PROCEEDINGS OF SCIENCE

Hybrid k_T-factorization at NLO

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We show how the hybrid high-energy factorization formula, in which one initial-state parton momentum is space-like and carries non-vanishing transverse components while the other is on-shell, can be promoted to next-to-leading order (NLO). All non-cancelling soft and collinear divergences in the real and virtual contribution for the partonic cross section are identified, and we observe that they force to change the interpretation of the factorization formula. Coincidentally, expressions for inclusive NLO quark-and gluon impact factor corrections known in literature are recovered.

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1. Hybrid k_T-factorization and the auxiliary parton method

At lowest order, the hybrid high-energy, or k_T-, factorization formula for cross sections in hadron scattering looks schematically like

$$d\sigma^{(0)} = \int dx_{in}\, d^2k_\perp\, d\bar{x}_{\overline{in}}\, F(x_{in}, |k_\perp|)\, f(\bar{x}_{\overline{in}})\, d\mathrm{B}^\star(x_{in}, k_\perp, \bar{x}_{\overline{in}}) \; . \tag{1}$$

The functions $F(x_{in}, |k_\perp|)$ and $f(\bar{x}_{\overline{in}})$ are parton density functions (PDFs). They both depend on a longitudinal momentum fraction, but only one of them depends on transverse momentum. The Born-level differential partonic cross section $d\mathrm{B}^\star$ is special compared to collinear factorization, as highlighted by the '$\star$', in the sense that one of its initial-state partons is space-like, and has momentum

$$k_{in}^\mu = x_{in} P_A^\mu + k_\perp^\mu \; , \tag{2}$$

where P_A is the light-like hadron momentum and $k_\perp$ is transverse, *i.e.* $k_\perp \cdot P_A = 0$. We consider the general case, for which the scattering process may involve a number of final-state jets, and/or massive quarks, and/or a Higgs boson etc. $d\mathrm{B}^\star$ is implied to depend on the momenta of those, and to include the differential phase space.

It is straightforward to define and calculate the tree-level amplitudes necessary to construct $d\mathrm{B}^\star$ with the auxiliary parton method [1]. We need a Sudakov decomposition of momenta as

$$K^\mu = x_K P^\mu + \bar{x}_K \bar{P}^\mu + K_\perp^\mu \tag{3}$$

where P and $\bar{P}$ are the colliding hadron momenta which have positive energy and satisfy

$$P^2 = \bar{P}^2 = 0 \quad , \quad 2P\cdot\bar{P} = \nu^2 > 0 \quad , \quad P\cdot K_\perp = \bar{P}\cdot K_\perp = 0 \; . \tag{4}$$

Now let the desired parton-level process be

$$g^\star(k_{in})\; \omega_{\overline{in}}(k_{\overline{in}}) \;\to\; \omega_1(p_1)\; \omega_2(p_2)\; \cdots\; \omega_n(p_n) \tag{5}$$

where $g^\star$ represents the space-like gluon, and the ω_i represent the other partons or particles involved in the process, with $k_{\overline{in}} = \bar{x}_{\overline{in}}\bar{P}$ in particular. In the auxiliary parton method, this process is obtained from the quark scattering process

$$q\big(k_1(\Lambda)\big)\; \omega_{\overline{in}}(k_{\overline{in}}) \;\to\; q\big(k_2(\Lambda)\big)\; \omega_1(p_1)\; \omega_2(p_2)\; \cdots\; \omega_n(p_n) \tag{6}$$

where

$$k_1^\mu = \Lambda P^\mu \quad , \quad k_2^\mu = p_\Lambda^\mu = (\Lambda - x_{in})P^\mu - k_\perp^\mu + \frac{|k_\perp|^2}{(\Lambda - x_{in})\nu^2}\,\bar{P}^\mu \; , \tag{7}$$

so $k_1^2 = k_2^2 = 0$ and $k_1 - k_2 = k_{in} + \mathcal{O}\big(\Lambda^{-1}\big)$ with k_{in} as in Eq. (2). The process of Eq. (6) with the auxiliary quarks is on-shell, and its squared matrix element is well-defined and it is known how to calculate it efficiently for any process. The squared matrix element of the desired process with the space-like gluon is obtained by taking

$$\Lambda \to \infty \; . \tag{8}$$

In [2] and [3] it was noted that instead of an auxiliary scattering quark, also an auxiliary scattering gluon can be used. At the level of squared matrix elements summed over color, one just needs to include a different overall factor to correct for the difference in color representation:

$$\frac{1}{g_s^2 C_{\mathrm{aux}}}\,\frac{x_{in}^2|k_\perp|^2}{\Lambda^2}\,\big|\overline{\mathcal{M}}^{\mathrm{aux}}\big|^2\big(\Lambda P, k_{\overline{in}}\,; p_\Lambda, \{p_i\}_{i=1}^n\big) \overset{\Lambda\to\infty}{\longrightarrow} \big|\overline{\mathcal{M}}^{\star}\big|^2\big(k_{in}, k_{\overline{in}}\,; \{p_i\}_{i=1}^n\big) \tag{9}$$

with

$$C_{\text{aux-q}} = \frac{N_c^2-1}{N_c} \quad , \quad C_{\text{aux-g}} = 2N_c\ . \tag{10}$$

The other factors on the left-hand side of Eq. (9) extract the leading power 2 of Λ, assure the correct power of the coupling constant, and the correct on-shell limit.

2. Promotion to NLO

In [4] it is described how the hybrid formula can be promoted to NLO. Schematically the cross section is expected to look like

$$\begin{aligned} d\sigma^{(1)} = \int dx_{in}\, d^2k_\perp\, d\bar{x}_{\overline{in}} \Big\{ & F(x_{in},|k_\perp|)\, f(\bar{x}_{\overline{in}}) \Big[d\mathrm{V}^\star(x_{in},k_\perp,\bar{x}_{\overline{in}}) + d\mathrm{R}^\star(x_{in},k_\perp,\bar{x}_{\overline{in}}) \Big] \\ & + \Big[F^{(1)}(x_{in},|k_\perp|)\, f(\bar{x}_{\overline{in}}) + F(x_{in},|k_\perp|)\, f^{(1)}(\bar{x}_{\overline{in}}) \Big] d\mathrm{B}^\star(x_{in},k_\perp,\bar{x}_{\overline{in}}) \Big\}\ , \end{aligned} \tag{11}$$

where, $d\mathrm{V}^\star$ represents the virtual contribution involving one-loop amplitudes, and $d\mathrm{R}^\star$ the real contribution involving an extra final-state parton. Both $d\mathrm{V}^\star$ and $d\mathrm{R}^\star$ are suppressed by an extra strong coupling constant compared to $d\mathrm{B}^\star$. The functions $f^{(1)}, F^{(1)}$ in the second line of Eq. (11) are higher-order corrections to the PDFs, and carry an extra power of the coupling constant compared to the leading-order ones.

The loop integrals in $d\mathrm{V}^\star$ and the phase space integrals in $d\mathrm{R}^\star$ cause soft and collinear divergences. Not all of these cancel, a well-known phenomenon in collinear factorization, for which the remnant divergences are absorbed by what is denoted $f^{(1)}$ in Eq. (11). In the auxiliary parton approach, extra divergences appear, both in the virtual and the real contribution.

2.1 Virtual contribution

In [5] one-loop amplitudes were calculated for some processes by applying the auxiliary parton method to existing expressions for on-shell one-loop amplitudes. It was found that for all of the considered processes, the amplitudes could be decomposed into two parts, referred to as *familiar* and *unfamiliar*. The former is independent of the type of auxiliary partons used, and exhibits a smooth limit for $|k_\perp| \to 0$ to the on-shell amplitude. The latter *does* depend on the type of auxiliary partons, does not exhibit a smooth on-shell limit, but is universal for the (space-like gluon) process Eq. (5). At the cross section level, the unfamiliar contribution is given by

$$d\mathrm{V}^{\star\,\mathrm{unf}}\big(x_{in},k_\perp,\bar{x}_{\overline{in}}\,;\{p_i\}_{i=1}^n\big) = a_\epsilon N_c\,\mathrm{Re}\big(\mathcal{V}_{\mathrm{aux}}\big)\, d\mathrm{B}^\star\big(x_{in},k_\perp,\bar{x}_{\overline{in}}\,;\{p_i\}_{i=1}^n\big)\ , \tag{12}$$

with

$$a_\epsilon = \frac{\alpha_s}{2\pi}\,\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \quad , \quad \mathcal{V}_{\mathrm{aux}} = \left(\frac{\mu^2}{|k_\perp|^2}\right)^{\epsilon}\left[\frac{2}{\epsilon}\ln\frac{\Lambda}{x_{in}} - \mathrm{i}\pi + \bar{\mathcal{V}}_{\mathrm{aux}}\right] + \mathcal{O}(\epsilon) + \mathcal{O}\big(\Lambda^{-1}\big)\ , \tag{13}$$

and

$$\bar{\mathcal{V}}_{\text{aux-q}} = \frac{1}{\epsilon}\frac{13}{6} + \frac{\pi^2}{3} + \frac{80}{18} + \frac{1}{N_c^2}\left[\frac{1}{\epsilon^2} + \frac{3}{2}\frac{1}{\epsilon} + 4\right] - \frac{n_f}{N_c}\left[\frac{2}{3}\frac{1}{\epsilon} + \frac{10}{9}\right], \tag{14}$$

$$\bar{\mathcal{V}}_{\text{aux-g}} = -\frac{1}{\epsilon^2} + \frac{\pi^2}{3} \,. \tag{15}$$

Notably, this part contains a contribution proportional to $\ln\Lambda$ that is expected from known high-energy limits of amplitudes. In [4] a more general argument was given for the expression of this unfamiliar contribution using collinear limits of on-shell one-loop auxiliary parton amplitudes.

2.2 Real contribution

For the real radiation, one can also identify an "unfamiliar" contribution that is not taken into account if one just adds a final-state parton to the Born process and allows a parton to become soft and/or a pair to become collinear. The *unfamiliar* contribution can be recognized before $\Lambda \to \infty$ at tree-level, and concerns the situation when the extra radiative parton also scales with Λ and takes a shared "auxiliary role". Then taking $\Lambda \to \infty$ leads to a factorization formula for the matrix element given by, labelling the auxiliary and radiative partons with q and r,

$$\frac{1}{C_{\text{aux}}}\left|\overline{\mathcal{M}}^{\text{aux}}\right|^2\big((\Lambda + x_{in})P, k_{\overline{in}}\,; x_r\Lambda P + r_\perp + \bar{x}_r\bar{P}\,,\, x_q\Lambda P + q_\perp + \bar{x}_q\bar{P}\,,\, \{p_i\}_{i=1}^n\big)$$
$$\overset{\Lambda\to\infty}{\longrightarrow}\ \mathcal{Q}_{\text{aux}}(x_q, q_\perp, x_r, r_\perp)\,\frac{\Lambda^2\left|\overline{\mathcal{M}}^\star\right|^2\big(x_{in}P - q_\perp - r_\perp, k_{\overline{in}}\,; \{p_i\}_{i=1}^n\big)}{x_{in}^2|q_\perp + r_\perp|^2}\,, \tag{16}$$

with

$$\mathcal{Q}_{\text{aux}}(x_q, q_\perp, x_r, r_\perp) = x_q x_r\,\mathcal{P}_{\text{aux}}(x_q, x_r)\,|q_\perp + r_\perp|^2$$
$$\times\left[\frac{c_{\bar{q}}}{|q_\perp|^2|r_\perp|^2} + \frac{1}{x_r|q_\perp|^2 + x_q|r_\perp|^2 - x_q x_r|q_\perp + r_\perp|^2}\left(\frac{c_r\,x_r^2}{|r_\perp|^2} + \frac{c_q\,x_q^2}{|q_\perp|^2}\right)\right], \tag{17}$$

and where $\mathcal{P}_{\text{aux}}(x_q, x_r)$ is the usual collinear splitting function for the relevant partons with $x_q = 1 - x_r$. The color factors $c_r, c_q, c_{\bar{q}}$ depend on the type of auxiliary-and radiative partons. Also the phase space factorizes, and the unfamiliar contribution $d\mathrm{R}^{\star\,\text{unf}}$ can be calculated analytically. Obviously, it depends on the type of auxiliary partons, and also it is proportional to $\left(\mu^2/|k_\perp|^2\right)^\epsilon d\mathrm{B}^\star$ and produces a term proportional to $\ln\Lambda$.

2.3 Unfamiliar contribution

We do not present $d\mathrm{R}^{\star\,\text{unf}}$ separately, but instead immediately the combination

$$d\mathrm{R}^{\star\,\text{unf}} + d\mathrm{V}^{\star\,\text{unf}} = \Delta_{\text{unf}}\,d\mathrm{B}^\star\,, \tag{18}$$

where

$$\Delta_{\text{unf}} = \frac{a_\epsilon N_c}{\epsilon}\left(\frac{\mu^2}{|k_\perp|^2}\right)^\epsilon\left[\mathcal{J}_{\text{aux}} + \mathcal{J}_{\text{univ}} + \mathcal{J}_{\text{univ}} - 2\ln\frac{2P\cdot\bar{P}\,x_{in}}{|k_\perp|^2} + \frac{1}{\epsilon}\right], \tag{19}$$

with

$$\mathcal{I}_{\text{univ}} = \frac{11}{6} - \frac{n_f}{3N_c} - \frac{\mathcal{K}}{N_c}(-\epsilon) \quad \text{writing} \quad \mathcal{K} = N_c\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5n_f}{9} , \tag{20}$$

and

$$\mathcal{I}_{\text{aux-q}} = \frac{3}{2} - \frac{1}{2}(-\epsilon) \quad , \quad \mathcal{I}_{\text{aux-g}} = \frac{11}{6} + \frac{n_f}{3N_c^3} + \frac{n_f}{6N_c^3}(-\epsilon) . \tag{21}$$

We recognize that $\mathcal{I}_{\text{aux-q}} + \mathcal{I}_{\text{univ}}$ and $\mathcal{I}_{\text{aux-g}} + \mathcal{I}_{\text{univ}}$ are identical to the target impact factor corrections calculated in [6]. Our extra term $\mathcal{I}_{\text{univ}}$ is related to the renormalization of the coupling constant, and appears because our virtual result is not UV-subtracted. The logarithm in Eq. (19) is the $\mathcal{O}(\alpha_s)$ contribution to the space-like gluon Regge trajectory. The $1/\epsilon$ within square brackets finally is an artefact of the double counting in the real radiation contribution, of the phase space region when the radiative momentum becomes soft *and* collinear to P. We want it to be just present in the familiar contribution, to cancel the virtual soft-collinear divergence, and must be removed here. It turns out that following a phase space restriction in the relevant unfamiliar real contribution that was also employed in [6], avoiding the radiation to become collinear to P without becoming soft, removes exactly just this $1/\epsilon$.

2.4 Divergencies in the familiar real contribution

We remind the reader that the familiar virtual contribution is the one with the smooth on-shell limit, and that the familiar real contribution is the "naïve" one by adding a final-state parton to the Born process, after the limit $\Lambda \to \infty$, and allowing a parton to become soft and/or a pair to become collinear. It is not difficult to see that all IR-divergences from these contributions, both soft and final-state collinear, cancel like usually in the on-shell case. There is an on-shell initial-state in hybrid factorization, the one with momentum $k_{\overline{in}} = \bar{x}_{\overline{in}}\bar{P}$. The collinear divergence in the familiar real contribution associated with it, for example for a gluon becoming collinear to a gluon, is given by

$$-\frac{a_\epsilon}{\epsilon}\int_{\bar{x}_{\overline{in}}}^1 dz \left\{2N_c\left[\frac{1}{[1-z]_+} + \frac{1}{z} - 2 + z(1-z)\right]\right\}\frac{1}{z}\frac{f(\bar{x}_{\overline{in}}/z)}{f(\bar{x}_{\overline{in}})}\, d\mathrm{B}^\star(x_{in}, k_\perp, \bar{x}_{\overline{in}}) . \tag{22}$$

It is the same as in collinear factorization, and obviously cannot cancel against a virtual contribution, as is well-known. We divide by $f(\bar{x}_{\overline{in}})$ so we can write it as a contribution from $d\mathrm{R}^\star$ in Eq. (11).

Remains the other, space-like, initial state. At first glance, one might think that there is no collinear singularity because of the space-like gluon, but in fact there is a singularity when the radiation becomes collinear to the longitudinal component P. We have

$$\begin{aligned}|\overline{\mathcal{M}}^\star|^2&(x_{in}P + k_\perp, k_{\overline{in}}; r, \{p_i\}_{i=1}^n)\\ &\overset{r\to x_rP}{\longrightarrow} \frac{2N_c}{P\cdot r}\,\frac{x_{in}^2}{x_r(x_{in}-x_r)^2}\,|\overline{\mathcal{M}}^\star|^2\big((x_{in}-x_r)P + k_\perp, k_{\overline{in}}; \{p_i\}_{i=1}^n\big) ,\end{aligned} \tag{23}$$

and we recognize the usual collinear splitting formula, but with a splitting function that only consists of $2N_c/z/(1-z)$. This leads to a collinear divergence

$$-\frac{a_\epsilon}{\epsilon}\int_{x_{in}}^1 dz \left\{2N_c\left[\frac{1}{[1-z]_+} + \frac{1}{z}\right]\right\}\frac{1}{z}\frac{F(x_{in}/z, k_\perp)}{F(x_{in}, k_\perp)}\, d\mathrm{B}^\star(x_{in}, k_\perp, x_{in}) . \tag{24}$$

It is completely analogous to the one-shell side, but with the simplified splitting function.

2.5 Cross section at NLO

So eventually, we find that Eq. (11) can be cast in the form

$$d\sigma^{(1)} = \int dx_{in}\, d^2k_\perp\, d\bar{x}_{\overline{in}} \Big\{ F(x_{in},|k_\perp|)\, f(\bar{x}_{\overline{in}}) \Big[d\mathrm{V}^\star(x_{in},k_\perp,\bar{x}_{\overline{in}}) + d\mathrm{R}^\star(x_{in},k_\perp,\bar{x}_{\overline{in}}) \Big]_{\text{cancelling}}$$
$$+ \Big[F^{(1)}(x_{in},|k_\perp|) + \Delta^\star_{\text{coll}} + F(x_{in},|k_\perp|)\Delta_{\text{unf}} \Big] f(\bar{x}_{\overline{in}})\, d\mathrm{B}^\star(x_{in},k_\perp,\bar{x}_{\overline{in}}) \qquad (25)$$
$$+ \Big[f^{(1)}(\bar{x}_{\overline{in}}) + \Delta_{\overline{\text{coll}}} \Big] F(x_{in},|k_\perp|) d\mathrm{B}^\star(x_{in},k_\perp,\bar{x}_{\overline{in}}) \Big\} .$$

The first line of Eq. (25) is free of any divergences, and also independent of the type of auxiliary partons used. The third line is also independent, but contains the well-known divergence also appearing in collinear factorization, in the $\overline{\text{MS}}$ scheme given by

$$\Delta_{\overline{\text{coll}}} = -\frac{a_\epsilon}{\epsilon} \int_{\bar{x}_{\overline{in}}}^1 dz \left\{ \mathcal{P}^{\text{reg}}_{\overline{in}}(z) + \gamma_{\overline{in}}\delta(1-z) \right\} \frac{1}{z} f\!\left(\frac{\bar{x}_{\overline{in}}}{z}\right) \qquad (26)$$

where, $\mathcal{P}^{\text{reg}}_{\overline{in}}(z)$ is the equivalent for general splittings of what the function between curly brackets in Eq. (22) is for gluon to gluons splitting, and

$$\gamma_{\text{q}} = \frac{3}{2}\frac{N_{\text{c}}^2-1}{2N_{\text{c}}} \quad , \quad \gamma_{\text{g}} = \frac{11N_{\text{c}} - 2n_f}{6} . \qquad (27)$$

Strictly speaking, what is referred to as $\mathrm{B}^\star$ must include various processes both with initial-state gluons and light quarks as the on-shell initial-state partons. Regarding the space-like initial state, this is not necessary, and we can stick to a gluon because the singularity of Eq. (23) does not appear if a final-state quark becomes collinear to P.

The second line of Eq. (25) contains divergences that are different than the ones appearing in collinear factorization. Still similar is

$$\Delta^\star_{\text{coll}} = -\frac{a_\epsilon}{\epsilon} \int_{x_{in}}^1 dz \left\{ \frac{2N_{\text{c}}}{[1-z]_+} + \frac{2N_{\text{c}}}{z} + \gamma_{\text{g}}\delta(1-z) \right\} \frac{1}{z} F\!\left(\frac{x_{in}}{z},|k_\perp|\right) . \qquad (28)$$

Also appearing, however, is Δ_{unf} from Eq. (19).

3. Conclusion

In conclusion, while $\Delta^\star_{\text{coll}}$ could still be interpreted like in the collinear case and is to be absorbed by the PDF correction $F^{(1)}$, the appearance of Δ_{unf} goes beyond such an interpretation. The factorized form can still be maintained at NLO, but not anymore purely into PDFs and partonic cross section, and we see the high-energy type of factorization manifest itself in terms of impact factors and Green function. The auxiliary parton dependent color factor at LO must be interpreted as related to the target impact factor, and at NLO its non-trivial corrections appear.

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