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Radiative corrections to mechanical properties of bound states

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Recent work on radiative corrections to gravitational form factors of bound particles is reviewed. Gravitational and electromagnetic form factors are compared and contrasted. Application of asymptotic expansions of Feynman diagrams is advocated for future work on higher order radiative corrections to gravitational form factors.

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1. Introduction

It may come as a surprise that there is a connection between the scattering of gravitons and current medium-energy particle experiments. Gravitons are hypothetical quanta whose interaction with subatomic particles is at best feeble. However, they couple to the energy-momentum density and this quantity can be probed in contemporary experiments. For example, a recent measurement of charmonium photoproduction, a process shown in Fig. 1, was interpreted as a determination of the gluonic gravitational form factor of the proton [1, 2]. This and related experiments go beyond earlier electromagnetic probes of nucleons that studied charge and spin distributions. They provide access to new characteristics of hadrons such as the mass radius, in addition to the precisely determined charge radii [3, 4].

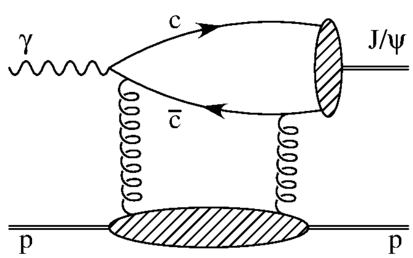

Figure 1: Production of J/ψ by a photon scattering on a proton. At least two gluons (shown with spring lines) are exchanged to ensure a color singlet $c\bar{c}$ pair. The lowest-order probability amplitude is quadratic in the gluon field strength, like the energy-momentum density.

Such experimental progress, together with future plans at the Electron-Ion Collider [5], motivate theoretical studies of gravitational form factors of bound states. Even though scattering of gravitons will not be experimentally accessible in the foreseeable future, it makes sense to consider it theoretically in order to understand the form factors related to the energy-momentum tensor density, to which gravitation couples. Such considerations are simpler in atoms than in hadrons because we can fully determine the atomic wave function. For this reason, we focus here on the simplest case, that of the hydrogen atom.

A recent paper [6] has determined $O(\alpha)$ corrections to the so-called D-term (see below); $\alpha \simeq 1/137$ is the fine-structure constant. That pioneering study analyzed in detail contributions to the D-term of effective field theory operators appearing in non-relativistic (NR) quantum electrodynamics (QED). In order to understand the significance of that result, we compare the structure of electromagnetic and gravitational couplings. Since spin does not play a role in Ref. [6], we neglect the spin of the electron and model the electron as a charged scalar particle. (The spin of the proton is of course also neglected. The proton is treated is a static source of the electric field.)

Electromagnetic and gravitational interactions of a charged scalar are illustrated in Fig. 2. We introduce average momentum P and the momentum transfer q, $P = \left(p_i + p_f\right)/2$, $q = p_f - p_i$. The structure of the interactions is

$$\left\langle p_f \left| j^\mu (0) \right| p_i \right\rangle \sim F\left(q^2\right) P^\mu \text{ (electromagnetic)}, \tag{1}$$

$$\left\langle p_f \left| T^{\mu\nu} (0) \right| p_i \right\rangle \sim A\left(q^2\right) P^\mu P^\nu + D\left(q^2\right)\left(q^\mu q^\nu - q^2 g^{\mu\nu}\right) \text{ (gravitational)}, \tag{2}$$

where j^μ is the electric charge current density and $T^{\mu\nu}$ is the energy-momentum tensor. The argument (0) refers to the position space and the possible tensor structures are limited by gauge invariance: contraction with the photon/graviton momentum q_μ should give zero if external scalar states satisfy the equation of motion.

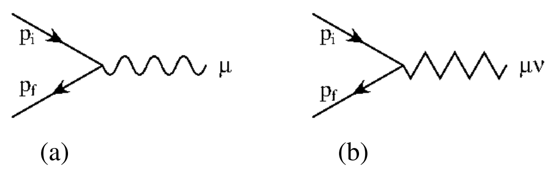

Figure 2: Coupling of a charged scalar particle to a photon (panel (a)) and to a graviton (panel (b)). Momenta of scalars and Lorentz indices of the photon and the graviton are shown next to particle lines. From now on gravitons are denoted by zigzag and photons by wavy lines.

Since the graviton is a spin-2 particle, it carries two indices and we can form more gauge-invariant structures than in the spin-1 photon case. Thus we have two form gravitational form factors but only one electromagnetic. Values of all form factors at zero momentum transfer describe some global properties probed by the photon and the graviton. Thus $F(0)$ is related to the electric charge and $A(0)$ to the mass. What does $D(0)$ correspond to? This is the so-called D-term. It turns out [7] that it can be related to the second moment of a distribution of a diagonal element $T^{ii}(r)$ that in macroscopic systems is interpreted as pressure; for a spherically-symmetric system,

$$D \sim \int_0^\infty \mathrm{d}r r^4 T^{ii}(r) \,. \tag{3}$$

The first moment of this distribution vanishes for a mechanically stable system; this is the so-called von Laue stability condition [8],

$$\int_0^\infty \mathrm{d}r r^2 T^{ii}(r) = 0. \tag{4}$$

This means that $T^{ii}(r)$ must change sign at least once at some distance from the origin. Comparison of these two equations (3) and (4) shows that D can have either sign, depending on whether negative T^{ii} contributions dominate at large or at small r [9]. The former case is realized in liquid droplets stabilized by the surface tension. The latter case includes atoms held together by tension between opposite charges of the nucleus and electrons. We note that the sign of D was controversial until recently [10, 11].

In the remainder of this work we focus on radiative corrections to the D term.

2. Scattering of gravitons off a free scalar and off a hydrogen atom

Full QED corrections to the graviton scattering were considered, for free electrons, in Refs. [12, 13]. For free scalars, more recent studies include Ref. [14, 15], where references to earlier works can be found.

In case of a bound particle, electromagnetic radiative correction to D have recently been determined in the one-loop order [6, 11]. The result of this study can be very well approximated by retaining only the logarithmically enhanced term,

$$D \simeq D_0 \left(1 + \frac{4\alpha}{3\pi} \ln \alpha\right), \tag{5}$$

where D_0 is the tree-level value. Given that even the one-loop evaluation was rather complicated, it is interesting to consider whether a simpler method is viable, so that in the future also higher-order loop corrections can be determined.

As pointed out in [6, 11], the coefficient of the logarithmic term in Eq. (5) is related to that of $\ln \frac{q^2}{m^2}$ in case of a free particle of mass m (see for example Eq. (17) in [14]). For a scalar, that infrared log arises from diagrams in Fig. 3.

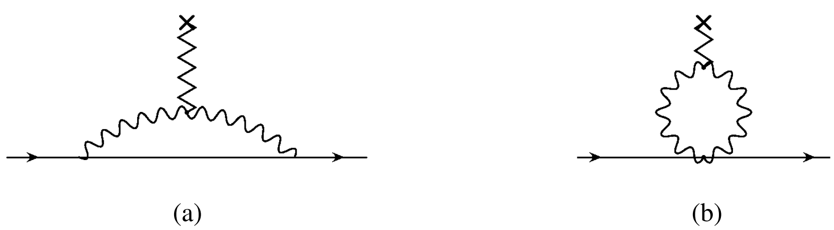

Figure 3: Diagrams responsible for the logarithmic enhancement of the radiative correction to the graviton-scalar scattering. The same diagram as in panel (a) contributes to the graviton-fermion scattering; panel (b) is specific to scalars.

Determination of such effects, especially at higher-loop orders, is greatly simplified with the method of asymptotic expansions [16]. If we are interested only in form factors at small momentum transfer q, diagram in Fig. 3(a) has two characteristic regions of the loop momentum k: hard $k \sim m$ and soft $k \sim q$. In the hard region we can expand all propagators in a Taylor series in the small momentum q. Propagator-type diagrams result. More interesting is the soft region. Here we expand the scalar propagator in terms quadratic in the loop momentum k and obtain an eikonal propagator $\sim k \cdot q$ [17].

The diagram in Fig. 3(b) has only one region, soft with $k \sim q$. All other diagrams, where the graviton couples to the scalar line or to any scalar-photon vertex, have only hard regions. In dimensional regularization, poles $1/\epsilon$ (ϵ parametrizes the deviation of the dimension from 4) multiply fractional powers of characteristic scales; even when the poles cancel, the cancellation involves regions with different characteristic scales and a logarithm of the ratio of those scales remains.

For a bound charged particle such as the electron in a hydrogen atom, examples of graviton scattering diagrams are shown in Fig. 4.

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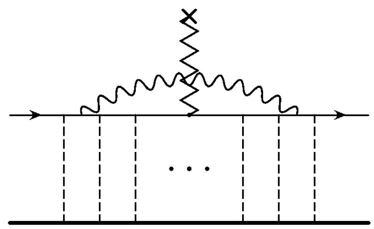

Figure 4: Graviton scattering off a hydrogen atom. The bottom thick line denotes the nucleus.

Here the charged particle is slightly off the mass shell due to the interaction with the Coulomb field. This shift prevents the infrared divergence. When we take the limit $q \to 0$, no logarithm involving q^2 appears but rather a logarithm of the particle virtuality $\sim (\alpha m)^2$ divided by the hard scale m^2; this is the origin of $\ln \alpha$ in Eq. 5.

What is the most efficient way to determine both logarithmic and non-logarithmic radiative corrections in such a bound state? The problem is more difficult than for example that of finding the Lamb shift [18] or the hyperfine splitting [19] because of the presence of the external gravitational field. However, bound states in external fields have already been successfully studied [20] in the case of the magnetic field, see Fig. 5.

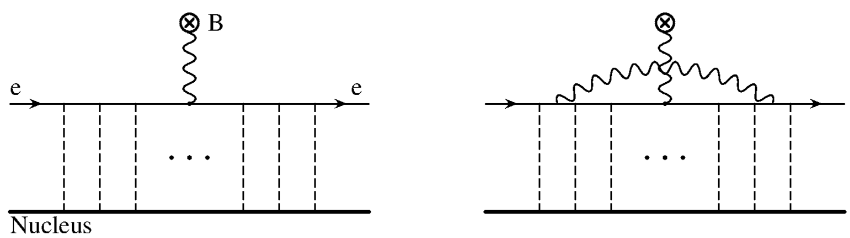

Figure 5: Hydrogen atom in an external magnetic field B.

Very likely, methods developed to determine the bound-electron gyromagnetic factor can be applied to the D-term as well. Determination of the logarithmically-enhanced part should be the first step and would provide a proof of principle.

3. Conclusions

In this contribution, we have reviewed the recent work related to the so-called D-term in gravitational form factors of a bound charged particle. Recent calculation of its one-loop radiative correction seems to have been very involved. We speculate that methods developed to characterize atoms in an external magnetic field can be applied to the gravitational field as well.

It seems that a calculation organized in this way would involve a smaller number of Feynman diagrams. Their evaluation would be simplified by examining characteristic regions of the loop momentum that contribute to the desired order in binding effects.

If this approach works, it should be possible to obtain two-loop corrections to the D-term, as has been done for the bound-electron gyromagnetic factor (g_{bound}) [21, 22].

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4. Appendix

This Appendix collects Feynman rules for graviton interactions needed to compute the logarithmically enhanced diagrams in Fig. 3.

4.1 Graviton-scalar vertex

The Lagrangian of a scalar field is

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2, \tag{6}$$

so its energy-momentum tensor is

$$\begin{aligned} T_\phi^{\mu\nu} &= \frac{\partial\mathcal{L}_\phi}{\partial\partial_\mu\phi}\partial^\nu\phi - \eta^{\mu\nu}\mathcal{L}_\phi & (7)\\ &= \partial^\mu\phi\partial^\nu\phi - \eta^{\mu\nu}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2\right) & (8)\\ &= \frac{1}{2}\left[\partial^\mu\phi\partial^\nu\phi + \partial^\nu\phi\partial^\mu\phi - \eta^{\mu\nu}\left(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right)\right]. & (9)\end{aligned}$$

The resulting Feynman rule for the graviton-scalar vertex is

$$\frac{i\kappa}{2}\left[-p_1^\mu p_2^\nu - p_1^\nu p_2^\mu + \eta^{\mu\nu}\left(p_1\cdot p_2 + m^2\right)\right]. \tag{10}$$

4.2 Graviton-photon vertex

For the electromagnetic field,

$$\begin{aligned} T_A^{\mu\nu} &= F^\mu{}_\alpha F^{\alpha\nu} - \frac{\eta^{\mu\nu}}{4}F_{\beta\alpha}F^{\alpha\beta} & (11)\\ &= \frac{1}{2}\left(F^\mu{}_\alpha F^{\alpha\nu} + F^\nu{}_\alpha F^{\alpha\mu} - \frac{\eta^{\mu\nu}}{2}F_{\beta\alpha}F^{\alpha\beta}\right) & (12)\\ &= \frac{1}{2}\left(\delta^\mu_\rho\delta^\nu_\beta + \delta^\nu_\rho\delta^\mu_\beta - \frac{\eta^{\mu\nu}\eta_{\rho\beta}}{2}\right)\eta_{\sigma\alpha}F^{\rho\sigma}F^{\alpha\beta} & (13)\\ &= \frac{1}{2}\left(\delta^\mu_\rho\delta^\nu_\beta + \delta^\nu_\rho\delta^\mu_\beta - \frac{\eta^{\mu\nu}\eta_{\rho\beta}}{2}\right)\eta_{\sigma\alpha}\left(\partial^\rho A^\sigma - \partial^\sigma A^\rho\right)\left(\partial^\alpha A^\beta - \partial^\beta A^\alpha\right). & (14)\end{aligned}$$

The Feynman rule for the graviton-photon vertex is, with μ, ν denoting the graviton indices, and with photons characterized by the momenta and indices p_1 and ρ and p_2 and σ,

$$i\kappa\big(- g^{\mu\nu} g^{\rho\sigma} p_1 \cdot p_2 + g^{\mu\nu} p_1^\sigma p_2^\rho + g^{\mu\rho} g^{\nu\sigma} p_1 \cdot p_2 - g^{\mu\rho} p_1^\sigma p_2^\nu \tag{15}$$

$$+ g^{\mu\sigma} g^{\nu\rho} p_1 \cdot p_2 - g^{\mu\sigma} p_1^\nu p_2^\rho - g^{\nu\rho} p_1^\sigma p_2^\mu - g^{\nu\sigma} p_1^\mu p_2^\rho \tag{16}$$

$$+ g^{\rho\sigma} p_1^\mu p_2^\nu + g^{\rho\sigma} p_1^\nu p_2^\mu\big). \tag{17}$$

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