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Progress towards full next-to-leading logarithmic accuracy to scattering at high energy

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We discuss briefly the steps necessary to control all sources of next-to-leading logarithmic corrections to processes in the high energy limit.

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1. Introduction

The volume of data collected at the LHC now allows for detailed studies of both Higgs boson and electroweak boson production in association with two jets (called VBF and VBS respectively). These processes are of interest mainly because of their electroweak component, which offers a direct window to the electroweak symmetry breaking and the triboson couplings. However, the process $pp \rightarrow Vjj$ has contributions at both α_w^3 (VBS) and $\alpha_s^2\alpha_w$ (QCD) respectively, so a precise prediction of the QCD component is necessary in order to achieve a satisfactory determination of the VBS component from the measurement of both components combined (with the interference small[1–3]). However, the signal processes are quark initiated and peak at large invariant mass between the produced jets. In this region the QCD process receives large perturbative corrections $\alpha_s \log(s/pt^2)$ which slow the convergence of the perturbative series.

This observation motivates the systematic study of perturbative corrections of the form $\alpha_s \log(s/pt^2)$ to standard model processes within the framework of *High Energy Jets* (HEJ)[4–8]. Technically, this formalism achieves the same leading logarithmic accuracy as BFKL[9, 10], but achieves this while also maintaining gauge invariance (not just asymptotically) and crossing symmetry for each of the components of the high-energy factorised amplitude. Furthermore, since the formalism is formulated with a standard numerical phase space integral for each multiplicity (including emissions generated through the resummation), the results can be matched point by point in phase to tree-level accuracy for each jet multiplicity and the expansion to next-to-leading order of the amplitude allows for NLO matching of kinematic distributions[8].

2. Issues at next-to-leading logarithmic accuracy

The calculation of the NLL pieces of the BFKL kernel and impact factors (for an overview see [11] and references therein) addresses the accuracy of the *amplitudes* needed in the resummation. However, as pointed out in reference [12] and further elaborated in references [16–18], these NLL pieces are still insufficient to ensure overall NLL accuracy of the cross sections. The issue can be distilled down to the fact that the analytic resummation does not constrain the energy of the emissions needed to ensure one fixed overall $\sqrt{s}$. At NLL this becomes a problem since a change in the argument s of the leading logarithms induces NLL change etc. This problem is further illustrated by the fact that the calculation of the real emission corrections to each of the leading log pieces of the impact factor and the kernel itself involves an integral over the energies of the two-particle configurations with only the sum of the transverse momentum fixed, and is therefore itself not constrained to a single $\sqrt{s}$. These issues of phase space integrals relate to the calculation both of the impact factors and the central emission vertices, which in the analytic approach are expressed in terms of just transverse momenta (with unconstrained longitudinal momenta).

A final concern raised in reference[12] is the fact that within BFKL the rapidity integrals of the real emissions are correlated with logarithms of the virtual corrections by identifying $\Delta y \sim \log(s/t)$, which is correct only up to terms of order t/s so again generates differences of NLL order.

3. Resolution of these issues within High Energy Jets

The phase-space related issues can be resolved by performing the resummation using a Monte Carlo phase space generation for each multiplicity. This explicit generation of the phase space points allows for the correct evaluation of the centre of mass energy. Obviously, however, this requires the use of just the particle amplitudes, not the BFKL kernel or impact factors, since these at NLL contain a phase space integral over a range of energies (see e.g. [18]). Not just does that create problems for the logarithmic accuracy, but it also invalidates any jet clustering to form jet observables relevant for the LHC.

In order to obtain just the correct logarithmic accuracy one could then proceed just from the amplitudes derived in ref.[13] with appropriate regularisation and numerical integration. The approach taken in *High Energy Jets* is yet more ambitious and requires a new calculation of the components of the high energy factorisation of the amplitudes to repair a few undesirable properties of the amplitudes of ref.[13] and thus restore :-

a. **Gauge Invariance**: The amplitudes of ref.[13] are gauge invariant only in the strict limit of multi-regge kinematics, but there are sub-leading gauge-dependent terms. These terms are relevant in sub-leading regions and were studied using e.g. the *kinematic constraint* of ref.[14, 15].

   It is instead possible to derive a set of factorised amplitudes which are truly gauge invariant in the sense they fulfil the Ward identity for any choice of momentum for the emitted gluons. This is achieved by applying only a milder set of approximations in the calculations. For example the $1 \to 2$ impact factors are derived from the full $2 \to 3$ QCD amplitudes by applying approximations only to the set of momenta which are not part of the relevant impact factors. This approach is used in *HEJ*[4].

b. **Crossing symmetry**: Some of the essential analytic properties of the full scattering amplitudes are spoiled in the traditional approach. This includes e.g. crossing symmetry. This is easily seen since e.g. the impact factors depend only on the light-cone momentum of the incoming particles. If crossed to the final state these states therefore cannot regain any dependence on transverse momentum.

   The amplitudes used in *HEJ* respect full crossing symmetry in the impact factors (i.e. in the only part of the amplitude which has any states which can be crossed from the initial to final state).

As a by-product of respecting gauge invariance and crossing symmetry the new high energy approximation of the full amplitudes receives much smaller matching corrections when matched order-by-order to the exact full QCD amplitudes. Put different, they are a better approximant in the regions of phase space relevant for LHC dynamics, even if of course the asymptotic MRK limit agrees with the amplitudes derived in ref.[13]. The same is true for the full QCD amplitudes, even if of course they differ from their MRK limit in all off phase space.

4. First results incorporating NLL pieces

The first results with *HEJ* beyond pure leading-logarithmic accuracy were obtained in ref.[19, 20]. We will focus here on the results obtained the perturbative corrections in the process $pp \to Wjj$. Ref.[20] calculated the high-energy factorised components of the amplitude for all the new partonic channels appearing at NLL. For $pp \to Wjj$ this includes "impact factors" (technically four-vectors or currents in *HEJ*) for $q \to qgg^*$ and $q \to q'g(W \to)e\bar{\nu}g^*$ ($g \to q\bar{q}g^*$ and $g \to q\bar{q}'g(W \to)e\bar{\nu}g^*$ can be found through crossing). g^* denotes the off-shell t-channel gluon exchange, which will be contracted with another current. There currents are derived with no approximations applied in the momenta of the constituents of the "impact factor". This means it contains all the soft and collinear divergences between the partons involved from the full QCD amplitude of a process which can be built from these "impact factors". Since the virtual corrections are not yet implemented these pieces can be used only in their separate 2-jet phase space and therefore they start contributing in the 3-jet phase space (since at least one other jet is produced from the other impact factor in the event). Additionally, central emission vertices were derived for $g^*g^* \to q\bar{q}$ and $g^*g^* \to q\bar{q}'(W \to)e\bar{\nu}$. Again, the corresponding virtual corrections were not (yet) included, so these central pieces should be used only in their 2-jet configuration, and therefore they start contributing from a 4-jet process.

It had previously been observed that while the leading-logarithmic description (matched to full high-multiplicity Born level in HEJ) would describe well observables aligning themselves with the kinematics of the high energy limit[20]. However, observables as $\mathrm{d}\sigma/\mathrm{d}p_{\perp 1}$ were poorly described, since increasing $p_{\perp 1}$ is the opposite of the multi-regge-kinematic limit. As illustrated in figure 1, the inclusion of the NLL components described above is sufficient to repair the description of data in the regions described poorly with just leading logarithmic accurate results. The description with the combined NLL pieces and NLO matching (obtained by expanding the resummed calculation to NLO accuracy) is very good indeed.

Work continues to include all the relevant virtual corrections (which again will differ from the those of the standard high-energy approaches in order to satisfy e.g. crossing symmetry) such that a resummation at full NLL accuracy can be performed.

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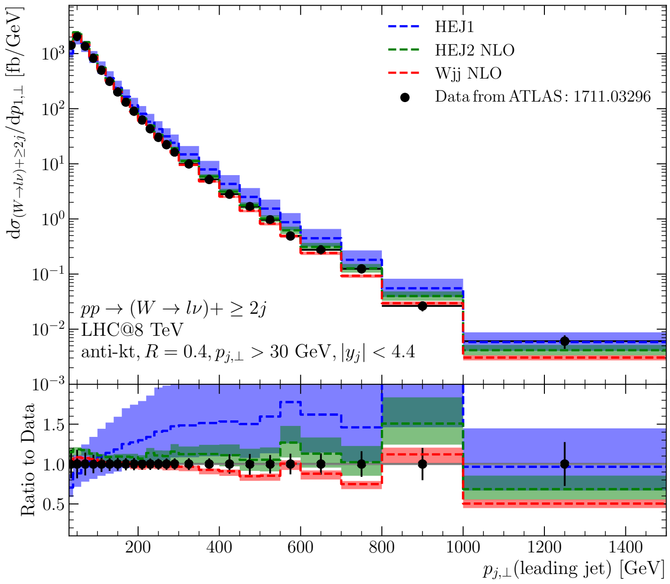

Figure 1: $d\sigma/dp_{\perp,1}$ for $pp \to Wjj$ for fixed NLO (red), high energy LL resummation and matching in (blue) and the result by including the NLL pieces described (green). The NLL components ensure a good description also away from the high energy limit. The NLO matching implemented in HEJ2 also reduced the scale variation (indicated by the size of the band) of the predictions compared to HEJ1, which was matched only at leading order. Figure taken from ref.[20].

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