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Mixed QCD-EW two-loop amplitudes for charged current Drell-Yan production

Narayan Rana[a,*]

[a] *School of Physical Sciences, National Institute of Science Education and Research, An OCC of Homi Bhabha National Institute, 752050 Jatni, India*

E-mail: narayan.rana@niser.ac.in

In this proceeding, we present the mixed QCD-EW two-loop virtual corrections for the charged current Drell-Yan production. The presence of one additional mass compare to the neutral current case makes the computation of the two-loop amplitudes extremely challenging, specially the two-loop Feynman integrals. Our approach to evaluate the relevant two-loop Feynman integrals using semi-analytical method, allows us to obtain the renormalized two-loop amplitudes. We perform the subtraction of the universal infrared singularities mostly analytically and evaluate the hard function numerically as a grid.

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1. Introduction

The Drell-Yan (DY) production of a pair of leptons plays a crucial role in the precision determinations of the parton density functions (PDFs) and of the electroweak (EW) parameters of the Standard Model (SM). Precise experimental measurements and equally precise theoretical predictions, both are necessary for these precision studies. The level of precision for the corresponding observables reached till date, is only possible due to the high-quality experimental data collected at the Fermilab Tevatron by the CDF and D0 experiments, and at the CERN LHC by the ATLAS, CMS, and LHCb experiments, and precise theoretical predictions. The future hadron colliders will achieve precision higher than ever, and hence it is very crucial to reach a similar accuracy for theoretical estimates. The experimental data can be compared to the theoretical estimates only when both are equally precise. This comparison can indicate possible deviations from the SM physics, thus providing signals for beyond the SM physics.

The precision for theoretical predictions can be increased by including corrections from the higher orders in the perturbative expansion. For DY production, the perturbative Quantum Chromodynamics (QCD) corrections have been computed up to third order in the strong coupling constant (α_s) in refs. [1–6]. The next-to-next-to-leading-order (NNLO) QCD differential distributions have been obtained in refs. [7–11]. The first estimates of the next-to-next-to-next-to-leading-order (N^{3}LO) QCD fiducial cross sections for the neutral current (NC) DY have been obtained in ref. [12]. The threshold behaviour at the third and higher order in α_s has been studied in refs. [13–21]. The NLO EW corrections in the electromagnetic coupling constant (α) have been computed in refs. [22–31]. The third order QCD corrections provide *per mill* effects, implying necessary inclusion of the mixed QCD-EW corrections, as the later one is also expected to be of same magnitude as the former one. Additionally, the QCD corrections only control the uncertainties arising from the unphysical scales, while the large effects from the Sudakov logarithms are controlled only by considering the EW corrections. Hence, to obtain a robust estimate, it is necessary to include the mixed QCD-EW corrections.

In several works [32–40], attempts were taken either to compute the necessary ingredients or to obtain a rough estimate through various approximations. In refs. [41–45], an analytic computation was performed to obtain the complete NNLO QCD-EW corrections to Z boson inclusive production. The differential cross-sections for the on-shell Z and W production at this perturbative order have been computed in refs. [46] and [47], respectively. The first step was taken in refs. [48, 49] to obtain the $O(\alpha_s\alpha)$ corrections to the full DY production considering the pole approximation [50]. Also a partial result has been computed in ref. [51] by obtaining the $O(n_F\alpha_s\alpha)$ terms. Towards the computation of the complete NNLO QCD-EW corrections to the NC DY process, the contributing master integrals (MIs) were obtained in several publications [52–55]. In ref. [56], the two-loop helicity amplitudes for NC DY was obtained for massless leptons. In ref. [57], the two-loop virtual contributions were obtained considering massive leptons, with the small mass limit. Finally the complete NNLO QCD-EW corrections to the NC DY process has been obtained in refs. [58, 59], including the exact two-loop virtual contributions. The mixed QCD-EW corrections to the charged-current (CC) DY process has been computed in ref. [60], with the reweighted two-loop virtual corrections in the pole approximation and the rest in exact form. To obtain the complete result for the CC DY process, the only missing part is the two-loop virtual contribution.

In this proceeding, we present the computational procedure we have followed to obtain the mixed QCD-EW two-loop virtual corrections for the CC DY production. The presence of one additional mass compare to the NC DY process, makes the computation of the two-loop amplitudes extremely challenging, specially the two-loop Feynman integrals. Our approach to evaluate the relevant two-loop Feynman integrals using semi-analytic method, allows us to obtain the renormalized two-loop amplitudes. We perform the subtraction of the universal infrared singularities mostly analytically and evaluate the hard function numerically as a grid.

2. Theoretical framework

We consider the CC DY process which is given, at the partonic level, by

$$u(p_1) + \bar{d}(p_2) \to \nu_l(p_3) + l^+(p_4)\,, \tag{1}$$

with the on-shell conditions of the external particles given by

$$p_1^2 = p_2^2 = p_3^2 = 0;\;\; p_4^2 = m_l^2\,. \tag{2}$$

m_l is the lepton mass. The Mandelstam variables for this process are defined as follows:

$$s = (p_1 + p_2)^2,\; t = (p_1 - p_3)^2,\; u = (p_2 - p_3)^2 \text{ with } s + t + u = m_l^2\,. \tag{3}$$

We also define here μ_W and μ_Z, the complex masses of the W and Z bosons respectively, as follows:

$$\mu_V^2 = M_V^2 - iM_V\Gamma_V\;. \tag{4}$$

The mass and decay width M_V, Γ_V are real and the pole quantities. The bare amplitude (denoted by the *hat*) can be expanded into a double perturbative series in the two coupling constants as follows

$$|\hat{\mathcal{M}}\rangle = \sum_{m,n=0}^{\infty} \left(\frac{\hat{\alpha}_s}{4\pi}\right)^m \left(\frac{\hat{\alpha}}{4\pi}\right)^n |\hat{\mathcal{M}}^{(m,n)}\rangle\,. \tag{5}$$

The two-loop amplitudes are not physical quantities, and hence they contain divergences of ultraviolet (UV) and infrared (IR) origin. We use the method of the dimensional regularization with arbitrary space-time dimension $d = 4 - 2\epsilon$ to regulate the divergences. However, the EW interactions are chiral, and hence we need to define γ_5, an inherently four-dimensional object, in arbitrary d-dimensions. In ref. [56], it has been explicitly checked for the NC DY process at $O(\alpha\alpha_s)$ that the prescriptions to define γ_5 by 't Hooft and Veltman [61], and, Breitenlohner and Maison [62–64] and Kreimer et al. [65, 66], both yield different results for the scattering amplitudes, but same result for the IR-subtracted finite remainder. We follow the prescription by Kreimer et al, keeping the anti-commutation relation together with the relations

$$\gamma_5^2 = 1\,,\quad \gamma_5^\dagger = \gamma_5 \tag{6}$$

and we use a fixed point for the Dirac traces. For the remaining single γ_5, we perform the replacement

$$\gamma_5 = i\frac{1}{4!}\varepsilon_{\nu_1\nu_2\nu_3\nu_4}\gamma^{\nu_1}\gamma^{\nu_2}\gamma^{\nu_3}\gamma^{\nu_4}\,, \tag{7}$$

where, $\varepsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita tensor in d dimensions.

The two-loop virtual amplitudes contain both UV and IR divergences. We perform appropriate UV renormalization of the fields and parameters to remove the UV divergences. The renormalization procedure is similar to the one in the NC DY production, details of which have been presented in ref. [51]. To perform the renormalization, we need the on-shell electric charge counterterm at $O(\alpha^2)$ and $O(\alpha\alpha_s)$ [67], the mass counterterms [51] in the complex mass scheme [68], the finite correction Δr [69–71], appearing in the relation between the Fermi constant G_μ and the fine structure constant α, and the two-loop $O(\alpha\alpha_s)$ self-energy expressions of the gauge boson propagators and the counterterms [51, 70, 71].

The UV renormalized amplitudes contains only IR divergences. However, the IR structure of multiloop scattering amplitudes is universal and can be predicted by studying QCD factorization. The IR structure was well studied up to two-loop level [72–75] for a massless gauge theory. In case of a mixed gauge group, it was first presented in refs. [76, 77], particularly for mixed QCD⊗QED corrections to NC DY production with massless leptons. In case of the amplitudes containing massive particles in the final states, the IR structure changes substantially. Such studies have been performed rigorously in refs. [78–83]. For the specific process of top quark pair production in the hadron colliders, a detailed study has been performed in refs. [84–87]. This result has been abelianized [88] to obtain the IR structure for the mixed QCD-EW corrections to NC DY and CC DY production considering massive leptons. The IR structure of the scattering amplitudes has a perturbative structure and the subtraction operator $\mathcal{I}^{(m,n)}$ acts as the basic building block at each order of α_s. $\mathcal{I}^{(m,n)}$s are process-independent, but can differ from each other by a finite constant for different subtraction schemes. We use here the $\mathcal{I}^{(m,n)}$s which have been defined in the framework of the q_T-subtraction formalism [89]. The one-loop IR subtraction functions for CC DY at the renormalization scale $\mu_R^2 = s$, are given by

$$\mathcal{I}^{(1,0)} = C_F \left(\frac{2}{\epsilon^2} + \frac{1}{\epsilon}(3 + 2i\pi) - \zeta_2 \right) , \tag{8}$$

$$\mathcal{I}^{(0,1)} = \frac{Q_u^2 + Q_d^2}{2} \left(\frac{2}{\epsilon^2} + \frac{1}{\epsilon}(3 + 2i\pi) - \zeta_2 \right) - \frac{4}{\epsilon}\Gamma_l^{(0,1)} , \tag{9}$$

where

$$\Gamma_l^{(0,1)} = -\frac{Q_u Q_l}{2} \log\left(-\frac{u}{s} \right) + \frac{Q_d Q_l}{2} \log\left(-\frac{t}{s} \right) + \frac{Q_l^2}{4}\left(-1 + i\pi + \log\left(\frac{s}{m_l^2} \right) \right) . \tag{10}$$

Q_l and Q_u are the electric charges of the lepton and of the initial-state quark, and the Casimir of the fundamental representation of SU(N), C_F, is given by $C_F = \frac{N^2-1}{2N}$. Using the one-loop subtraction functions, we obtain the finite contributions from the one-loop QCD and EW amplitudes, respectively, as follows:

$$|\mathcal{M}^{(1,0),fin}\rangle = |\mathcal{M}^{(1,0)}\rangle + \mathcal{I}^{(1,0)}|\mathcal{M}^{(0)}\rangle , \tag{11}$$

$$|\mathcal{M}^{(0,1),fin}\rangle = |\mathcal{M}^{(0,1)}\rangle + \mathcal{I}^{(0,1)}|\mathcal{M}^{(0)}\rangle . \tag{12}$$

The mixed two-loop subtraction operator at the renormalization scale $\mu_R^2 = s$ is given by

$$\mathcal{I}^{(1,1)} = -C_F \Bigg[\frac{Q_u^2 + Q_d^2}{2} \bigg(\frac{4}{\epsilon^4} + \frac{1}{\epsilon^3}(12 + 8i\pi) + \frac{1}{\epsilon^2}(9 - 28\zeta_2 + 12i\pi) + \frac{1}{\epsilon}\Big(-\frac{3}{2} + 6\zeta_2 - 24\zeta_3 - 4i\pi\zeta_2 \Big) \bigg) + \bigg(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon}(3 + 2i\pi) + \zeta_2 \bigg) \frac{4}{\epsilon} \Gamma_l^{(0,1)} \Bigg]. \tag{13}$$

Using eqs. (11-13), we obtain the subtracted and finite two-loop amplitude

$$|\mathcal{M}^{(1,1),fin}\rangle = |\mathcal{M}^{(1,1)}\rangle + \mathcal{I}^{(1,1)}|\mathcal{M}^{(0)}\rangle + \tilde{\mathcal{I}}^{(0,1)}|\mathcal{M}^{(1,0),fin}\rangle + \tilde{\mathcal{I}}^{(1,0)}|\mathcal{M}^{(0,1),fin}\rangle\,. \tag{14}$$

$\tilde{\mathcal{I}}^{(i,j)}$ is obtained by dropping the term proportional to ζ_2 in $\mathcal{I}^{(i,j)}$. This conventional choice defines the finite part of our subtraction term.

2.1 Details of computational procedure

In this section, we provide a summary of the computational procedure to obtain the two-loop matrix elements. These computations are technically challenging and very much involved. Hence, we employ two independent set of programs to compute the matrix elements for cross-checking. In one set, we use QGRAF [90] to generate the Feynman diagrams, followed by a set of in-house FORM [91] routines, to perform the Lorentz and Dirac algebra. The appearing scalar integrals have been reduced to the MIs by means of integration-by-parts (IBP) identities [92, 93], through LiteRed [94, 95] and Reduze2 [96, 97]. In the second set of programs, we have used ABISS, based on FeynArts [98] to generate the Feynman diagrams and perform the algebras. The IBP reduction has then been performed using Kira [99].

As the IBP reduction and subsequently the computation of the MIs become extremely challenging for multiple mass scales, we consider massless lepton to compute the amplitudes. Later, we use the universality of QCD amplitudes and massification to obtain the results in the small lepton mass limit i.e. m_ℓ is negligible compared to μ_W, μ_Z and to the energy scales of the process. In this way, we get the collinear contributions from the lepton mass which appear as $\log(m_\ell^2/s)$. The dropped terms are proportional to $O(m_l^2)$ and negligible compared to the total contributions.

Following the IBP reduction, the unrenormalized two-loop amplitudes can be expressed as

$$\langle \hat{\mathcal{M}}^{(0)} | \hat{\mathcal{M}}^{(1,1)} \rangle = \sum_k c_k(s,t,\{m_i\},\varepsilon) I_k(s,t,\{m_i\},\varepsilon) \tag{15}$$

where both the rational coefficients c_k and the MIs I_k depend on ε, the kinematical variables s, t and the masses $\{m_i\}$ of the gauge bosons and fermions. The MIs can be broadly grouped depending on the presence of the masses. The massless two-loop form factor type MIs were computed in ref. [100]. The MIs with only one mass scale (m_W) are available in refs. [101, 102] for the form factor type, and in refs. [52–54] for the box type. However, the MIs with two mass scales (m_W and m_Z) are not known. We compute them using our Mathematica based program SeaSyde [103] semi-analytically as described in the next section. Additionally to observe the analytic cancellation of the IR poles with the universal IR structure, we also compute the necessary poles of these two-mass MIs analytically.

2.2 Computation of the master integrals

The massless and one-mass MIs has been computed by using the method of differential equations [104–110], obtaining the solution in terms of generalised harmonic polylogarithms (GHPLs), also known as Goncharov Polylogarithms (GPLs) [111–113]. However, the computation of two-different-mass MIs using the method of differential equations are extremely challenging. Hence we adopt the following semi-analytical approach. We obtain the MIs by solving the corresponding differential equations using series expansion method (See for instance [114–120]). To do so, we consider the complete system of differential equations for these two-different-mass MIs. We implemented the method, as presented in ref. [121] and the MATHEMATICA code DIFFEXP [122], in an independent public MATHEMATICA package SEASYDE [103], generalising it in order to perform the analytic continuation on the complex plane. First, we make an ansatz of the homogeneous solution of the corresponding differential equation as a Laurent series expansion around the initial boundary condition. The unknown coefficients of the series expansion can be determined by plugging it into the homogeneous system and by solving the set of algebraic equations. After obtaining the homogeneous solution, we use the method of variation of constant to compute the particular solution. The solutions can be computed to arbitrary precision, only limited by the precision of the initial conditions. We have extensively used SEASYDE for the known results of NC DY to cross-check. Also, we have performed a thorough check comparing DIFFEXP and SEASYDE for the same system of differential equations, and finding excellent agreement. We also have checked our results with AMFLOW [123] output for few chosen kinematical points. Additionally, we have computed the two-different-mass MIs up to the $\frac{1}{\epsilon^2}$ pole analytically, by solving the differential equations. This allows us to obtain complete analytic results up to the $\frac{1}{\epsilon^2}$ pole, and hence analytic cancellation with the universal IR contributions.

2.3 Massification

Additional scales in Feynman integrals make both the IBP reduction and computation of the MIs, challenging. Hence, we have performed both these operations considering massless lepton. Once, we obtain the renormalized two-loop amplitudes for massless lepton, we use the massification procedure [78, 82] i.e. we multiply the ratio of the massive and massless jet function for each lepton to obtain the results for massive leptons, but in the small mass limit. Through massification, the massive ($|\mathcal{M}_m\rangle$) and massless ($|\mathcal{M}_0\rangle$) amplitudes with k being the number of leptons to be massified, are related by

$$|\mathcal{M}_m\rangle = (Z_q^{(m|0)})^{\frac{k}{2}} |\mathcal{M}_0\rangle \,. \tag{16}$$

3. Results and conclusions

In this proceeding, we present the computational details of the mixed QCD-EW two-loop virtual corrections for the CC DY production. Compare to the NC DY production, the presence of one additional mass makes the computation of the two-loop amplitudes extremely challenging, specially the two-loop Feynman integrals. We address the issue using semi-analytical approach. We compute the necessary MIs using our MATHEMATICA package SEASYDE with great precision and successfully perform the subtraction of IR poles at double-precision. Additionally, we compute the required MIs analytically and obtain analytical results up to the $\frac{1}{\epsilon^2}$ pole, which allow us to perform the subtraction analytically to that order in ϵ. We obtain the hard function after performing the IR subtraction and numerically evaluate it in the form of a grid. The results will be presented in a future publication.

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