pi)^0, (\rho_L\rho_L)^0$ the 16 time-ordered double-decay rate intensities to (f, g) depend on the relative phase between the f- and g-decay amplitudes given by γ at tree level. Several constraining consistencies appear. An intrinsic accuracy of the method at the level of $\pm 1°$ could be achievable at Belle-II with an improved determination of the penguin amplitude to g channels from existing facilities.

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1. Introduction

The quark charged current couplings are proportional to the Cabibbo-Kobayashi-Maskawa (CKM) matrix V

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad (1)$$

by rephasing the up and down quark phases one can eliminate five phases out of nine in such a way that we can always write

$$V = \begin{pmatrix} |V_{ud}| & |V_{us}|e^{i\chi'} & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} \qquad (2)$$

because the rephasing invariant quartet $Im\left(V_{ij}V_{il}^{*}V_{kj}V_{kl}^{*}\right)$ for $i \neq k, j \neq l$ is universal (Chau, Keung, Jarlskog…) up to a sign, it is easy to show relatively, that

$$\beta, \gamma \sim O(1) \;\; ; \;\; \beta_s \sim \lambda^2 \;\; ; \;\; \chi' \sim \lambda^4 \qquad (3)$$

where $\lambda \sim 0.22$ is the Wolfenstein CKM expansion parameter. Therefore, γ

$$\gamma = \phi_3 = \; arg\,(-V_{ud}V_{cb}V_{ub}^{*}V_{cd}^{*}\,) \qquad (4)$$

is the unique non-small phase that can appear at tree level, obviously in B decays.

In fact, a minimal ingredient to measure γ is in the interference of the decays $B^- \to D^0 + K^-$ and $B^- \to \bar{D}^0 + K^-$ but to have interference we need a common final state f for $D^0 \to f$ and $\bar{D}^0 \to f$. Technically we have to compare $B^- \to D_{\nrightarrow f}^{\perp} + K^-$ with $B^+ \to D_{\nrightarrow f}^{\perp} + K^+$, where $D_{\nrightarrow f}^{\perp}$ is the state filtered by the final state f that is the orthogonal to the state that does not decay to f: $D_{\nrightarrow f}$. These states can be shown to be [13]

$$\begin{aligned} |D_{\nrightarrow f}\rangle &= \bar{c}_f|D^0\rangle - c_f|\bar{D}^0\rangle \\ |D_{\nrightarrow f}^{\perp}\rangle &= c_f^{*}|D^0\rangle + \bar{c}_f^{*}|\bar{D}^0\rangle \end{aligned} \qquad (5)$$

with $\left|c_f\right|^2 + \left|\bar{c}_f\right|^2 = 1$ and, for $f = K^+K^-$, the corresponding D decay amplitudes fix the ratio

$$\frac{c_f}{\bar{c}_f} = \frac{A(D^0 \to f)}{A(\bar{D}^0 \to f)} = \frac{V_{us}V_{cs}^{*}}{V_{us}^{*}V_{cs}} \qquad (6)$$

In fact, the corresponding decay amplitudes are

$$\begin{aligned} A^- &= A(B^- \to D^0 + K^-)A(D^0 \to f) + A(B^- \to \bar{D}^0 + K^-)A(\bar{D}^0 \to f) \\ &= aV_{cb}V_{us}^{*}V_{us}V_{cs}^{*} + bV_{ub}V_{cs}^{*}V_{cs}V_{us}^{*} = a|V_{us}|^2V_{cb}V_{cs}^{*} + b|V_{cs}|^2V_{ub}V_{us}^{*} \end{aligned} \qquad (7)$$

in such a way that CP the violating difference of decay rates verify

$$\begin{aligned} |A^-|^2 - |A^+|^2 &\propto Im(ab^{*})Im(V_{us}V_{cb}V_{ub}^{*}V_{cs}^{*}) \\ Im(V_{us}V_{cb}V_{ub}^{*}V_{cs}^{*}) &\propto \sin(\chi' + \gamma) \sim sin(\gamma) \end{aligned} \qquad (8)$$

where the piece $Im(ab^{*})$ encodes the need for relative strong final state interactions. Behind this comparison are the different ways to measured γ [1] proposed by Gronau, London, Wyler (GLW), Atwood, Dunietz, Soni (ADS), Giri, Grossman, Soffer, Zupan (GGSZ) and many more [2-4].

2. Using entanglement to measure gamma

The use of the EPR [5] correlation to study CP violation was proposed by Wolfenstein, Gavela et al, Falk and Petrov and Alvarez and Bernabeu [6-9] among others for several decay channels in the B factories. The method for γ consists in the observation of the coherent double decay of $\Upsilon(4s)$ (1^{--}) to the CP eigenstates (f,g), with $f = J/\psi K_S(0^{-+}), J/\psi K_L(0^{--})$ (in short S or L) and $g = h^+h^-, h^0h^0(0^{++})$ and $h = \pi, \rho_L$. In such a way that

$$\begin{aligned} \Upsilon(4s) &\to (J/\psi K_S)_B (hh)_B \ ; \quad \text{is CP allowed} \\ \Upsilon(4s) &\to (J/\psi K_L)_B (hh)_B \ ; \quad \text{is CP forbidden} \end{aligned} \tag{9}$$

The necessary interference between amplitudes [10] containing the $V_{cd}V_{cb}^*$ and $V_{ud}V_{ub}^*$ sides of the unitarity triangle is automatic from the two terms of the entangled $B^0 - \bar{B}^0$ system (from $\Upsilon(4s)$):

$$\begin{aligned} |\Psi_0\rangle &= \frac{1}{\sqrt{2}}\left(|B_d^0\rangle|\bar{B}_d^0\rangle - |\bar{B}_d^0\rangle|B_d^0\rangle\right) = \\ &= \frac{1}{2\sqrt{2}pq}\left(|B_H\rangle|B_L\rangle - |B_L\rangle|B_H\rangle\right) \end{aligned} \tag{10}$$

with $B_H = pB^0 + q\bar{B}^0$, $B_L = pB^0 - q\bar{B}^0$ the eigenstate of the system with eigenvalues $\mu_{H,L}$ and $A_{H,L}^f = \langle f|\mathrm{T}|B_{H,L}\rangle$ where $A_f = \langle f|\mathrm{T}|B^0\rangle$; $\bar{A}_f = \langle f|\mathrm{T}|\bar{B}^0\rangle$, and it can be seen in the double decay amplitudes

$$\langle f, t_0; g, t_0 + t|\mathrm{T}|\Psi_0\rangle = \frac{e^{-i(\mu_H+\mu_L)t_0}}{2\sqrt{2}pq}\left(e^{-i\mu_H t}A_L^f A_H^g - e^{-i\mu_L t}A_H^f A_L^g\right) \tag{11}$$

So, the double decay rate for $\Psi_0 \to (f, t_0; g, t_0 + t)$ to the state f at t_0 and to the state g at $t_0 + t$ integrated for t_0 is given by [11]

$$\begin{aligned} I(f,g;t) &= \frac{e^{-\Gamma|t|}}{16\Gamma|pq|^2}\left|e^{-i\Delta Mt/2}A_L^f A_H^g - e^{+i\Delta Mt/2}A_H^f A_L^g\right|^2 \\ &= \frac{e^{-\Gamma|t|}}{16\Gamma|pq|^2}\left|\begin{matrix} \cos\left(\frac{\Delta M}{2}t\right)\left(A_L^f A_H^g - A_H^f A_L^g\right) \\ -i\sin\left(\frac{\Delta M}{2}t\right)\left(A_L^f A_H^g + A_H^f A_L^g\right) \end{matrix}\right|^2 \end{aligned} \tag{12}$$

where $\Delta M, \Gamma$ have the usual relations to the real and imaginary parts of $\mu_{H,L}$ and we are using $Im\mu_H = Im\mu_L$. For convenience we normalize (12) in terms of the averaged – for particle and antiparticle- decay rate $\langle\Gamma_f\rangle$. This way we define a reduced double decay rate [12]:

$$\begin{aligned} \hat{I}(f,g;t) &\equiv \frac{\Gamma}{\langle\Gamma_f\rangle\langle\Gamma_g\rangle} I(f,g;t) = \\ &= e^{-\Gamma|t|}\left(I_d^{fg}\cos^2\left(\frac{\Delta Mt}{2}\right) + I_m^{fg}\sin^2\left(\frac{\Delta Mt}{2}\right) + I_{od}^{fg}\sin(\Delta Mt)\right) \end{aligned} \tag{13}$$

which has been proven [13] to verify an exact connection with the observables related to the evolution and transitions among B states.

$$\hat{I}(f,g;t) = \left|\langle B_{\nrightarrow g}^{\perp}|B_{\nrightarrow f}(t)\rangle\right|^2 \tag{14}$$

This reduced double decay rate is equal to the rate at which -our initial $B_{\nrightarrow f}$ meson state tagged by the first decay $B \to f$- evolves after t to the B-meson filtered at the final meson state $B_{\nrightarrow f}^{\perp}$ by the second decay $B \to g$.

In eq (13) we have introduce the "intensity parameters" I_d^{fg}, I_m^{fg} and I_{od}^{fg} for every pair of decay channel (f,g). The first one appears at $t = 0$ so clearly is a signal of the direct correlation between the decay amplitudes to the two channels (f,g), I_d^{fg} will not depend on the mixing parameters appearing in the evolution in time of the $|B_{\nrightarrow f}\rangle$. We use d for direct, m for mixing induced and od for odd under t.

3. Consistency conditions

Our observables will be the coefficients in eq (13): I_d^{fg}, I_m^{fg} and I_{od}^{fg}. These terms enjoy interesting properties useful for the experimental measurements. A first property verified formally by eq (13) is the invariance of the reduced double decay rate under the simultaneous reversing of the decay channels $(f,g) \to (g,f)$ and $t \to -t$

$$\hat{I}(f,g;t) = \hat{I}(g,f;-t) \qquad (15)$$

It implies the following consistency conditions

$$I_d^{fg} = I_d^{gf}\,;\; I_m^{fg} = I_m^{gf}\,;\; I_{od}^{fg} = -I_{od}^{gf} \qquad (16)$$

Of much more subtle origin is our second set of consistency conditions. Remembering that we are using S for $J/\psi K_S$ and L for $J/\psi K_L$ and noting that $|B_{\nrightarrow S}^{\perp}\rangle$ and $|B_{\nrightarrow L}^{\perp}\rangle$ are orthogonal we get

$$|B_{\nrightarrow S}^{\perp}\rangle\langle B_{\nrightarrow S}^{\perp}| + |B_{\nrightarrow L}^{\perp}\rangle\langle B_{\nrightarrow L}^{\perp}| = I \qquad (17)$$

Therefore

$$\begin{aligned}\hat{I}(g,S;t) + \hat{I}(g,L;t) &= \left|\langle B_{\nrightarrow S}^{\perp} | B_{\nrightarrow g}(t)\rangle\right|^2 + \left|\langle B_{\nrightarrow L}^{\perp} | B_{\nrightarrow g}(t)\rangle\right|^2 \\ &= \langle B_{\nrightarrow g}(t) | B_{\nrightarrow g}(t)\rangle = e^{-\Gamma|t|}\end{aligned} \qquad (18)$$

And applying this result to eq (13) we get the second set of consistency conditions

$$I_d^{Sg} + I_d^{Lg} = 1\,;\; I_m^{Sg} + I_m^{Lg} = 1\,;\; I_{od}^{Sg} + I_{od}^{Lg} = 0 \qquad (19)$$

The importance of eq (16) relies in the fact that for some of the intensity parameters is not necessary to distinguish if the f decay has occurred before or after the g decay. The non-linearity of equations (19) gives a controlled connection between the CP forbidden and CP-allowed time-dependent transitions for any of the four decay products g. For practical purposes by adjusting the data sample to these constraints one can measure all three observables $I_{d,m,odd}^{Sg}$ for all the (f,g) and (g,f) channels $f = J/\psi K_S, J/\psi K_L$ and $g = (\rho_L^+\rho_L^-)$, $(\rho_L^0\rho_L^0), (\pi^+\pi^-), (\pi^0\pi^0)$. To be more explicit, by choosing and measuring the two opposite CP channels (S,g) and (L,g) one can recover all the $I_{d,m,odd}^{Sg}$ intensity parameters just by measuring the four ratios $I_{m,odd}^{Sg}/I_d^{Sg}$ and $I_{m,odd}^{Lg}/I_d^{Lg}$.

4. The observables and the connection with γ

In full generality the magnitude where it enters our phase γ is the ratio of amplitudes $\bar{A}_g = A(\bar{B} \to g)$ and $A_g = A(B \to g)$:

$$\frac{\bar{A}_g}{A_g} \equiv \rho_g e^{-2i\phi_g} \qquad (20)$$

If the B decays to our g channels were dominated by a single amplitude -without pinguin pollution- we would have $\rho_g = 1$ and $\phi_g = \gamma$, for example this is the case for the $\pi\pi$ channel in the isospin $I = 2$ state:

$$\frac{\bar{A}_{(\pi\pi)_{I=2}}}{A_{(\pi\pi)_{I=2}}} = e^{-2i\gamma} \qquad (21)$$

Therefore, the main target magnitude will be ϕ_g. Of course, we will also need the analogous to equation (20) for the cannels $g = L, S$:

$$\frac{\bar{A}_L}{A_L} = -\frac{\bar{A}_S}{A_S} = 1 \qquad (22)$$

A final ingredient, appearing in the B evolution in time, is the mixing parameter, that in our case is:

$$\frac{q}{p} = e^{-2i\phi_M} \qquad (23)$$

Where $\phi_M = \beta$ and β is the well-known CP violating phase in $B \to J/\psi K_S$. It is traditional to introduce when time evolution is relevant the interfering mixing induced quantities

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f} \qquad (24)$$

In such a way that we have

$$\lambda_S = -\lambda_L = -e^{-2i\phi_M}$$
$$\lambda_g = \rho_g e^{-2i(\phi_g+\phi_M)} \qquad (25)$$

For the observable present at $t = 0$ we get

$$I_d^{L,Sg} = \frac{\left|\frac{\bar{A}_{L,S}}{A_{L,S}} - \frac{\bar{A}_g}{A_g}\right|^2}{\left(1+|\lambda_{L,S}|^2\right)\left(1+|\lambda_g|^2\right)} = \frac{1}{2}\left[1 \mp \frac{2\rho_g\cos(2\phi_g)}{(1+\rho_g^2)}\right] \qquad (26)$$

It is independent of ϕ_M and for the channels without pinguin pollution $(\phi_g = \gamma,\ \rho_g = 1)$ it confirms that the channel (L, g) is CP forbidden and $(S\ , g)$ is CP allowed:

$$I_d^{Lg} = \sin^2\gamma\ ;\ I_d^{Sg} = \cos^2\gamma \qquad (27)$$

For the other observables we get

$$I_m^{L,Sg} = \frac{|1-\lambda_{L,S}\lambda_g|^2}{\left(1+|\lambda_{L,S}|^2\right)\left(1+|\lambda_g|^2\right)} = \frac{1}{2}\left[1 \mp \frac{2\rho_g\cos(2\phi_g+4\phi_M)}{(1+\rho_g^2)}\right]$$
$$I_{od}^{L,Sg} = \frac{2Im\left[(1-\lambda_{L,S}\lambda_g)(\lambda_g^* - \lambda_{L,S}^*)\right]}{\left(1+|\lambda_{L,S}|^2\right)\left(1+|\lambda_g|^2\right)} = \mp\frac{(1-\rho_g^2)}{(1+\rho_g^2)}\sin(2\phi_M) \qquad (28)$$

The quantities to be extracted from $I_{d,m,odd}^{L,Sg}$, for each channel g, are ϕ_g, ρ_g and ϕ_M. The one we are mainly interested is ϕ_g. The way of getting γ from ϕ_g is by the analogous of the Gronau and London isospin analysis.

5. Isospin analysis

In general, we will have for each $g = h^+h^-, h^0h^0$, $h = \pi, \rho_L$ a departure from the universal γ value

$$\epsilon_g = \gamma - \phi_g \qquad (29)$$

The charged decay amplitudes $A_{+0} = A(B^+ \to h^+h^0)$ and $\bar{A}_{+0} = A(B^- \to h^-h^0)$ have a final state with Isospin 2 and therefore, only $\Delta I = 3/2$ tree level amplitudes contribute with the weak phase γ: $\bar{A}_{+0}/A_{+0} = e^{-2i\gamma}$. With the definitions

$$a_g = \frac{A_g}{A_{+0}}\ ;\ \bar{a}_g = \frac{\bar{A}_g}{\bar{A}_{+0}} \qquad (30)$$

We get

$$\frac{\bar{a}_g}{a_g} = \rho_g e^{-2i\phi_g}e^{2i\gamma} = \rho_g e^{2i\epsilon_g} \qquad (31)$$

With these complex ratios the isospin triangular relations are [14-15]

$$\frac{1}{\sqrt{2}}a_\pm = 1 - a_{00}\ ;\ \frac{1}{\sqrt{2}}\bar{a}_\pm = 1 - \bar{a}_{00} \qquad (32)$$

Therefore, we can get a_g and $\bar{a}_g$ from the branching ratios of the processes $B^{\pm} \to h^{\pm}h^0$; $B^0 \to h^+h^-, h^0h^0$ fixing ϵ_g and ρ_g. The summary of our isospin analysis with PDG data [16] is presented in the following table

g	ρ_g	ϵ_g
$\rho_L^+\rho_L^-$	1.007 ± 0.076	$\mathbf{0.008 \pm 0.091}$
$\rho_L^0\rho_L^0$	0.972 ± 0.241	0.007 ± 0.345
$\pi^+\pi^-$	1.392 ± 0.062	$\pm(0.307 \pm 0.170)$
$\pi^0\pi^0$	1.306 ± 0.206	$\pm(0.427 \pm 0.172)$

Table I. Result of the actual isospin analysis

Because the $\rho_L^+\rho_L^-$ channel is the one with the largest branching ratio, $\delta\epsilon_{\rho_L^+\rho_L^-} = 0.091 = 5.2°$ give us an estimate of the uncertainty due to present knowledge of the penguin pollution, in the determination of γ. Note that important improvements are expected from Belle II and LHCb.

6. Estimate of the accuracy of the method

The intrinsic accuracy of the proposed method is controlled by the ability to extract value ϕ_g. Under the assumption that Belle II [17-19] can collect 1000 $\rho_L^+\rho_L^-$ events in the categories $(L, \rho_L^+\rho_L^-)$, $(S, \rho_L^+\rho_L^-)$, $(\rho_L^+\rho_L^-, L)$, $(\rho_L^+\rho_L^-, S)$, 50 $\rho_L^0\rho_L^0$, 200 $\pi^+\pi^-$ and 50 $\pi^0\pi^0$, we generate simulated data. For each *g*, we generate values of *t,* the events, distributed according to the four double-decay intensities. To incorporate the experimental time resolution, each t is randomly displaced following a normal distribution with zero mean and $\sigma = 1ps$. Efficiencies are not included (but remember only ratio information is used). Generation proceeds until the chosen number of events. Events are binned. The procedure is repeated in order to obtain mean values and standard deviations in each bin: these constitute our simulated data. Results are shown for 20 bins; there are no significant differences if one considers, for example, 15 or 10 bins. Our fits to the simulated data result in:

g	ϕ_g	ρ_g
$\rho_L^+\rho_L^-$	$\mathbf{1.222 \pm 0.020}$	1.00 ± 0.06
$\rho_L^0\rho_L^0$	1.22 ± 0.09	1.00 ± 0.24
$\pi^+\pi^-$	1.57 ± 0.12	1.35 ± 0.12
$\pi^0\pi^0$	1.57 ± 0.18	1.35 ± 0.24
	$\phi_M = 0.384 \pm 0.031$	

Table II. Result of the fit to the simulated data for the four channels

The simulated data, for the benchmark channel are shown bellow

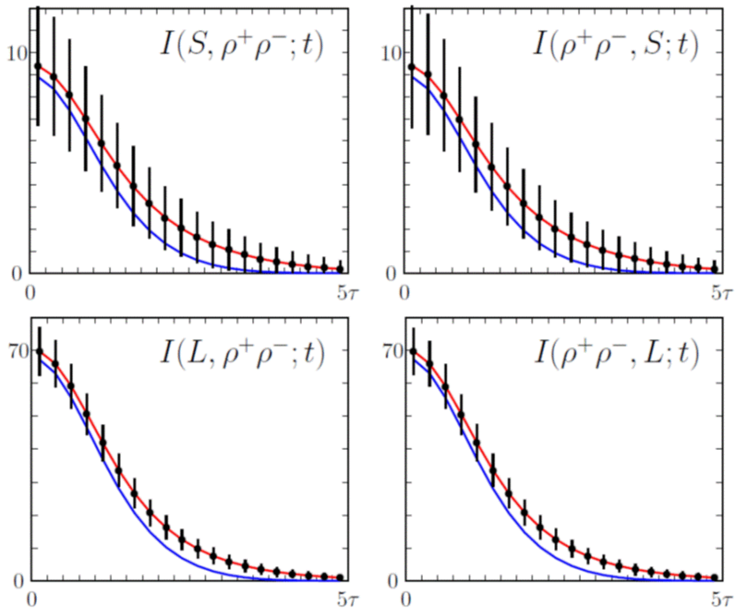

FIG. 1. Simulated data, 1000 events, benchmark $\rho_L^+\rho_L^-$. Black dots with bars indicate mean values and associated uncertainties; the red curves are the extracted double-decay intensities, while the blue curves correspond to the I_d^{fg} term in each intensity

We conclude that, since $\gamma = \phi_g + \epsilon_g$, the error $\delta\phi_{\rho_L^+\rho_L^-} = 0.020 = 1.1°$ gives an idea of the intrinsic statistical limiting error we would expect in the determination of γ for the assumed number of events.

7. Conclusions

We have shown that $B^0 - \bar{B}^0$ entanglement at the $\Upsilon(4s)$ peak gives two decay paths to measure interfering phases. With decays to CP eigenstates, we can choose CP allowed and CP forbidden decays. The possibility of measuring γ appears: the phases entering in the time evolution, coming from mixing are not needed. The channels in the double decay rate $\Upsilon(4s) \to (f, g)$ with $f = J/\psi K_S, J/\psi K_L$ and $g = \pi\pi, \rho_L\rho_L$ have a tree level common relative phase γ. $\rho_L^+\rho_L^-$ is the benchmark channel. General constraints allow the full measurement of all the observables combining the channels $(J/\psi K_S, g)$ and $(J/\psi K_L, g)$. To extract γ, the proposal has to be completed with an isospin analysis of $B \to \rho_L\rho_L, \pi\pi$. The intrinsic accuracy we estimate, according to Belle II design and expectations is of 1° [17-19]. The accuracy associated to isospin analysis is 5° according to the actual data (to be improved by LHCb and Belle II...)

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