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Origin of mass scales in scale-symmetric extension of Standard Model

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Quantum corrections in ultraviolet drive Higgs boson mass to higher values than we measure in experiments. This is the so called hierarchy problem. One can solve it by imposing conformal or scale symmetry. Nevertheless, Higgs mass must be somehow generated, which may be obtained by spontaneous scale symmetry breaking. We present a scale-symmetric extension of SM Higgs scalar sector in which mass scales are not generated by radiative corrections, but spontaneous breaking of scale symmetry when its Goldstone (dilaton) acquires a vacuum expectation value. We present how this vev can be reached via evolution of the fields in hot Universe and we show that coupling this theory to gravity stabilizes the solution.

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1. Introduction

One of the unresolved problems posed by the Standard Model is the so called- hierarchy problem. Including quantum corrections causes rise of the Higgs mass m_H and one needs to set up very precise cancellations between a priori unrelated contributions to the effective potential of the scalar sector in order to maintain m_H hierarchically lower than the Planck mass scale M_P. Avoiding this issue is possible in theories with scale symmetry, where all mass scales, especially m_H, are set to zero. Then, scale symmetry can be broken, e.g. spontaneously or explicitly, and therefore, mass scales can be generated dynamically.

Proposals of scale symmetric extensions of the SM have been made already in the past. One possibility is to add a scalar singlet and couple it to gravity [1–15]. This new scalar, dilaton ϕ_0, is then responsible for generating all mass scales in the model and scale symmetry is broken spontaneously by the dilaton vacuum expectation value. We do not consider nature of quantum corrections in presented model since scale symmetry can be maintained at the quantum level, when renormalization scale is a function of the field $\mu = \mu(\phi_0)$. What is more, such idea, proposed e.g. in [16–21], leads to small corrections to m_H and additional fine tuning to maintain small Higgs mass is not required.

To simulate field time evolution in expanding hot Universe, we study temperature corrections to classically scale invariant potential for the Higgs and dilaton. Since temperature is a mass scale, it breaks scale symmetry explicitly. Such analysis can show, how SSB of scale symmetry can occur and how dilaton settles at its vev to generate mass scales.

Extended version of the following work can be found on arXiv [25]. Here, we present only the main aspects of our analysis.

2. Scale symmetric extension of a Higgs scalar sector and origin of mass scales

Let us consider a model with two scalar fields coupled to gravity: ϕ_0 singlet, which we will call dilaton and which is a new sector coupled to the Higgs neutral component ϕ_1. We consider a Lagrangian:

$$\frac{\mathcal{L}}{\sqrt{g}} = -\frac{1}{12}\left(\xi_0\phi_0^2 + \xi_1\phi_1^2\right)R + \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 + \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - V(\phi_0,\phi_1), \tag{1}$$

where R is Ricci scalar and ξ_i are non-minimal couplings. In scale symmetric scenario, the Higgs mass parameter is vanishing, $m_H^2 = 0$, so tree-level potential takes form:

$$V(\phi_0,\phi_1) = \lambda_0\phi_0^4 + \lambda_1\phi_0^2\phi_1^2 + \lambda_2\phi_1^4. \tag{2}$$

The couplings are dimensionless and fulfill certain hierarchy:

$$\lambda_2 \gg |\lambda_1| \gg \lambda_0, \qquad \lambda_2 > 0, \quad \lambda_1 < 0, \quad \lambda_0 > 0, \tag{3}$$

so the Higgs quartic coupling is the strongest and the new dilaton sector is weakly coupled to the Higgs scalar.

We choose FLRW metric $(1, -a(t)^2, -a(t)^2, -a(t)^2)$ with $\sqrt{g} = \sqrt{|\det g|} = a(t)^3$. Equations of motion from (1) for each field and $g_{\mu\nu}$ are:

$$\begin{aligned}
\phi_0: &\quad \ddot{\phi_0} + 3H\dot{\phi_0} + \frac{\xi_0}{6}\phi_0 R + 4\lambda_0\phi_0^3 + 2\lambda_1\phi_0\phi_1^2 = 0 \\
\phi_1: &\quad \ddot{\phi_1} + 3H\dot{\phi_1} + \frac{\xi_1}{6}\phi_1 R + 4\lambda_2\phi_1^3 + 2\lambda_1\phi_0^2\phi_1 = 0 \\
g_{\mu\nu}: &\quad \frac{1}{12}\left(\xi_0\phi_0^2 + \xi_1\phi_1^2\right)R - \frac{1}{2}\dot{\phi_0}^2 - \frac{1}{2}\dot{\phi_1}^2 + 2\left(\lambda_0\phi_0^4 + \lambda_1\phi_0^2\phi_1^2 + \lambda_2\phi_1^4\right) = 0
\end{aligned} \tag{4}$$

where $H = \frac{\dot{a}}{a}$ is Hubble parameter. Adding the condition for zero cosmological constant at the ground state:

$$V(\langle\phi_0\rangle, \langle\phi_1\rangle) = 0.$$

we obtain stationary solutions with a flat direction:

$$\langle\phi_1^2\rangle = -\frac{\lambda_1}{2\lambda_2}\langle\phi_0^2\rangle, \qquad \lambda_0 = \frac{\lambda_1^2}{4\lambda_2}, \qquad \langle R\rangle = 0. \tag{5}$$

The mass matrix:

$$M^2 = \begin{pmatrix} \lambda_1\left(2\phi_1^2 + \frac{3\lambda_1}{\lambda_2}\phi_0^2\right) & 4\lambda_1\phi_1\phi_0 \\ 4\lambda_1\phi_1\phi_0 & 2\left(6\lambda_2\phi_1^2 + \lambda_1\phi_0^2\right) \end{pmatrix}, \tag{6}$$

has two eigenvalues (at the ground state):

$$m_G^2 = 0, \qquad m_H^2 = -4\lambda_1\left(1 - \frac{\lambda_1}{2\lambda_2}\right)\langle\phi_0^2\rangle, \tag{7}$$

so one has massless Goldstone associated with scale symmetry and a massive "Higgs".

One more mass scale should appear in this model: Planck mass M_P. $\xi_i\phi_i^2 R$ term can be compared to the Einstein-Hilbert term:

$$-\frac{1}{12}\left(\xi_0\phi_0^2 + \xi_1\phi_1^2\right)R \quad \Longleftrightarrow \quad -\frac{1}{2}M_P^2 R. \tag{8}$$

Then ϕ_i fields, especially at the ground state, play a role of Planck mass:

$$\frac{1}{6}\left(\xi_0\phi_0^2 + \xi_1\phi_1^2\right) \xrightarrow{\text{ground state}} \frac{1}{6}\left(\xi_0 - \frac{\lambda_1}{2\lambda_2}\xi_1\right)\langle\phi_0^2\rangle = M_P^2 \tag{9}$$

With a scale symmetric theory, only ratios of mass scales can be determined and $\langle\phi_0\rangle$ is arbitrary. The scale symmetry is broken when ϕ_0 acquires its vev and flat direction no longer exists. Because Higgs vev $\langle\phi_1\rangle$, Higgs mass m_H^2 and Planck mass M_P^2 are proportional to $\langle\phi_0\rangle$, so dilaton generates all mass scales.

2.1 Higgs potential parameters and Planck mass:

To limit possible values of λ_1, λ_2 we can use conditions: hierarchy (3), λ_0 relation from (5) and the exact numerical values for the Higgs mass and Higgs vev:

$$m_H^2 = (125 \text{ GeV})^2, \qquad \langle\phi_1\rangle = 250 \text{ GeV}. \tag{10}$$

Having:

$$m_H^2 = -4\lambda_1\Big(1 - \frac{\lambda_1}{2\lambda_2}\Big)\langle\phi_0^2\rangle, \quad \langle\phi_1^2\rangle = -\frac{\lambda_1}{2\lambda_2}\langle\phi_0^2\rangle. \tag{11}$$

we get a relation:

$$\lambda_2 = \frac{1}{32}\Big(1 + 16\lambda_1\Big), \qquad -\frac{1}{48} \le \lambda_1 \le 0 \tag{12}$$

with example values:

$$\lambda_2(\lambda_1 = -10^{-6}) \approx \lambda_2(\lambda_1 = -10^{-11}) \approx 0.03125, \tag{13}$$

and dilaton vev $\langle\phi_0\rangle$:

$$\langle\phi_0^2\rangle = -\frac{2\lambda_2}{\lambda_1}\langle\phi_1^2\rangle = -\frac{2\lambda_2}{\lambda_1}\cdot(250\text{ GeV})^2. \tag{14}$$

To force constraints on non-minimal couplings ξ_i we use (9):

$$\frac{1}{6}\left(\xi_0 - \frac{\lambda_1}{2\lambda_2}\xi_1\right)\langle\phi_0^2\rangle = M_P^2. \tag{15}$$

There are justifications, [**?**], that ξ_0 should be much stronger than ξ_1 ($\xi_1 \ll \xi_0$). Combining (12), (14) and (15) we obtain λ_1, ξ_0, ξ_1 relation:

$$\lambda_1 = \frac{-0.0625\cdot\xi_0}{\xi_0 - \xi_1 + 1.43\cdot 10^{34}}, \tag{16}$$

with example values:

$$\begin{aligned}
\xi_0 = 10^5, \quad \xi_1 = 0.1 \quad &\Rightarrow \quad \lambda_1 = -4.37\cdot 10^{-31}, \\
\xi_0 = 10^{10}, \quad \xi_1 = 0.1 \quad &\Rightarrow \quad \lambda_1 = -4.37\cdot 10^{-26}, \\
\xi_0 = 10^{15}, \quad \xi_1 = 0.1 \quad &\Rightarrow \quad \lambda_1 = -4.37\cdot 10^{-21}.
\end{aligned} \tag{17}$$

3. Temperature corrections

For the details on exact theoretical description of thermal field theory, we refer the reader to the most basic textbooks and articles in this subject: [22–24] from which thermal potentials formulas used in below analysis come from.

Temperature corrections are implemented by adding to potential two temperature dependent parts:

$$V(\phi_0, \phi_1) \longrightarrow V_{full}(\phi_0, \phi_1, T) = V(\phi_0, \phi_1) + \delta V_T(\phi_0, \phi_1, T) + \delta V_{ring}(\phi_0, \phi_1, T), \tag{18}$$

where δV_T are first order temperature corrections:

$$\delta V_T(\phi_0, \phi_1, T) = \frac{T^4}{2\pi^2}\left[\sum_{i=\text{bosons}} n_i \cdot J_B\Big(\frac{m_i^2(\phi_k)}{T^2}\Big) + \sum_{j=\text{fermions}} n_j \cdot J_F\Big(\frac{m_j^2(\phi_k)}{T^2}\Big)\right], \tag{19}$$

where n_i and n_j are numbers of degrees of freedom of considered boson or fermion particle with field-dependent mass $m_i(\phi_k)$ and J_B and J_F are thermal bosonic (B) or fermionic (F) functions represented by integrals:

$$J_B\Big(\frac{m^2}{T^2}\Big) = \int_0^\infty dx \cdot x^2 \log\Big(1 - e^{-\sqrt{x^2+\frac{m^2}{T^2}}}\Big), \tag{20}$$

$$J_F\Big(\frac{m^2}{T^2}\Big) = \int_0^\infty dx \cdot x^2 \log\Big(1 + e^{-\sqrt{x^2+\frac{m^2}{T^2}}}\Big). \tag{21}$$

δV_{ring} is infrared contribution from the so called daisy resummed diagrams [22]:

$$\delta V_{ring} = -\frac{T}{12\pi}\Big(m_{eff}(\phi_i,T)^3 - m_i(\phi_i)^3\Big). \tag{22}$$

with temperature-dependent masses $m_{eff}(\phi_i,T)$, obtained from high temperature expansion of:

$$V + \delta V_T\Big|_{m/T\ll 1}. \tag{23}$$

Particle content and thermal masses

Beside two mass eigenstates of (6) with field dependent masses:

$$\begin{aligned} m_G^2 &= 2\lambda_1\phi_1^2 + O(\lambda_1^2) \\ m_H^2 &= 12\lambda_2\phi_1^2 + 2\lambda_1\phi_0^2 + O(\lambda_1^2). \end{aligned} \tag{24}$$

and $n_G = n_H = 1$, there are other important contributions coming from the SM. These are:

- $W^\pm$ boson: $m_W^2 = \frac{1}{4}g_2^2\phi_1^2$, $n_w = 6$,
- Z boson: $m_Z^2 = \frac{1}{4}(g_1^2 + g_2^2)\phi_1^2$, $n_Z = 3$
- top quark: $m_t^2 = \frac{1}{2}h_t^2\phi_1^2$, $n_t = -12$,

where $g_1 \approx 0.35$, $g_2 \approx 0.65$ and $h_t \approx 1$ are weak, strong and top yukawa coupling constants.

From high temperatue expansion (23) we get the mass matrix:

$$\begin{pmatrix} m_{00} & m_{10} \\ m_{01} & m_{11} \end{pmatrix}_{eff} = \begin{pmatrix} \frac{\partial^2 V}{\partial\phi_0^2} & \frac{\partial^2 V}{\partial\phi_1\partial\phi_0} \\ \frac{\partial^2 V}{\partial\phi_0\partial\phi_1} & \frac{\partial^2 V}{\partial\phi_1^2} \end{pmatrix} + \begin{pmatrix} \Big(\frac{\lambda_1}{6} + \frac{\lambda_1^2}{4\lambda_2}\Big)T^2 & 0 \\ 0 & \Big(\lambda_2 + \frac{\lambda_1}{6} + \frac{g_1^2}{16} + \frac{3g_2^2}{16} + \frac{h_t^2}{4}\Big)T^2 \end{pmatrix}, \tag{25}$$

with eigenvalues as the thermal masses m_{eff}^2 for scalars:

$$\begin{aligned} (m_G^2)_{eff} &= 2\lambda_1\phi_1^2 + \frac{\lambda_1}{6}T^2 + O(\lambda_1^2), \\ (m_H^2)_{eff} &= 12\lambda_2\phi_1^2 + 2\lambda_1\phi_0^2 + \Big(\lambda_2 + \frac{\lambda_1}{6} + \frac{g_1^2}{16} + \frac{3g_2^2}{16} + \frac{h_t^2}{4}\Big)T^2 + O(\lambda_1^2). \end{aligned} \tag{26}$$

PoS(CORFU2022)045

3.0.1 Numerical analysis

We present plots of temperature dependent potential (18) and individual ϕ_i directions. The results show how thermal part breaks the scale symmetry. We choose $\lambda_1 = -10^{-6}$ ($\lambda_2 = 0.03125$) to make plots more clear. From Figure (2) is nicely visible, that for non-zero temperatures the flat direction no longer exists. There's a local minimum for $\phi_1 = 0$ and dilaton:

$$\phi_0^2 = \Bigg[\frac{(9.89\alpha - 3.63\lambda_2 - 6.91 \cdot 10^{-16} g_1^2 - 2.76 \cdot 10^{-15} h_t^2)}{\big(- 39.48\alpha + \lambda_2(\alpha \log(-\lambda_1) + 3.48\alpha + 43.53)\big)} + \\ + \frac{\lambda_2 \big(6.58\alpha - 43.53\lambda_2 - 2.72 g_1^2 - 8.16 g_2^2 - 10.88 h_t^2\big)}{\lambda_1\big(- 39.48\alpha + \lambda_2(\alpha \log(-\lambda_1) + 3.48\alpha + 43.53)\big)} \Bigg] \cdot T^2, \tag{27}$$

where

$$\alpha = \sqrt{48\lambda_2 + 3g_1^2 + 9g_2^2 + 12h_t^2}. \tag{28}$$

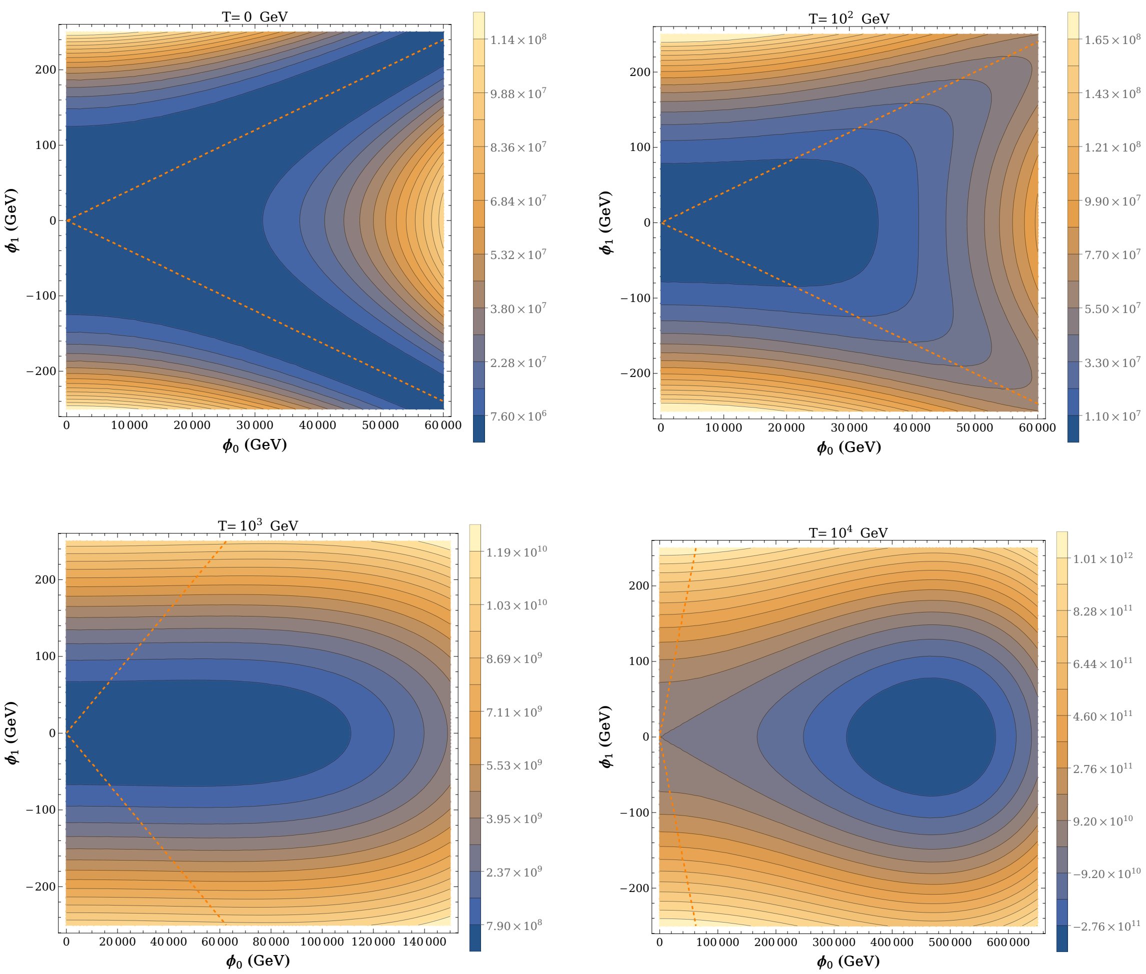


Figure 1: Plots of $V_{full}(\phi_0, \phi_1, T)$ for different temperatures and $\lambda_1 = -10^{-6}$. Orange dashed line marks flat direction $\phi_1^2 = -\frac{\lambda_1}{2\lambda_2}$. It is easy to see that as the temperature increase, the flat direction no longer exists and the scale symmetry is broken.

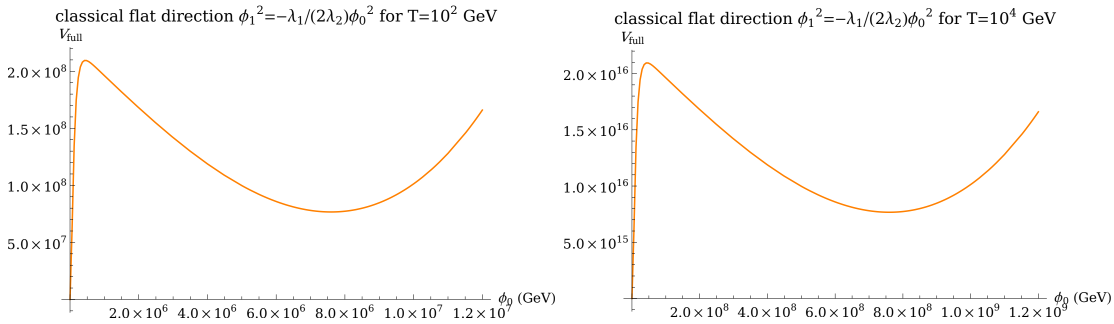


Figure 2: Classical flat direction for non-zero temperatures.

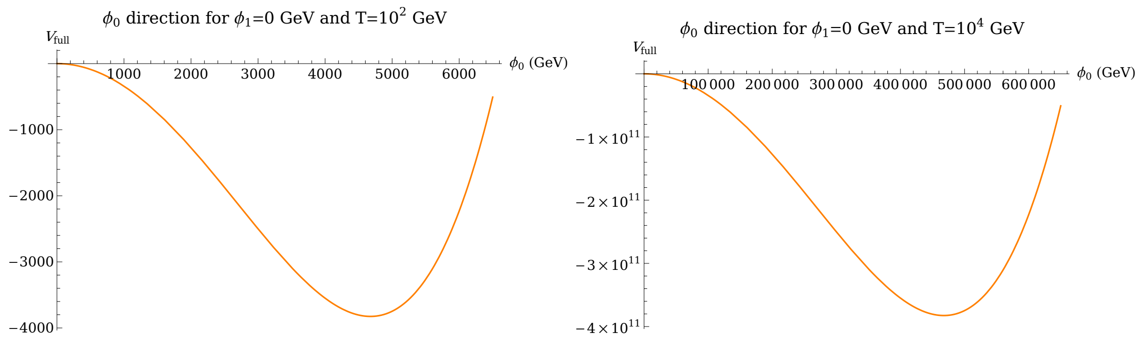


Figure 3: ϕ_0 direction of $V_{full}(\phi_0, \phi_1, T)$ for 2 different temperatures and $\phi_1 = 0$. There's visible minimum at $\phi_0 \approx 4.7 \cdot 10 \cdot T$, which corresponds to equation (27).

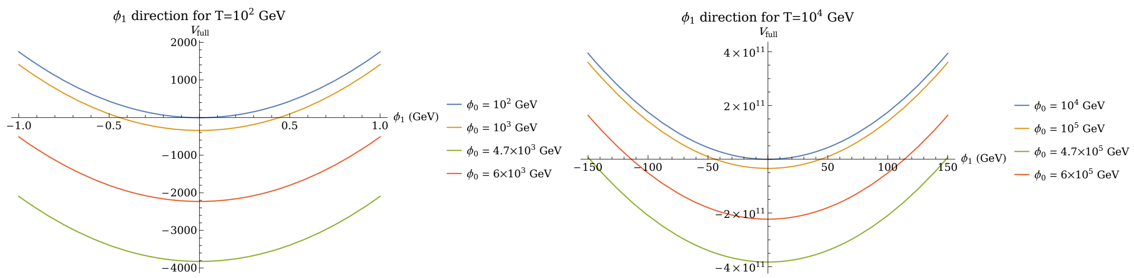


Figure 4: ϕ_1 direction of $V_{full}(\phi_0, \phi_1, T)$ for 2 different temperatures and various ϕ_0 values. The lowest curve corresponds to ϕ_0 from high temperature minimum (27).

4. Time evolution of the fields

We'll look numerically into the development of the scale-symmetric scalar sector in the expanding universe in this part. We demonstrate that there are plausible initial conditions that, at late stages of evolution, result in the physically appropriate vacuum configuration i.e. Higgs vev $v = 250$ GeV. In this paper we show results only for the realistic scenario of parameter space (i.e. fullfilling requirements from section 2.1). For more detailed discussion, we refer the reader to our paper [25].

Zero temperature:

We use equations of motion (4) with relation $R = 12H^2 + 6\dot{H}$:

$$\begin{aligned}&\ddot{\phi}_0 + 3H\dot{\phi}_0 + 2\xi_0\phi_0^2H^2 + \xi_0\phi_0^2\dot{H} + 4\lambda_0\phi_0^3 + 2\lambda_1\phi_0\phi_1^2 = 0\\&\ddot{\phi}_1 + 3H\dot{\phi}_1 + 2\xi_1\phi_1^2H^2 + \xi_1\phi_1^2\dot{H} + 4\lambda_2\phi_1^3 + 2\lambda_1\phi_0^2\phi_1 = 0\\&\frac{1}{2}\left(\xi_0\phi_0^2 + \xi_1\phi_1^2\right)(2H + \dot{H}) - \frac{1}{2}\dot{\phi_0}^2 - \frac{1}{2}\dot{\phi_1}^2 + 2\left(\lambda_0\phi_0^4 + \lambda_1\phi_0^2\phi_1^2 + \lambda_2\phi_1^4\right) = 0.\end{aligned} \tag{29}$$

Non-zero temperature:

To examine evolution of fields ϕ_0 and ϕ_1 in a hot universe we add to the potential V the most leading terms in temperature:

$$V_{eff} \approx V + \frac{1}{2}\phi_1^2 \cdot \left(\lambda_2 + \frac{\lambda_1}{6} + \frac{g_1^2}{16} + \frac{3g_2^2}{16} + \frac{h_t^2}{4}\right)T^2. \tag{30}$$

For λ_1 value used in this analysis, dilaton is out of thermal equilibrium, hence its sector doesn't acquire thermal corrections. Temperature dependence is ruled by radiation:

$$T(t) = \frac{A}{\sqrt{\left(t + t_0\right) \cdot \text{GeV}}}, \qquad A = 1.6 \cdot 10^9 \text{ GeV}, \tag{31}$$

where t_0 is chosen to fit the desired initial temperature of evolution $T_0 = 10^4$ GeV. If one assumes radiation dominated era, its contribution to H is so small that we can neglect it, so we treat the Hubble parameter both in the zero and non-zero temperature simulations as an independent variable, which dynamics is ruled by equations (29). Fixed points of (29) for $T = 0$ lay on the flat direction (5). For $T \neq 0$ after sufficiently long time of evolution temperature goes down and fixed point in this case are also (5).

We choose realistic parameter space i.e. λ_i and ξ_i values, fullfiling requirements from section 2.1. Their values are:

$$\lambda_2 = 0.03125, \quad \lambda_1 = -4.37 \cdot 10^{-26}, \quad \xi_0 = 10^{10}, \quad \xi_1 = 0.1 \tag{32}$$

Initial conditions for ϕ_i fields, their time derivatives $\dot{\phi}_i$ and H, were the same for both $T = 0$ (Figure 5) and $T \neq 0$ (Figure 6) case, namely:

$$\phi_0(0) = 8 \cdot 10^{13} \text{ GeV}, \quad \dot{\phi}_0(0) = 5 \cdot 10^{13} \text{ GeV}^2, \quad \phi_1(0) = 0 \text{ GeV}, \quad \dot{\phi}_1(0) = 10 \text{ GeV}^2 \tag{33}$$

and two different $H(0) = H_0$ values: 0.1 GeV and 0.5 GeV. Initial temperature for $T \neq 0$ case is $T_0 = 10^4$ GeV. Since H can be interpreted as a parameter describing how fast ϕ_i fields loose their energy, the bigger H_0, the faster they slow down and settle in lower values. Direction of $\dot{\phi}_i$ was chosen arbitrarily.

Zero temperature

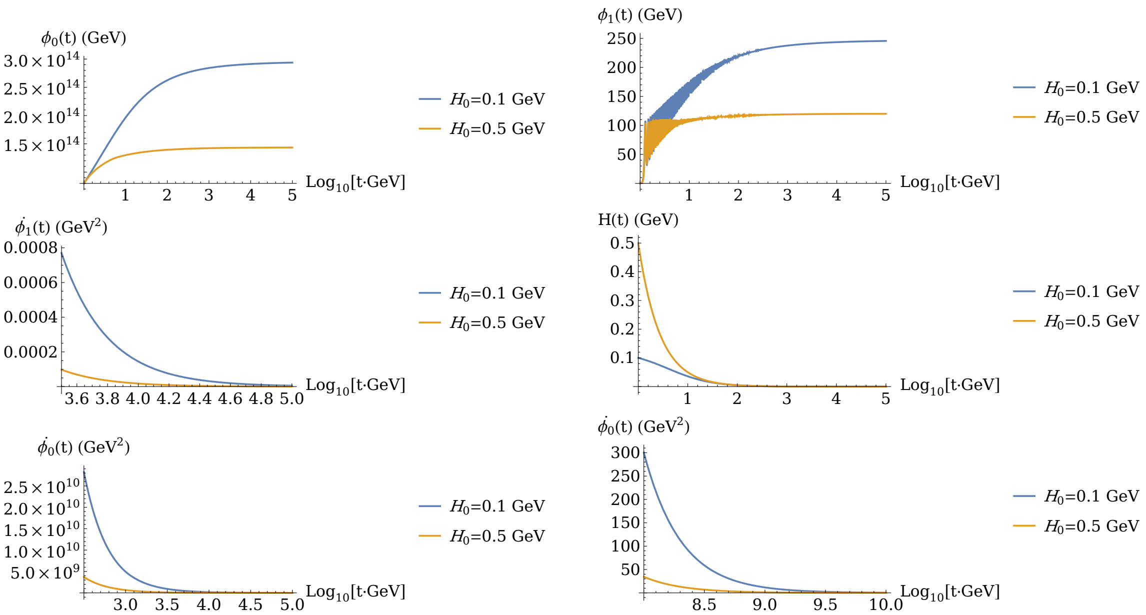


Figure 5: Evolution of ϕ_i fields and H with time for zero temperature and with coupling constants values fulfilling requirements from section 2.1. The bigger the H_0, the faster fields stop and acquire their vevs. There are two plots for $\dot{\phi}_0(t)$, one for later times to show that dilaton indeed stops and the system is stable.

Non-zero temperature

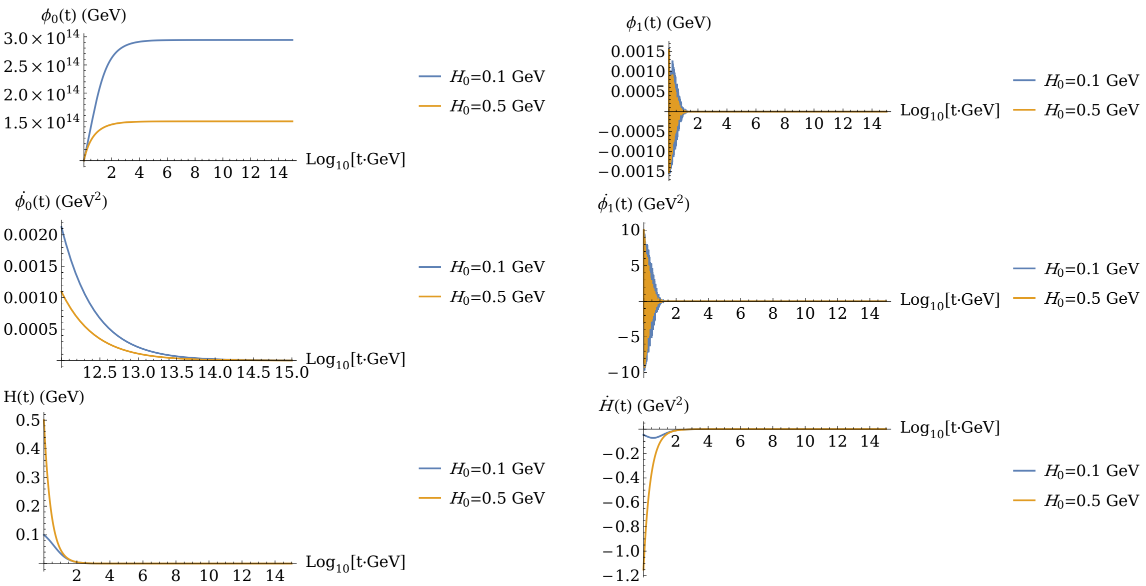


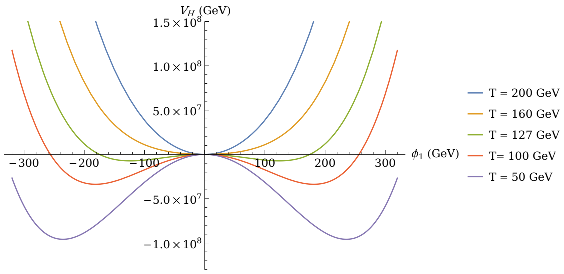
Figure 6: Evolution of ϕ_i fields and H with time for non-zero temperature. Because dilaton sector is not affected by temperature corrections, its final value is the same as in $T = 0$ case. However ϕ_1 field is driven to the origin and stays there till late times.

In $T \neq 0$ case, dilaton is unaffected by temperature and settles in the same final value as in $T = 0$. Higgs field behaviour can be easily understood with analysis of the plots from Figure 7.

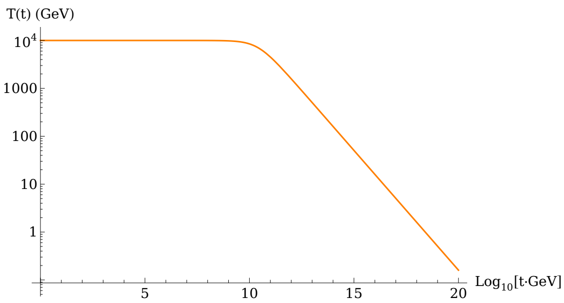
For temperatures above around 130 GeV, Higgs potential poses only one minimum, $\phi_1 = 0$. When temperature drops down, after sufficiently long time, two degenerated minima start to appear. However, $\dot{\phi}_1$ is zero after this time and because the potential is flat at the origin, Higgs field stays at zero value in our simulation. Now, for fields in in thermal equilibrium in temperature T, there's related a fluctuation [26]:

$$\delta\phi_1 = \frac{T}{\sqrt{24}}. \tag{34}$$

We implement simulation of further evolution for ϕ_1 field with $\phi_1(t_0) = \delta\phi_1$. Time $t_0 = 10^{14.2}/\mathrm{GeV}$ refers to temperature $T = 127$ GeV, for which two degenerated minima appear. Initial conditions for ϕ_0 and H are their values at t_0 from simulation from the Figure 6 plots. But their values don't change in this further evolution, hence we provide plot only for the $\phi_1(t)$ on Figure 8.



(a) Higgs potential $V_H(\phi_1, T)$ for different temperatures in time.



(b) Temperature dependence (31) for $T_0 = 10^4$ GeV.

Figure 7

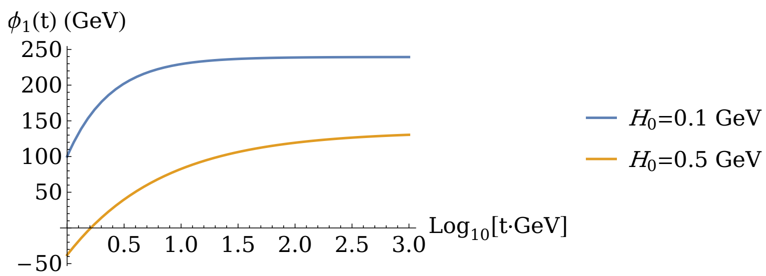


Figure 8: Further Higgs field evolution in time with initial values $\phi_1 = \delta\phi_1 = 37$ GeV and $\dot{\phi}_1 = 0$ GeV^2. Because ϕ_1 oscillates around its minimum in this evolution, the plot shows ϕ_1 average in time. H_0 values correspond to the initial conditions from Figure 6.

5. Summary

Scale symmetry offers a way to understand the origin of mass scales. It can be spontaneously broken when the dilaton acquires its vev during its evolution in expanding hot Universe. We showed how temperature changes the scale symmetric classical potential for dilaton and the Higgs scalar. Since for $T \neq 0$ Higgs value is driven to zero, one obtains restoration of the Electroweak Symmetry. We presented parameter space and initial conditions that can lead to physical values of mass scales in considered model.

Acknowledgments

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