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Higgs boson production at the next-to-next-to-leading power

Alexander A. Penin[a,*]

[a] *Department of Physics, University of Alberta, Edmonton AB T6G 2J1, Canada*

E-mail: penin@ualberta.ca

This presentation is based on Ref. [1] where the amplitude of the light quark mediated Higgs boson production via gluon fusion in the high-energy limit is analysed at the next-to-next-to leading power in the quark mass m_q. For the two-gluon Higgs boson form factor we obtain a complete analytic result for the three-loop $O(m_q^3)$ double-logarithmic term while the all-order analysis is performed in the large-N_c limit of QCD and for the abelian gauge group. An estimate of the high-order high-power light quark mass effect in the Higgs boson production and decay is given.

16th International Symposium on Radiative Corrections: Applications of Quantum Field Theory to Phenomenology (RADCOR2023)
28th May - 2nd June, 2023
Crieff, Scotland, UK

*Speaker

PoS(RADCOR2023)003

1. Introduction

Quantum corrections are known to significantly alter the high-energy properties of the gauge theory scattering amplitudes. The asymptotic behavior of the amplitudes which are not suppressed by the ratio of a characteristic infrared scale to the process energy is governed by the "Sudakov" radiative corrections enhanced by the second power of the large logarithm of the scale ratio per each power of the coupling constant. Sudakov logarithms exponentiate and result in a strong universal suppression of the scattering amplitudes in the limit when all the kinematic invariants of the process are large [2–10]. The structure of the power suppressed logarithmically enhanced contributions is by far more complex and the corresponding renormalization group analysis poses a serious challenge to the modern effective field theory. One of the important problems in this category is the analysis of the scattering amplitudes involving massive particle in the limit of small mass or high energy. The mass effects on the leading-power contributions have been extensively studied in the context of the high-order electroweak and QED radiative corrections [11–21]. The next-to-leading power contributions for a number of key processes in QED and QCD have been analysed in the leading (double) [22–29] and the next-to-leading logarithmic approximation [30, 31].

In the processes with massive fermions already at the next-to-leading power the origin of the logarithmic corrections and the asymptotic behavior of the amplitudes drastically differ from the leading-power Sudakov case. The double-logarithmic terms in this case are related to the effect of the eikonal (color) charge nonconservation in the process with soft fermion exchange and result in asymptotic exponential enhancement for a wide class of amplitudes and in a breakdown of a formal power counting [24, 28, 29, 32]. Thus, it is of a primary theoretical interest to get insight into the asymptotic behavior of the next-to-next-to-leading power contributions and determine whether any qualitatively new phenomenon appears in this order. The renormalization group analysis has not yet been extended beyond the next-to-leading power for any kind of power corrections to the high-energy processes. In this proceedings we present such an analysis of the simplest but fundamental and phenomenologically important amplitude of the light quark mediated Higgs boson production in gluon fusion [1]. The results of the analysis are used to get a quantitative estimate of the accuracy of the fixed-order calculations [33, 34] and the calculations based on the small-mass expansion [35, 36] of the light quark contribution to the Higgs boson production and decays.

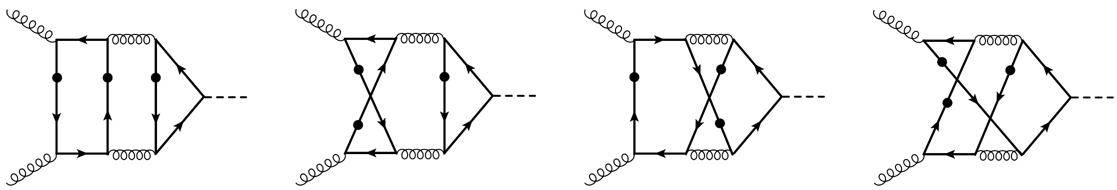
2. Higgs boson production in gluon fusion

A quark loop mediated ggH amplitude can be written as follows

$$\mathcal{M}^q_{ggH} = T_F \frac{\alpha_s}{\pi} \frac{y_q m_q}{m_H^2} \left(p_1^\mu p_2^\nu - g^{\mu\nu}(p_1 p_2)\right) A^a_\nu(p_1) A^a_\mu(p_2) H M^q_{ggH}\,, \tag{1}$$

where y_q is the quark Yukawa coupling, m_H is the Higgs boson mass, $p_i^2 = 0$, $(p_1 p_2) = -m_H^2/2$, the gauge condition $\partial^\mu A^a_\mu = 0$ is implied and one can choose the transversal polarization of the gluon fields. In the heavy quark limit $m_q \gg m_H$ the scalar amplitude approaches the value $M^q_{\gamma\gamma H} = -2/(3\rho)$, where now $\rho = m_q^2/m_H^2$ is a Minkowskian parameter. In the opposite limit of light quark $m_q \ll m_H$ it can be expanded in an asymptotic series

$$M^q_{ggH} = Z_g^2 \sum_{n=0}^{\infty} \rho^n M^{(n)}_{ggH}\,, \tag{2}$$



(a) (b) (c) (d)

Figure 1: The three-loop Feynman diagrams for the Higgs boson two-photon decay amplitude with triple soft quark exchange.

where the coefficients $M^{(n)}_{ggH}$ are finite and

$$Z_g^2 = \exp\left[-\frac{C_A s^{-\varepsilon}}{\varepsilon^2}\frac{\alpha_s}{2\pi}\right] \tag{3}$$

with $s = m_H^2$ is the universal Sudakov factor for the external on-shell gluon lines which incorporates all the infrared divergencies of the amplitude. Note that the amplitude is loop generated and in the high-energy (small-mass) limit is suppressed by the quark mass due to chirality flip at the Higgs boson vertex. The $O(m_q)$ next-to-leading power scalar amplitude in double-logarithmic approximation reads [28]

$$M^{(0)}_{ggH} = \ln^2\!\rho\, g(z)\,, \tag{4}$$

where

$$g(z) = 2\int_0^1 \mathrm{d}\xi \int_0^{1-\xi} \mathrm{d}\eta e^{2z\eta\xi} = {}_2F_2\,(1,1;3/2,2;z/2) \tag{5}$$

is the generalized hypergeometric function with the Taylor expansion

$$g(z) = 2\sum_0^\infty \frac{n!}{(2n+2)!}(2z)^n\,. \tag{6}$$

At $O(m_q^3)$ the double-logarithmic terms can be cast into three classes. The *factorizable* contribution results from the corrections to the $gg \to q\bar{q}$ amplitude in mass and soft quark momentum, which do not affect the structure of the leading-power soft gluon emission. The corresponding term in $M^{(1)}_{ggH}$ reduces to $-4M^{(0)}_{ggH}$, where $M^{(0)}_{ggH}$ is given by Eq. (4).

Starting with three loops the diagrams with *triple soft quark* exchange, Figs. 1(a-d), may contribute to $O(m_q^3)$ amplitude. The double-logarithmic part of the diagrams Figs. 1(b-d) vanishes after taking the spinor trace over the closed quark loop. At the same time the diagram Fig. 1(a) includes a two-loop subdiagram corresponding to the double-logarithmic off-shell scalar form factor, which can be obtained by generalization of the on-shell analysis [29]. In this way we get the double-logarithmic corrections to the coefficient $M^{(1)}_{ggH}$

$$\ln^2\!\rho\,\frac{T_F C_F}{45}x^2 h(z)\,, \tag{7}$$

where the function $h(z)$ has the following integral representation

$$h(z) = 6!\int_0^1 \mathrm{d}\eta \int_0^{1-\eta} \mathrm{d}\xi \int_0^{\eta} \mathrm{d}\eta_2 \int_0^{\xi} \mathrm{d}\xi_2 \int_0^{\eta_2} \mathrm{d}\eta_1 \int_0^{\xi_2} \mathrm{d}\xi_1\, e^{2z(\eta\xi-\eta_2\xi_2+\eta_1\xi_1)}\,. \tag{8}$$

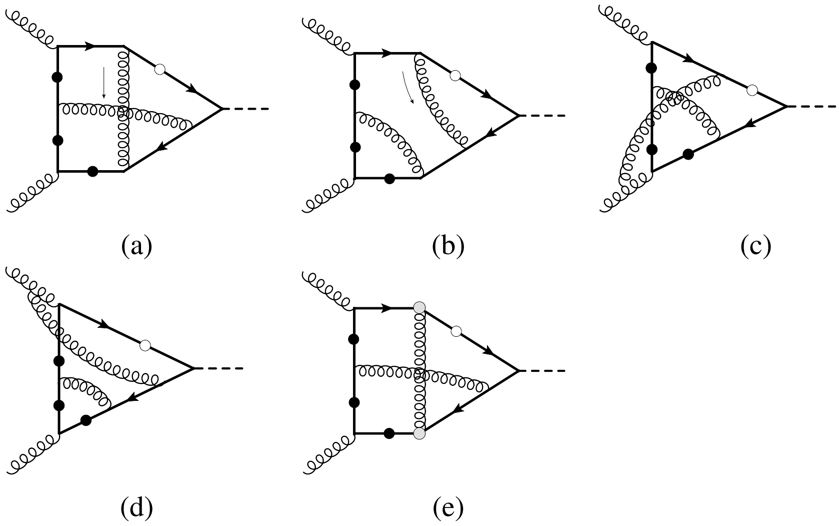


Figure 2: (a)-(d) the Feynman diagrams representing the three-loop correction to the ggH amplitude with an additional eikonal gluon emitted by the soft quark. (e) the Feynman diagram with the effective soft gluon exchange, which represents the total QCD three-loop non-Sudakov double-logarithmic correction associated with the eikonal gluon emission, Eq. (9).

The coefficients of the Taylor series $h(z) = 1 + \sum_{n=1}^{\infty} h_n z^n$ can be computed for any given n corresponding to the $(n+3)$-loop double-logarithmic contribution.

All the double-logarithmic contributions we have considered so far factored out into the (effective) corrections to the Higgs boson vertex. In three loops a new source of the *nonfactorizable* double-logarithmic corrections opens up with an additional eikonal gluon connecting one of the eikonal and the soft quark lines. The corresponding abelian and nonabelian diagrams are given in Figs. 2(a,b) and (c,d), respectively. Note that for the planar topology Fig. 2(b) due to a cancellation specific to three loops the double-logarithmic contribution vanishes. After separating the infrared divergencies in the same way as it has been done for the functions $f(z)$ and $g(z)$, the remaining infrared finite double-logarithmic contribution is described by the diagram Fig. 2(e) with the effective soft gluon exchange. The corresponding Feynman integral is the same as for the abelian diagram in Fig. 2(a), which gives the following contribution to $M^{(1)}_{ggH}$

$$- \ln^2\rho \, \frac{(C_A - C_F)\,(C_A - 2C_F)}{9} x^2 \,. \tag{9}$$

The color structure of Eq. (9) is quite peculiar. As it has been previously discussed the factor $C_A - C_F$ accounts for the eikonal color charge variation caused by a soft quark emission. The remaining factor $C_A - 2C_F$ reflects the change of the eikonal quark and antiquark state into color octet after the emission of the eikonal gluon.

The higher-order double-logarithmic corrections of this type are obtained by dressing the diagram in Figs. 2(a)-(d) with multiple soft gluons. This results in multiplication of Eq. (9) by a function of the double-logarithmic variable $j(z) = 1 + \sum_{n=1}^{\infty} j_n z^n$. Thus the complete double-logarithmic approximation for the next-to-next-to-leading power coefficient can be written as follows

$$M^{(1)}_{ggH} = \ln^2\rho \left[-4g(z) + \left(\frac{T_F C_F}{45}h(z) - \frac{(C_A - C_F)(C_A - 2C_F)}{9} j(z)\right) x^2\right]. \tag{10}$$

Calculation of the functions $j(z)$ requires a systematic factorization of the soft emissions with respect to the emission of the additional eikonal gluon. For QCD this is a rather complicated computational problem due to the soft interaction of the eikonal gluon, which starts to contribute in four loops. The full QCD analysis, however, goes beyond the scope of the present paper. Instead, we consider two complementary limits where such a complication is absent. First we discuss QCD with the large number of colors $N_c \to \infty$. In this case the color factor of the diagram Fig. 2(a) vanishes and the double logarithmic approximation is entirely determined by the function $g(z)$ where $z = N_c x/2$. In the opposite abelian limit $C_A = 0$ the gluon self-coupling is absent but the analysis of the factorization is nevertheless quite nontrivial. For $C_A = 0$ we get the following integral representation of the function $j(z)$

$$\begin{aligned} j^{\rm ab}(z) &= 72\int_0^1 {\rm d}\eta \int_0^{1-\eta} {\rm d}\xi \int_0^{1-\xi} {\rm d}\eta_1 \int_0^{1-\eta_1-\xi} {\rm d}\xi_1\, \eta\xi_1 e^{2z\eta(\xi+\xi_1)} \\ &\times \left[1 + \frac{e^{-2z\eta\xi} - 1}{2} + \frac{e^{-2z\eta\xi} - 1 + 2z\eta\xi}{4z\eta\xi_1}\right], \end{aligned} \tag{11}$$

where in the abelian approximation the double-logarithmic variable reduces to $z = -C_F x$. The perturbative expansion of Eq. (10) reads

$$\begin{aligned} M^{(1)}_{ggH} &= \ln^2\rho \left[-4 - \frac{2}{3}(C_A - C_F)x + \left(\frac{T_F C_F}{45} - \frac{14}{45}C_F^2 + \frac{23}{45}C_F C_A - \frac{9}{45}C_A^2\right)x^2\right. \\ &+ \left. c_4 x^3 + \ldots\right], \end{aligned} \tag{12}$$

where the four-loop coefficient is $c_4 = -N_c^3/840$ in the large-N_c approximation and

$$c_4 = -\frac{T_F C_F^2}{210} + \frac{13}{90}C_F^3 \tag{13}$$

in the abelian approximation. The series Eq. (12) can be compared to the existing fixed-order results. The two-loop term agrees with the expansion of the exact analytic result [37]. The high-energy expansion of the three-loop ggH amplitude has been obtained numerically in Ref. [33]. Eq. (12) corresponds to the following coefficient of the L_s^6/z^2 term in Eq. (C.1) of [33]

$$\frac{1}{23040}\left(-T_F C_F + 14C_F^2 - 23C_F C_A + 9C_A^2\right), \tag{14}$$

which agrees with its numerical value 0.0005738811728. The result Eq. (10) for the gluon fusion amplitude can be transformed into the one for the amplitude of the Higgs boson two-photon decay by changing the color charge of the external lines from C_A to zero. This results in the replacement $C_A - C_F \to -C_F$ in the definition of the double-logarithmic variable z and in the coefficient of Eq. (9). By adopting the notations similar to the gluon fusion case we get

$$M^{(1)}_{H\gamma\gamma} = \ln^2\rho \left[-4 + \frac{2}{3}C_F x + \left(\frac{T_F C_F}{45} - \frac{14}{45}C_F^2 + \frac{C_F C_A}{9}\right)x^2 + \ldots\right]. \tag{15}$$

The three-loop term can be compared to the numerical result for the high-energy expansion of the amplitude given in Ref. [34]. It corresponds to the coefficient

$$-\frac{1}{3840}\left(T_F C_F - 14C_F^2 + 5C_F C_A\right) \tag{16}$$

of the L_s^6/z^2 term in Eq. (C.1) and agrees with its numerical value 0.001099537037.

Let us consider the all-order asymptotic behavior of the $O(m_q^3)$ amplitude in the high-energy (small-mass) limit. In the large-N_c approximation it reads

$$M_{ggH}^{(1)} = -4\ln^2\!\rho\, g\left(\frac{N_c x}{2}\right), \tag{17}$$

where

$$g(z) \sim \left(\frac{2\pi e^z}{z^3}\right)^{\frac{1}{2}} \tag{18}$$

at $z \to \infty$, *i.e.* the amplitude is exponentially enhanced. Note that the limit $N_c \to \infty$ is taken first and in general may not commute with the kinematical limit $z \to \infty$. In the abelian approximation the relevant asymptotic behavior of the functions in Eq. (10) at $z \to -\infty$ reads

$$g(z) \sim -\frac{\ln(-2z) + \gamma_E}{z}, \qquad h(z) = O(1/z^3), \qquad j^{\rm ab}(z) \sim \frac{9}{2z^2}\,. \tag{19}$$

Thus the coefficient asymptotically approaches the value $M_{ggH}^{(1)} = -\ln^2\rho$, *i.e.* the double logarithmic corrections effectively reduce the leading-order coefficient by factor four.

Now we can estimate the effect of the high-order $O(m_q^3)$ terms for the physical values of the parameters. The relative correction to the $O(m_q)$ amplitude is given by the factor

$$1 + \rho\left[-4 + \left(\frac{T_F C_F}{45}h(z) - \frac{(C_A - C_F)(C_A - 2C_F)}{9}j(z)\right)\frac{x^2}{g(z)}\right]. \tag{20}$$

In the large-N_c approximation Eq. (20) reduces to $1 - 4\rho$ with $\rho \approx 1.6 \cdot 10^{-3}$, which amounts of approximately 0.64% negative correction to the $O(m_q)$ contribution. It does not depend on x and is the same for the gluon and photon external lines. Hence it gives a universal all-order estimate of the next-to-next-to-leading power corrections both for the production and decay amplitudes.

3. Summary

We have presented the analysis of the high-energy asymptotic behavior of the light quark loop mediated Higgs boson production in the third order of the small quark mass expansion. To our knowledge this is the first example of the renormalization group analysis of the next-to-next-to-leading power amplitudes.

The double-logarithmic corrections to the $O(m_q^3)$ Higgs boson production and decay amplitudes are induced by single and triple soft quark exchanges. This is the first example where the mass suppression of the double-logarithmic contribution is not entirely associated with the chirality flip on a fermion line. Starting with three loops a new source of the double-logarithmic corrections opens up with an emission of an additional virtual eikonal gluon by the soft quark. Our analytic result agrees with the previous numerical evaluation of the three-loop QCD corrections to the Higgs boson production [33] and two-photon decay [34]. Beyond three loops the all-order double-logarithmic asymptotic behavior of the amplitudes has been derived in two complementary approximations. In the large-N_c limit, which is supposed to catch the qualitative behavior of real QCD, the structure of the double-logarithmic corrections significantly simplifies and becomes identical to the one of the leading $O(m_q)$ contribution, which is exponentially enhanced for the large values of the double-logarithmic variable. The opposite abelian limit $C_A = 0$, though less phenomenologically relevant, reveals a more complex structure of the double-logarithmic contributions and represents the general case for the mass-suppressed amplitudes at the next-to-next-to-leading power.

We have also presented a quantitative estimate of the accuracy of the high-order calculations based on the small-mass expansion for the Higgs boson production and decays mediated by the bottom quark loop, which may become relevant with the permanently increasing accuracy of the QCD predictions. On the basis of the double-logarithmic analysis we conclude that neglecting the terms suppressed by the mass ratio m_b^2/m_H^2 in such a calculation introduces a relative error at the scale of one percent in every order of the perturbative expansion.

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