PROCEEDINGS OF SCIENCE

Two-loop QCD and QED corrections to light-by-light scattering

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In this talk we discuss the analytic results for two-loop QCD as well as QED corrections to light-by-light scattering including contributions due to massive internal fermions.

16th International Symposium on Radiative Corrections: Applications of Quantum Field Theory to Phenomenology (RADCOR2023)
28th May - 2nd June, 2023
Crieff, Scotland, UK

*Speaker

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1. Introduction & Motivation

The study of high-energy photon-photon processes in e^+e^- and e-p collisions has a history of over thirty years. In the last two decades, the advent of the Large Hadron Collider (LHC) has propelled research in this domain, thanks to its extended center-of-mass energies. The LHC's multi-TeV energies and high-luminosity beams, capable of accelerating heavy ions with charges up to $Z = 82$ for lead (Pb) ions, have led to novel measurements of $\gamma\gamma$-collisions in ultraperipheral collisions (UPCs) involving proton-proton (p-p), proton-nucleus (p-A), and nucleus-nucleus (A-A) interactions [1]. One of the fundamental processes contributing only through quantum corrections is light-by-light scattering. At the leading order, the cross section is of $O(\alpha^4)$ through one-loop box Feynman diagrams. At the next-to-leading order, we need to calculate two-loop box diagrams.

This article discusses the analytic computation of higher-order corrections to light-by-light scattering in Pb-Pb collisions. The electromagnetic field of any charged particle accelerated at high energies can be approximated in the equivalent photon approximation (EPA) [2] as a flux of quasi-real photons whose intensity is proportional to the square of the electric charge, Z^2. This Z^2 photon-flux boost results in $\gamma\gamma$ cross sections being significantly enhanced, by factors of up to $Z^4 \approx 50 \times 10^6$ for Pb-Pb, with respect to p-p. Among various photon-fusion processes [3], we focus on $\gamma\gamma \to \gamma\gamma$. This process has recently been observed by ATLAS [4]. It is intriguing not only in the Standard Model (SM) but also in probing physics beyond the SM.

So far, theoretical studies of photon-photon physics in UPCs at RHIC, LHC, and FCC have primarily used dedicated Monte Carlo (MC) event generators, with a subset of selected physical processes previously coded at leading-order (LO) accuracy. The automated event generation of arbitrary exclusive final states produced via photon fusion in UPCs of protons and/or nuclei, $AB \to A\,\mathrm{X}\,B$, has been implemented in the GAMMA-UPC code [1]. Cross sections can be calculated in the EPA using γ fluxes derived both from electric dipole and charge form factors. In addition, the code has hadronic survival probabilities. In the EPA framework, the exclusive production cross section of a final state X via photon fusion, in a $\gamma\gamma$ UPC of hadrons A and B, factorizes into a product of the elementary cross section at a given centre of mass energy, $\sigma_{\gamma\gamma\to X}(W_{\gamma\gamma})$, convolved with the two-photon differential distribution of the colliding beams. The elementary cross section consists solely of virtual corrections, and this talk primarily focuses on their analytic computation.

2. Two-loop amplitudes for light-by-light scattering

We calculate the two-loop amplitudes for light-by-light scattering analytically, including massive corrections arising from the internal propagators. Two-loop QCD and QED corrections were previously studied in the ultra-relativistic limit [5], where they were found to be small in that limit. Light-by-light scattering was also investigated in supersymmetric QED [6]. The two-loop integrals for these corrections with massive internal propagators were studied in [7], revealing the complications in analytic computations due to the presence of massive propagators.

The process of interest can be expressed as:

$$\gamma(p_1, \lambda_1) + \gamma(p_2, \lambda_2) + \gamma(p_3, \lambda_3) + \gamma(p_4, \lambda_4) \to 0, \tag{1}$$

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where all the momenta p_i of the photons are incoming, and their helicities are denoted as λ_i. The amplitude is expressed as:

$$\mathcal{M} = \left(\prod_{i=1}^{4} \varepsilon_{\lambda_i,\mu_i}(p_i) \right) \mathcal{M}^{\mu_1\mu_2\mu_3\mu_4}(p_1, p_2, p_3, p_4). \tag{2}$$

The Lorentz decomposition of the scattering tensor $\mathcal{M}^{\mu_1\mu_2\mu_3\mu_4}$ is given as (cf. eq.(3.3) in [6]):

$$\begin{aligned}\mathcal{M}^{\mu_1\mu_2\mu_3\mu_4} =& A_1 g^{\mu_1\mu_2} g^{\mu_3\mu_4} + A_2 g^{\mu_1\mu_3} g^{\mu_2\mu_4} + A_3 g^{\mu_1\mu_4} g^{\mu_2\mu_3} \\ &+ \sum_{j_1,j_2=1}^{3} (B^1_{j_1 j_2} g^{\mu_1\mu_2} p^{\mu_3}_{j_1} p^{\mu_4}_{j_2} + B^2_{j_1 j_2} g^{\mu_1\mu_3} p^{\mu_2}_{j_1} p^{\mu_4}_{j_2} + B^3_{j_1 j_2} g^{\mu_1\mu_4} p^{\mu_2}_{j_1} p^{\mu_3}_{j_2} \\ &+ B^4_{j_1 j_2} g^{\mu_2\mu_3} p^{\mu_1}_{j_1} p^{\mu_4}_{j_2} + B^5_{j_1 j_2} g^{\mu_2\mu_4} p^{\mu_1}_{j_1} p^{\mu_3}_{j_2} + B^6_{j_1 j_2} g^{\mu_3\mu_4} p^{\mu_1}_{j_1} p^{\mu_2}_{j_2}) \\ &+ \sum_{j_1,j_2,j_3,j_4=1}^{3} C_{j_1 j_2 j_3 j_4} p^{\mu_1}_{j_1} p^{\mu_2}_{j_2} p^{\mu_3}_{j_3} p^{\mu_4}_{j_4}. \end{aligned} \tag{3}$$

The coefficients A_i, B^i_{jk}, and C_{ijkl} are functions of the Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_2 + p_3)^2$, and $u = (p_1 + p_3)^2$, as well as the masses of the particles in the loops. Thanks to parity invariance in QCD and QED, the term proportional to the Levi-Civita tensor is prohibited. Equation (3) comprises 138 independent functions, which can be related through transversity, Bose symmetry, and Ward identities resulting from gauge symmetry, reducing the number of independent functions to 3 [6].

Upon obtaining the independent form factors, we represent all the involved integrals in terms of master integrals using integration by parts identities (IBPs). For the two-loop corrections, we generated 60 diagrams through QGRAF [8] and FEYNARTS [9]. Prior to applying the IBPs, we identified 7798 integrals in the amplitudes. Through the use of IBPs, we subsequently express all the diagrams as 18 top-level topologies, as shown in Figure 1.

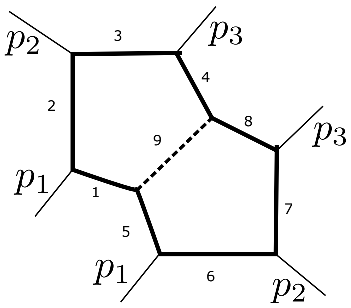

Figure 1: Top-level diagram sufficient to generate all the families needed for the NLO corrections to light-by-light scattering. Thin lines represent massless particles whereas solid lines represent massive internal propagators.

3. Analytic computation of master integrals

In this talk, we discuss the analytical computation of scattering amplitudes, focusing on NLO QCD and QED corrections for light-by-light scattering. Our approach involves expressing the

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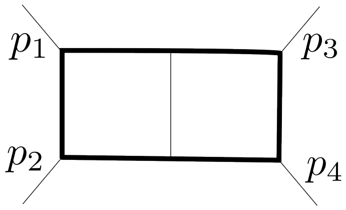

Figure 2: Two-loop top-sector diagram appearing in the scattering amplitudes. Thin lines represent massless particles whereas solid lines represent massive internal propagators.

results of the master integrals in a format suitable for phenomenological applications. To start, we define a set of two-loop Feynman integrals using the following expression:

$$I_{a_1,\cdots,a_9} = \left(\frac{e^{\epsilon\gamma_E}}{i\pi^{\frac{d}{2}}}\right)^2 \int \prod_{i=1}^{2} d^d k_i \frac{D_8^{a_8} D_9^{a_9}}{D_1^{a_1} D_2^{a_2} D_3^{a_3} D_4^{a_4} D_5^{a_5} D_6^{a_6} D_7^{a_7}}. \tag{4}$$

Here, $d = 4 - 2\epsilon$ represents the space-time dimension, and $a_j \in \mathbb{Z}$ are integer parameters. The loop momenta are denoted as k_i, the Euler-Mascheroni constant as γ_E, and the inverse propagators as $D_j = q_j^2 - m_j^2 + i0^+$, where q_j and m_j respectively denote momentum and mass. The integral family is constructed using a set of inverse propagators:

$$\begin{aligned}&\big\{k_1^2 - m^2, (k_1 + p_1)^2 - m^2, (k_1 + p_1 + p_2)^2 - m^2, (k_2 - k_1)^2, (k_2 - p_1 - p_2)^2 - m^2,\\ &(k_2)^2 - m^2, (k_2 + p_1 + p_2 + p_3)^2 - m^2, (k_1 + p_1 + p_2 + p_3)^2, (k_2 + p_1)^2\big\}\,,\end{aligned} \tag{5}$$

In this set, m represents the mass of the quarks that appears in the loop. We established a system of IBPs using KIRA [10] and LiteRed [11] (as implemented in FiniteFlow) [12], resulting in 103 master integrals. Among these 103 master integrals, only 30 need to be calculated, while the rest can be casted into these 30 integrals. One of the 30 integrals is a box one-loop integral times a tadpole one-loop integral, which can be easily computed. Our next task is to evaluate the remaining 29 master integrals. To derive the analytic expressions for these master integrals, we employed the method of differential equations. Given the presence of massive propagators, it is inevitable to encounter square roots. The square roots we encountered in our context include $\sqrt{s(s-4m^2)}$, $\sqrt{t(t-4m^2)}$, $\sqrt{st(st-4m^2(s+t))}$, and $\sqrt{s(m^4 s - 2m^2 t(s+2t) + st^2)}$.

For simplicity, we chose to set m^2 to 1, creating a two-variable system of differential equations for us to solve. Our choice of a canonical basis is based on [7]. To simplify our expressions, we simultaneously rationalized the first three square roots by selecting a specific set of variables, as detailed in [7]

$$\begin{aligned} s &= \frac{-4(w-z)^2}{(1-w^2)(1-z^2)},\\ t &= \frac{-(w-z)^2}{(wz)}. \end{aligned} \tag{6}$$

However the square root r, given by $\sqrt{-2wz + z^2 + w^4 z^2 - 2w^3 z^3 + w^2(1 + z^2 + z^4)}$, cannot be rationalized further. Nevertheless, we present the analytic results in terms of iterated integrals

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with polylogarithmic kernels, with explicit representation in terms of Goncharov's polylogarithms wherever possible.

For the analytic representation of our scattering amplitudes we use Chen's iterated integrals. Chen's iterated integrals are defined as follows. Let $\mathbb{M}$ be an n-dimensional manifold, where n is the number of coordinates and ω's be the one-forms. Let there be a path γ on $\mathbb{M}$, $\gamma : [0,1] \to \mathbb{M}$, where $\gamma(0)$ is the starting point and $\gamma(1)$ is the end point. Let the pullbacks of the one-forms to the interval $[0,1]$ be given as $f_j(\lambda)d\lambda = \gamma^*\omega_j$. The k-fold iterated integral over the one-forms is defined as

$$\begin{aligned} I_\gamma(\omega_1,...,\omega_k;\lambda) &= \int_0^\lambda d\lambda_1 f_1(\lambda_1) \int_0^{\lambda_1} d\lambda_2 f_2(\lambda_2) ... \int_0^{\lambda_{k-1}} d\lambda_k f_k(\lambda_k) \\ &= \int_0 d\lambda_1 f_1(\lambda_1) I_\gamma(\omega_2,...,\omega_k;\lambda_1), \end{aligned} \tag{7}$$

with $I_\gamma(;\lambda) = 1$ [13]. Feynman integrals often give rise to homotopy-invariant one-forms ω's, which are expressible in the form dlog $p_i(x_1,..,x_m)$ where $x_1,...,x_m$ are the coordinates and $p_i(x_1,...,x_m)$ is a rational or an algebraic function. Multiple polylogarithms (MPLs) are special cases of iterated integrals where p_i is rational and linear-reducible in the variables x_j. In this work, the iterated integrals are expressed in terms of dlog-forms with algebraic dependence on the coordinates x_i in the form of the square root r. To bring the canonical differemtial equation to a dlog-form we match them against a suitable ansatz and simplify the letters, as explained in [14]. The entries of the differential equation matrix are $\mathbb{Q}$-linear combination of dlog-forms of 15 letters. The scattering amplitudes is then expressed in a very compact way using these analytic representations.

For numerical evaluation of our analytic results, several methods are at our disposal. The iterated integral representation can be numerically integrated by expanding the one-forms along specific one-dimensional paths, as guided in [16]. It's worth noting that we can express all but seven integrals in terms of MPLs, and we maintain this notation wherever feasible. For the numerical evaluation of the master integrals that aren't represented in the MPLs, we employ a technique known as the "ibp trick," elucidated in the following references [7][17]. The fundamental concept behind this approach is the conversion of the last two-fold integrations into one-fold integrals. Additionally, we express the results of the first two orders of integration by matching symbols in both the w, z as well as s, t coordinate systems, in terms of logarithms and classical polylogarithms, following the methods outlined in [15]. The next task is to evaluate the whole analytic system in all the phase-space regions of interest. For us, the physical phase-space region of interested are:

$$\begin{aligned} &s < 0,\ t < 0,\ u > 0 \\ &s > 0,\ t < 0,\ u < 0 \\ &s < 0,\ t > 0,\ u < 0. \end{aligned} \tag{8}$$

For numerical integration in all these regions, we employ a technique more suitable for phenomenological applications. Rather than integrating our results within a single region and then relying on analytic continuation to cross the boundaries into other regions, we obtain distinct analytic results that are specifically valid within each of these regions. Consequently, our numerical evaluations of the integrals are tailored to the specific phase-space region. We also obtain analytic boundary constants in all these regions.

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4. Conclusions

In conclusion, we have discussed the two-loop QCD and QED corrections to light-by-light scattering in ultraperipheral Pb-Pb collisions. These corrections are essential for a precise theoretical understanding of $\gamma\gamma \to \gamma\gamma$, which have gained prominence in the context of modern high-energy physics experiments. We have provided insights into the analytic computation of these corrections, emphasizing the challenges posed by massive internal propagators. The results of this study will contribute to our knowledge of the theoretical framework underlying $\gamma\gamma$ interactions in ultraperipheral heavy-ion collisions and further our understanding of physics beyond the SM.

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