PROCEEDINGS OF SCIENCE

ZU-TH 55/23

Singlet and anomaly contributions to massive QCD form factors

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Massive form factors are important building blocks in higher-order corrections to various observables like heavy-quark production, top quark decays or muon-electron scattering, where they describe the virtual contributions. Furthermore, they show universal infrared behaviour which makes them interesting to study also in the context of the infrared structure of QCD amplitudes. In these proceedings we present our recent calculation of singlet and anomaly contributions to the massive quark form factors, which is based on expansions around singular and regular points of the master integrals and numerical matching. This calculation completes the calculation of the massive form factors for external vector, axial-vector, scalar and pseudoscalar currents up to $O(\alpha_s^3)$.

16th International Symposium on Radiative Corrections: Applications of Quantum Field Theory to Phenomenology (RADCOR2023)
28th May - 2nd June, 2023
Crieff, Scotland, UK

*Speaker

PoS(RADCOR2023)045

1. Introduction

Massive form factors describe the virtual corrections to a plethora of physical observables like heavy-quark and lepton pair production, top quark decays, but also electron-muon scattering at low energies. The massive form factors are known fully analytically up to $O(\alpha_s^2)$ (see Refs. [1–7]) and have been calculated up to higher orders in the dimensional regulator in Refs. [8–11]. The contributions to the form factors are usually split into the so-called non-singlet and singlet contributions. From the point of Feynman graphs they correspond to graphs where the external current couples to the external quark or to an internal quark loop, respectively. Since the anomaly of the axial-vector current only vanishes when a complete fermion doublet is considered, one has to take into account singlet diagrams where the current couples to heavy and light quarks. At $O(\alpha_s^3)$ analytic results are available for the non-singlet process in the limit of a large number of colors [9, 12], for contributions involving massless fermion loop insertions [12–15] and partially for contributions involving massive fermion loop insertions [16, 17]. Form factors for massless external and massive internal quarks have also been calculated up to $O(\alpha_s^3)$ [18] and the massless form factors have recently be obtained at $O(\alpha_s^4)$ fully analytically [19].

In these proceedings we report on the calculation of massive form factors at $O(\alpha_s^3)$ and focus on the singlet and anomaly contributions published in Ref. [20]. The non-singlet contributions have been presented in Refs. [21, 22]. In Sec. 2 we will introduce the notation and technical details of the computation, while in Sec. 3 we will present a package which allows to compute the massive form factors numerically. In Sec. 4 we conclude.

2. Notation and technical details

We calculate the three-point functions with two external quarks coupled to either vector (v), axial-vector (a), scalar (s) or pseudoscalar (p) currents. These quantities can be decomposed into scalar form factors according to

$$\begin{aligned} \Gamma_\mu^v &= F_1^v(q^2)\gamma_\mu - \frac{\mathrm{i}}{2m}F_2^v(q^2)\sigma_{\mu\nu}q^\nu \,, & \Gamma_\mu^a &= F_1^a(q^2)\gamma_\mu\gamma_5 - \frac{1}{2m}F_2^v(q^2)q_\mu\gamma_5 \,, \\ \Gamma_\mu^s &= mF^s(q^2) \,, & \Gamma_\mu^p &= \mathrm{i}mF^p(q^2) \,. \end{aligned} \tag{1}$$

The momenta q_1 and q_2 belong to the incoming and outgoing quarks, which are on-shell ($q_1^2 = q_2^2 = m^2$). Additionally, $q = q_1 - q_2$ is the momentum of the current with $q^2 = s$ and $\sigma_{\mu\nu} = \frac{\mathrm{i}}{2}\left[\gamma_\mu, \gamma_\nu\right]$. In these proceedings we are focusing on the singlet contributions, therefore we decompose the form factors into

$$F^k = F^k_{\mathrm{non-singlet}} + n_h F^k_{\mathrm{sing},h} + n_l F^k_{\mathrm{sing},l}. \tag{2}$$

The subscript h or l distinguishes the cases where the current couples to an internal light or massive quark. Some examples of Feynman diagrams contributing to the singlet form factors are shown in Fig. 1. The form factors can be obtained by applying projectors. The amplitudes are obtained in an automated set-up utilizing the programs `qgraf` [23], `tapir` [24], `exp` [25, 26] and `FORM` [27] and the resulting scalar integrals are reduced to master integrals with the help of `Kira` [28–31] (see Ref. [20] for details).

While for the calculation of the non-singlet contributions the naive anticommuting γ_5 prescription can be used, this is not the case for the singlet contributions. To treat γ_5 in d-dimensions we use the prescription of Ref. [32] and replace

$$\gamma^\mu \gamma_5 \to \frac{\mathrm{i}}{3!} \epsilon^{\mu\nu\rho\sigma} \gamma_{[\nu}\gamma_\rho\gamma_{\sigma]} \,, \qquad \gamma_5 \to \frac{\mathrm{i}}{4!} \epsilon^{\mu\nu\rho\sigma} \gamma_{[\mu}\gamma_\nu\gamma_\rho\gamma_{\sigma]} \,, \tag{3}$$

in the definition of the currents and the projectors. This introduces two ϵ tensors which are contracted according to

$$\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}\epsilon_{\beta_1\beta_2\beta_3\beta_4} = \det\left(g_{\alpha_i\beta_j}\right) \,, \tag{4}$$

where the metric tensors on the right hand side have to be interpreted in d dimensions again. Since this treatment of γ_5 violates helicity conservation, an additional finite renormalization is necessary for the different currents. The necessary renormalization constants are known [32]. It is worth mentioning that the singlet contributions as introduced above do not renormalize multiplicative, but only the non-singlet and the sum of non-singlet and singlet do. Therefore, one needs to calculate also the non-singlet contribution using the same γ_5 prescription as for the singlet diagrams in order to renormalize the results correctly. Formally the non-singlet contributions in this γ_5 scheme are only needed to lower orders in α_s. As a welcome cross check we computed them to three-loop order and verified that we reproduce the results of Refs. [21, 22] after performing the appropriate finite renormalizations. For a more extensive discussion about the finite renormalization associated to the γ_5 scheme we refer to Ref. [20].

Another check on our treatment of γ_5 and the finite renormalizations is given by the chiral Ward identity [33]

$$(\partial^\mu j^a_\mu)_\mathrm{R} = 2(j^p)_\mathrm{R} + \frac{\alpha_s}{4\pi}(G\tilde{G})_\mathrm{R} \,, \tag{5}$$

which holds on the level of renormalized operators as indicated by the subscript R. It establishes a relation between the derivative of the axial-vector current, the pseudoscalar current and the pseudoscalar gluonic operator

$$G\tilde{G} = \epsilon_{\mu\nu\rho\sigma} G^{a,\mu\nu} G^{a,\rho\sigma} \,, \tag{6}$$

where $G^{a,\mu\nu}$ is the field strength tensor of the gluon. It can therefore be used to test the non-trivial renormalization of the singlet form factors in the presence of γ_5. In Ref. [6] it has already been used to test the renormaliztion of the massive form factors to $O(\alpha_s^2)$. As can be seen from Eq. (5) it required the calculation of the pseudoscalar gluon form factor at $O(\alpha_s)$. For the present calculation we extended the calculation to $O(\alpha_s^2)$ analytically in order to check the three-loop form factors. The analytical calculation utilized techniques of Ref. [15] to solve the system of differential equations without the need of a special basis of the master integrals. We used the packages `Sigma` [34, 35] `OreSys` [36] and `HarmonicSums` [37–50] for the implementation. The boundary conditions were fixed in the limit $s \to 0$ using the method of regions [51]. The analytic results can be found in Ref. [20].

The master integrals for the three-loop form factors are not solved analytically but with the method as presented in Ref. [52] and subsequently used to calculate the non-singlet contributions in Refs. [21, 22]. Let us outline the steps of the algorithm:

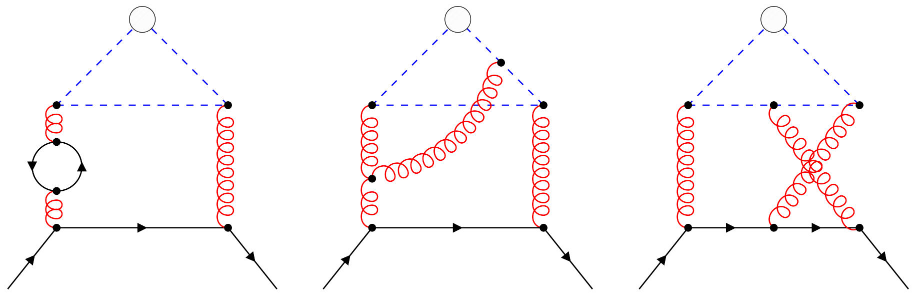

Figure 1: Sample Feynman diagrams contributing to the singlet parts of the massive quark form factors. Solid (black) lines denote the heavy quark, curly (red) lines denote gluons, dashed (blue) lines can represent either heavy or light quarks and the grey blob represents the interaction with the external current.

1. We calculate boundary conditions for the master integrals at a given point s_0.
2. We calculate symbolic expansions around the point s_0 by inserting a suitable ansatz into the system of differential equations of the master integrals. The system of linear equations we obtain by comparing coefficients in ϵ, $x = s - s_0$ and possibly $\ln(x)$ is solved with the help of `Kira` and `FireFly` [28–31] and the boundary constants can be fixed with the values from step 1.
3. We calculate a new symbolic expansion around s_1 and match it numerically to the expansion around s_0 in a point where both expansions converge e.g. $(s_1 + s_0)/2$.
4. We repeat steps 2 and 3 until the full physical phase space is mapped out with overlapping series expansions.

The boundary conditions for the singlet integrals are calculated analytically in the limit $s \to 0$ utilizing the method of regions for the singlet diagrams where the current couples to the massive quark line. In this limit the loop momenta can be either scale soft $k_i \sim \sqrt{s}$ or hard $k_i \sim \sqrt{m}$. Here only three out of four possible regions contribute due to the additional heavy-quark loop. In the case when the current couples to a massless quark all four regions can contribute and we were not able to determine all boundary conditions analytically. Instead we calculate the boundary conditions numerically using the program `AMFlow` [53] at the regular point $s/m^2 = -1$ and start the matching procedure from there. The subset of analytically computed boundary conditions at $s = 0$ are used as cross checks.

3. Numerical evaluation

We provide two ways to numerically evaluate the massive form factors numerically:

1. A standalone `Mathematica` package `formfactors3l` which evaluates the bare and finite (which means ultraviolet renormalized and infrared subtracted) form factors. It can be obtained from `https://gitlab.com/formfactors3l/formfactors3l`.

2. A Fortran library FF3l which evaluates the ultraviolet renormalized but infrared divergent form factors. It can be obtained from https://gitlab.com/formfactors3l/ff3l. The library can also be used in Mathematica using Wolfram's MathLink interface.

Let us show the usage of the Fortran library. After installing the library we can run the following program to obtain the $O(\epsilon^0)$ contributions to the ultraviolet renormalized form factor $F^{a}_{1,\text{sing},h}$ for $\frac{s}{m^2} \in (0, 80)$ (the function can provide values for all real values of s/m^2):

```
program axf1
  use ff3l
  implicit none
  double complex :: ff
  double precision :: s = 0.01
  integer :: eporder = 0, i

  call ff3l_nonsinglet_off()
  call ff3l_nlsinglet_off()
  call ff3l_nlsinglet_on()

  do i = 0,399
    s = s + 1.0D0 /5
    print *,"F1aNHsing0( s = ",s,") = ", ff3l_axF1(s,eporder)
  enddo
end program axf1
```

Note that the third-order corrections are expressed in terms of $\alpha_s^{(n_l+n_h)}(m)$ renormalized in the $\overline{\text{MS}}$ scheme, the mass is renormalized in the on-shell scheme and the default functions evaluate the QCD form factors setting the number of colors $N_C = 3$ and $n_l = 4$, $n_h = 1$. If the form factors for QED are needed the same functions supplemented by _qed are provided. The values provided by this script can be used to generate Fig 2. A more extensive documentation is available on the corresponding webpages.

4. Conclusions

In these proceedings we reported on the calculation of singlet and anomaly contributions to massive quark form factors at $O(\alpha_s^3)$. For the calculation of the three-loop master integrals we used a semi-analytic method which is based on series expansions in regular and singular points of the differential equation and numerical matching. A subset of initial values was calculated with the help of AMFlow. Compared to the non-singlet diagrams a more complicated renormalization due to the treatment of γ_5 on two different spin lines is necessary. We checked our implementation of the γ_5 prescription of Ref. [32] by explicitly checking the chiral Ward identity which relates the derivative of the axial-vector form factors, the pseudoscalar form factor and the pseudoscalar gluonic operator $G\tilde{G}$. For this check we calculated the latter quantity up to $O(\alpha_s^2)$ analytically. All results for the form factors are accessible in terms of a Mathematica and Fortran implementations.

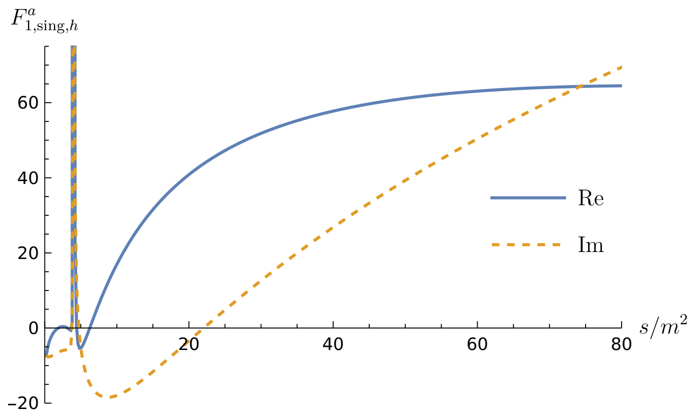

Figure 2: Plot of real and imaginary parts of the $O(\epsilon^0)$ contributions to the ultraviolet renormalized form factor $F^a_{1,\text{sing},h}$ for $\frac{s}{m^2} \in (0, 80)$.

The calculation of these contributions completes the full set of massive form factors at $O(\alpha_s^3)$ for one heavy quark flavor.

Acknowledgments

I want to thank Matteo Fael, Fabian Lange and Matthias Steinhauser for the fruitful collaboration on this project. This research was supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme grant agreement 101019620 (ERC Advanced Grant TOPUP). The Feynman diagrams were drawn with the help of `FeynGame` [54].

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