PROCEEDINGS OF SCIENCE

Graham Ross, Semi-Classical v Loops and Quantum Gravity

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My talk is in three parts. In the first I give a personal history of some of my research activities, with emphasis on the extent to which they were influenced by or the result of collaboration with, Graham Ross. In the remaining parts I describe my recent and continuing collaborations with Ian Jack and Marty Einhorn.

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1. Graham Ross

In 1973 I was a postgraduate in Oxford. I was perhaps fortunate in that my allocated supervisor (who had not undergone "conversion to QFT and gauge theories") went away on sabbatical and I persuaded John C. Taylor to take me on. At the time he was the one group member who worked on gauge theories, and indeed had already made immensely significant contributions.

My first paper [1] was an attempt to generalise the successful calculation of the electromagnetic contribution to the pion mass difference to inclusion of weak interactions in the newly popular gauge models and was submitted in Nov. 1973. I intended to move on to the proton-neutron mass difference, but then noticed the paper [2]. From then on I watched for papers by Graham in every preprint list!

Meanwhile asymptotic freedom had been discovered. Had I but realised it, I already had done the relevant calculation of renormalisation constants; John Taylor had suggested I verify the Slavnov Taylor identity relating them at one loop! So I embarked on my thesis calculation [3], which I later extended to the recently invented supersymmetric gauge theory, while a postdoc at Sussex. At that time considerable effort went into constructing models with Han-Nambu quarks, and ones that were asymptotically free but with the gauge symmetry spontaneously broken by the Higgs mechanism, in order to give the vector bosons masses. I recall for instance a paper which Graham wrote with one of the discoverers of asymptotic freedom [4]. I don't know how they came to collaborate.

After my PhD I went to Sussex as a postdoc where I wrote my first paper on supersymmetry, and also enjoyed collaborating with Alex Love and David Bailin. David remained a close friend; he died suddenly in 2018.

As the confinement paradigm took hold, I had returned to Oxford and worked on gauge theory models with Emanuel Derman and lattice gauge theories with Don Sinclair and Peter Scharbach. An interesting account of this period appears in Derman's autobiographical book 'My Life as a Quant".

Graham remained faithful to the continuum and wrote a lot of excellent papers with a variety of collaborators.

He spent time at CERN [5] and at Caltech [6].

1.1 Low Energy Supersymmetry

It was the development of low energy supersymmetry in 1981 which brought me back to mainstream gauge theory models, with the appearance of a paper by Dimopoulos, Raby and Wilczek [7].

I was working with Marty Einhorn in 1981. We were surprised by the omission in this paper of contributions to the β-functions from the Higgs multiplet, thereby leaving the prediction of $\sin^2\theta_W$ unchanged from the non-susy case. We set out to rectify this, extend the renormalisation group (RG) analysis to two loops and calculate the $\frac{m_b}{m_\tau}$ mass ratio. [8].

Regarding the one-loop analysis, there appeared just before us a paper by Ibanez and Graham [9]. This was part of part of a long series of significant papers by Graham on supersymmetric gauge theories.

Figure 1: with Carol Coteus, Paul Coteus and Mike Green

1.2 The strings revolution

In 1984 I was in Aspen working on pertubatively finite gauge theories. Green and Schwarz were there and one day made the crucial discovery which led to string theory taking centre stage. Mike Green hiked with us on a nearby trail called "Lost Man Loop".

My contribution to string theory was made in 1982 and relates to the equation

$$196884 = 196883 + 1. \quad \text{(Robert Griess)}$$

Robert Greiss (the famous mathematician responsible for the monster group) was at Michigan. One day in 1983 he rang me up and proposed we meet. Over lunch he told me of suspected connections between the monster group and string theory, which he illustrated by writing the above equation on a napkin. I kept the napkin for years afterwards. The LHS arises in a branch of mathematics related to modular forms which are relevant in string theory, while the number 196883 arises in classification of representations of the monster group. This apparent numerical accident was investigated for years, and eventually pinned down by Borcherds in 1992.

I told Greiss that if he took one thing away from the lunch, it was that string theory would never amount to anything in terms of physical relevance. I sometimes wonder if he remembered this conversation after 1984.

Graham embraced the strings revolution and in particular its impact on the standard model and its extensions [10]. I studied his papers in particular hoping for insight. We sometimes worked on similar things, such as models of flavour but it was interest in Anomaly Mediation that finally brought us together.

1.3 Anomaly Mediation

I am not sure when we first met. I spent a year at CERN 2003-4, and he was there in a neighbouring office. It was then that we began to discuss joint interests. At the time Ian Jack and I had been working for some years on RG aspects of supersymmetric theories, including some exact results for β-functions in the case of Anomaly Mediated Supersymmetry Breaking (AMSB). With Graham we developed and pursued the phenomenology of an extension of the MSSM wherein the tachyonic slepton problem was resolved by the introduction of a Fayet-Iliopoulos (FI) term. [11] The soft supersymmetry breakings take the following characteristic form:

$$\begin{aligned} M_i &= m_0\beta_{g_i}/g_i \quad \text{gaugino masses} \\ h_{t,b,\tau} &= -m_0\beta_{Y_{t,b,\tau}} \quad \phi^3 \text{ couplings} \\ (m^2)^i{}_j &= \tfrac{1}{2}m_0^2\mu\frac{d}{d\mu}\gamma^i{}_j + kY_i\delta^i{}_j \quad \phi^*\phi \text{ masses} \\ m_3^2 &= \kappa m_0\mu - m_0\beta_\mu \quad H_1H_2 \text{ mass} \end{aligned} \tag{1}$$

where the FI-term contribution is the kY_i term. .

One might think that if the FI term is associated with an additional U_1' broken at some high scale M, then by the decoupling theorem, all effects of the U_1' would be suppressed at energies $E << M$ by powers of $1/M$. We showed that with a FI term this is not the case and it is quite natural for there to be $O(M_{\rm susy})$ scalar mass contributions arising from the presence of the FI term, with $M >> M_{\rm susy}$.

The sparticle mass scale is determined by the mass parameters m_0 and k. Requiring the U_1' to be anomaly free results in various interesting sum rules.

1.4 Anomaly Mediation and Dimensional Transmutation

Graham and I both felt that the introduction of a second scale (the FI term) to the anomaly mediated MSSM was an unattractive feature, though it works! It dawned on us that we could start with a *scale invariant* $MSSM \otimes U_1'$ model, with the scale of U_1' breaking arising by dimensional transmutation, and the only explicit terms of dimension two and three in the Lagrangian being those associated with AMSB. The U_1' breaking scale also determines the right-handed neutrino masses, which in turn determine the observable neutrino masses via the usual see-saw mechanism.

A brief account of this appears in [12]

We planned to follow up but but somehow we drifted off in different directions. Time to revisit this model perhaps?

I will return to Graham at the end of my talk.

2. Loops with Ian Jack

In autumn 2019 I noticed the paper [13] and was impressed by the success of the comparison of semi-classical calculations (with which I was unfamiliar) with perturbative ones (with which I was). This paper carried out, in particular, a calculation in simple $\lambda\phi^6$ of the anomalous dimension of the operator ϕ^n to two loops. This struck me as fertile ground for Ian Jack and me to plough, though we did not begin until well into 2020.

Motivation: Many particle amplitudes are of obvious relevance to the LHC. Of course we would like to do quark and gluon amplitudes, but a lot can be learned from scalar theories, specifically scale invariant ones. The study of this in scale invariant scalar theories is also of interest in the Conformal Field theory context. There are three such theories with a ϕ^M interaction:

$$\phi^4(d = 4), \phi^3(d = 6), \phi^6(d = 3)$$

Approaches that extend the reach of (or even transcend the need for) perturbation theory have always been challenging, and are all the more interesting now because of the increased importance attached to multi-leg amplitudes, which can present formidable calculational obstacles at higher loop orders. Another motivation for studying this class of theories is their (classical) scale invariance (CSI). As remarked in [14] the Standard Model (SM) is "almost" CSI. Indeed, in 1973, Coleman and Weinberg (CW) had hoped to argue that the SM might indeed be viable with the omission of the Higgs (wrong-sign) (mass)^2 term, with *dimensional transmutation* generating the physical mass scale in the CSI theory. This attractive idea failed. Neglecting Yukawa couplings led to a Higgs mass prediction which was too small; and including the top quark Yukawa coupling destabilised the Higgs vacuum altogether. This conclusion is watertight, notwithstanding occasional claims to the contrary.

2.1 The $d = 4$ case and the semi-classical calculation

We consider the interaction $\lambda_0(\overline{\phi}\phi)^2$ which has a $U(1)$ symmetry The anomalous dimension of the operator ϕ^n has a perturbative expansion

$$\gamma_{\phi^n} = n \sum_{l=1} \lambda^l P_l(n)$$

where P_l is a polynomial of degree l. Consequently perturbation theory breaks down for large λn no matter how small λ is.

The semiclassical calculation proceeds via a saddle point appoach to the path integral:

$$< \overline{\phi}^n(x_f)\phi^n(x_i) >= \frac{\int \mathcal{D}\phi\mathcal{D}\overline{\phi}\, \overline{\phi}^n(x_f)\phi^n(x_i)exp[-S/\lambda_0]}{\int \mathcal{D}\phi\mathcal{D}\overline{\phi}exp[-S/\lambda_0]}$$

where

$$S = \int \left[\partial\overline{\phi}\partial\phi + (\overline{\phi}\phi)^2/4\right]$$

and we have rescaled $\overline{\phi}, \phi$ to make λ_0 a loop counting parameter.

We move the $\overline{\phi}\phi$ up into the exponential:

$$< \overline{\phi}^n(x_f)\phi^n(x_i) >= \frac{\int \mathcal{D}\phi\mathcal{D}\overline{\phi}exp[-(1/(\lambda_0)[S - \ \lambda_0 n(\ln\overline{\phi}(x_f) + \ln\phi(x_i))]]}{\int \mathcal{D}\phi\mathcal{D}\overline{\phi}exp[-S/\lambda_0]}$$

We can now do a power series expansion in λ, evaluating the integrals in the using the saddle point approximation, that is at the stationary configuration of the exponents. The result of doing this is a perturbation series in λ_0 rather than $\lambda_0 n$. It can then be compared with the straightforward perturbative calculation of γ_{ϕ^n}.

Ian Jack and I produced a series of papers [15]-[18] where in each of the three cases $M = 3, 4, 6$ we extended existing perturbative calculations to higher loops. Generally speaking, the results were consistent with the semi-classical calculations.

As an example, I show the graphs responsible for the leading n and next to leading contributions to the anomalous dimension of the ϕ^n operator at four loops:

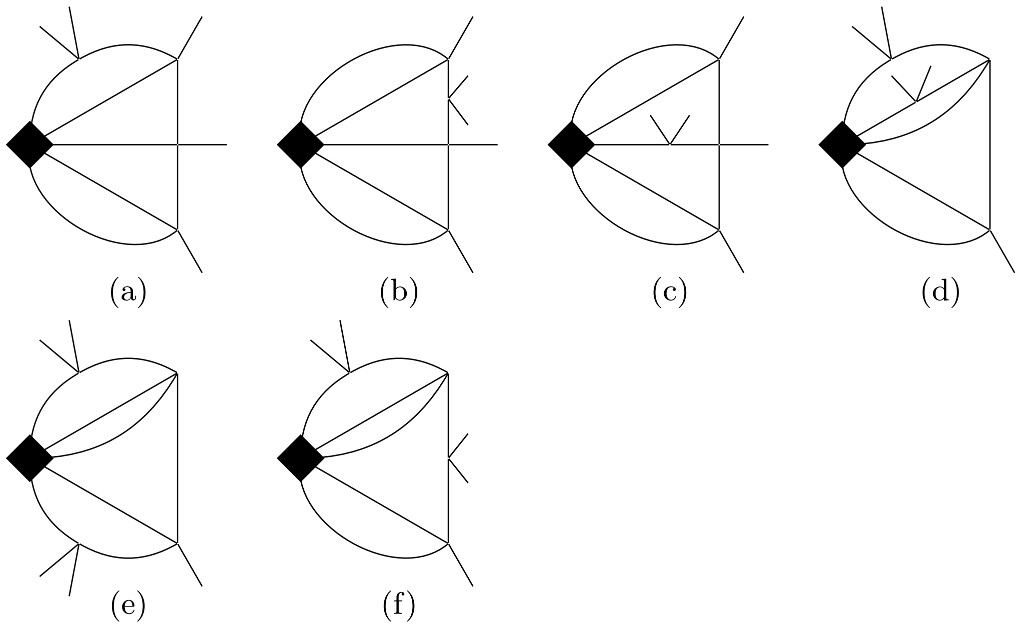

Figure 2: Four-loop diagrams for $\gamma_{T_{\bar{Q}}}$ contributing at leading n

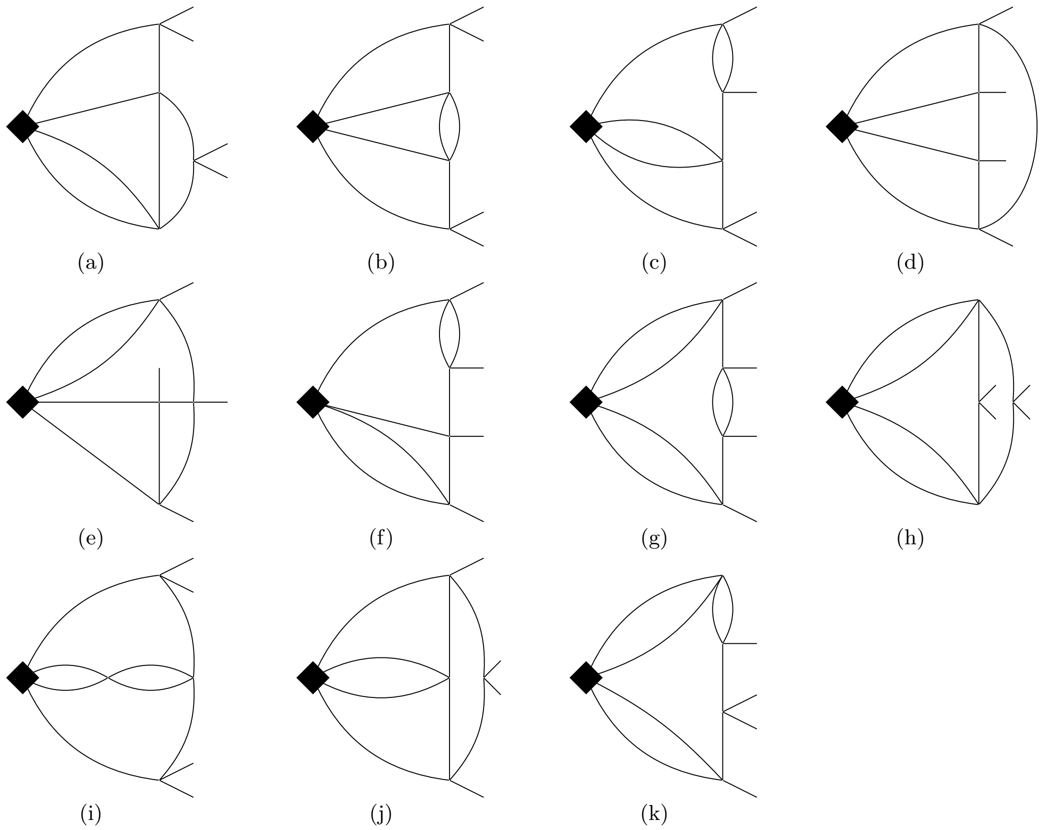

Figure 3: Some four-loop diagrams for $\gamma_{T_{\bar{Q}}}$ contributing at non-leading *n*

I will avoid a detailed description of our work. Suffice to say that, generally speaking our new results reinforced the agreement with the semi-classical calculations, where applicable. It would be interesting to pursue semi-classics and the comparison with perturbation theory for other scale invariant cases such as the Thirring model, the Wess-Zumino model or even QCD.

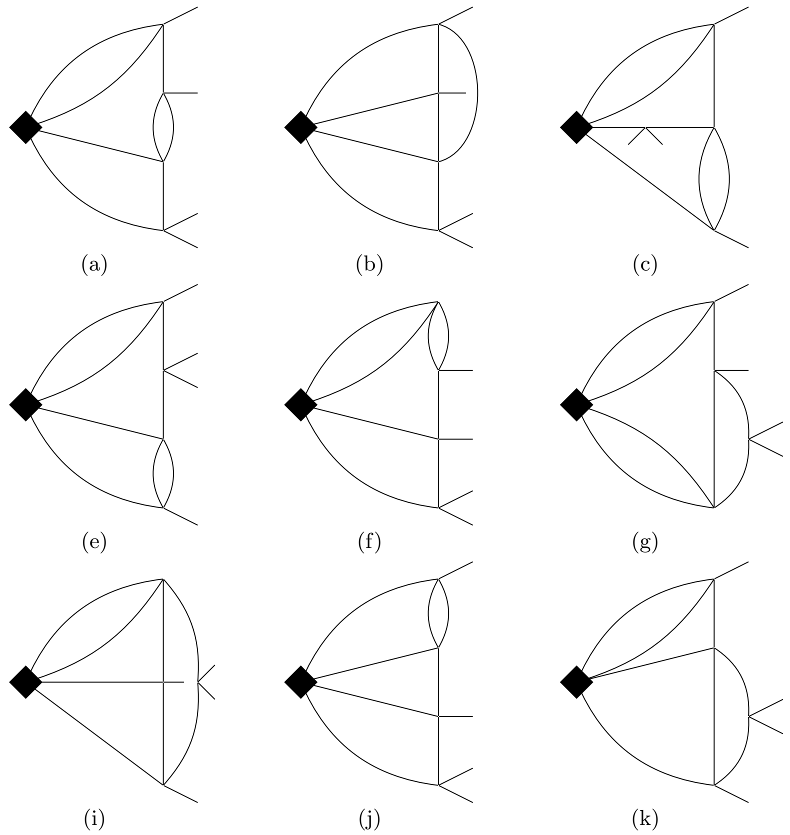

Figure 4: More four-loop diagrams for $\gamma_{T_{\tilde{Q}}}$ contributing at next-to-leading n

3. Quantum Gravity with Marty Einhorn

Marty Einhorn and I were very impressed by the paper [19] with its use of the scale invariant $\xi R\phi^2$ term to generate inflation via a large value for ξ. We pursued the implementation of this idea in the context of supergravity, [20].

3.1 Scale Invariant Gravity

Interest in the $\xi R\phi^2$ term got us interested in *scale invariant* quantum gravity, with the Einstein term $M_P^2 R$ is generated by a vacuum expectation value (vev) for a scalar field via dimensional transmutation (DT), much in the manner of the classic Coleman and Weinberg paper [21].

Scale Invariant Gravity takes the form

$$S = \int d^4x \sqrt{g} \left(M_P^2 R + \alpha R^2_{\mu\nu\rho\sigma} + \beta R^2_{\mu\nu} + \gamma R^2 \right) + S_{\text{matter}}$$

.

We explored coupling to scale invariant $\lambda\phi^4$ when the gravitational couplings can induce a vev for the scalar field, first for a single scalar field and then for scalar representations in gauge theories [22]-[28].

3.2 Outcomes

- Dimensionless transmutation can give a nonzero $\langle\phi\rangle$ in a theory with scalar fields coupled toR^2 gravity, and hence generate an Einstein term in the "low energy" theory.
- In the simplest model, the attraction basin of the only UV stable fixed point does not include the region in which DT minima occur, so in this region the theory becomes strongly coupled or must be modified at high scales.
- More complicated models can remedy this, and also the nonzero $\langle\phi\rangle$ can break a Grand Unified symmetry. For $SO(12)$, in a region of parameter space, Dimensional Transmutation occurs, with the adjoint vacuum expectation value breaking $SO(12) \to SU(6) \otimes U(1)$, and producing a Low Energy Effective Theory having Einstein-Hilbert gravity. Certain minima are locally stable and lie within the catchment basin of the ultraviolet fixed points. The scenario may be compatible with a form of Higgs inflation.
- Problems: Unitarity, the electroweak scale, naturalness, ...

3.3 Unitarity and First Order Formalism

In the hope of gaining insight into the Unitarity issue we have been working on the first order (Palantini) formalism as appled to R^2 gravity. First order formalism entails treating the metric and connection as independent fields.

In Einstein gravity, this was introduced by Einstein himself to simplify the derivation of the field equations. Thus from

$$S = M_P^2\sqrt{g}R,$$

where

$$R = g^{\mu\rho}R_{\mu\rho}, R_{\mu\rho} = R_{\mu\sigma\rho}{}^{\sigma}$$

and

$$\tfrac{1}{2}R_{\mu\nu\rho}{}^{\sigma}(\Gamma) \equiv \partial_{[\mu}\Gamma_{\nu]\rho}{}^{\sigma} + \Gamma_{\rho[\mu}{}^{\tau}\,\Gamma_{\nu]\tau}{}^{\sigma},$$

$g^{\mu\nu}$ becomes effectively an auxiliary field, and its equation of motion gives the Einstein equation:

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = 0.$$

The equation of motion for Γ then restores the usual definition of Γ in terms of g.

First Order formalism makes a big difference in R^2 gravity; for example the number of independent R^2 invariants is different:

$$\begin{aligned}
S_{RQG} \;=\; & \int d^4x\sqrt{g}\;[\alpha R^2 + \beta_1 R_{(\mu\nu)}R_{(\kappa\lambda)}g^{\mu\kappa}g^{\nu\lambda} - 2\beta_2 R_{(\mu\nu)}\tilde{R}_{\lambda}{}^{\mu}g^{\nu\lambda} \\
+ \; & \beta_3\tilde{R}_{\nu}{}^{\mu}\tilde{R}_{\mu}{}^{\nu} + \beta_6\tilde{R}_{\kappa}{}^{\mu}\tilde{R}_{\lambda}{}^{\nu}g_{\mu\nu}g^{\kappa\lambda} + \beta_8\tilde{R}_{\kappa}{}^{\mu}\overline{R}_{\mu\nu}g^{\kappa\nu} \\
+ \; & \beta_9\overline{R}_{\mu\nu}\overline{R}_{\kappa\lambda}g^{\mu\kappa}g^{\nu\lambda} + R_{\mu\nu\rho}{}^{\sigma}(\gamma_1 g^{\rho\kappa}g^{\mu\tau}R_{\kappa\sigma\tau}{}^{\nu} + \gamma_3 g^{\nu\lambda}g^{\mu\kappa}R_{\kappa\lambda\sigma}{}^{\rho} \\
+ \; & \gamma_4 g^{\rho\lambda}g^{\mu\kappa}R_{\kappa\lambda\sigma}{}^{\nu} \\
+ \; & \gamma_5 g^{\nu\tau}g^{\mu\kappa}R_{\kappa\sigma\tau}{}^{\rho} + \gamma_6 g_{\sigma\upsilon}g^{\rho\lambda}g^{\nu\tau}g^{\mu\kappa}R_{\kappa\lambda\tau}{}^{\upsilon} + \gamma_7 g^{\rho\tau}g^{\mu\kappa}R_{\kappa\sigma\tau}{}^{\nu})].
\end{aligned}$$

Here $\tilde{R}_\mu{}^\sigma \equiv g^{\nu\rho} R_{\mu\nu\rho}{}^\sigma(\Gamma)$, $\overline{R}_{\mu\nu} \equiv R_{\mu\nu\rho}{}^\rho$ and $R \equiv g^{\rho\mu} R_{(\mu\rho)}(\Gamma)$.

Note that the Riemann tensor does not respect all the familiar identities and $\overline{R}_{\mu\nu}$ is an antisymmetric tensor.

There have been previous studies, notably Ref [29].

The result is a vastly more complicated analysis of the RG evolution of the dimensionless couplings, and of the issue of Dimensional Transmutation. Our main motive is to get insight into the issue of Unitarity. Work in progress........

4. Graham again

I enjoyed my only too brief period of collaborating with Graham, and wish we had worked together more. Also that I had been capable of giving him more opposition on the squash court! He was MUCH too good for me.

He was a great scientist and a good friend. He and Ruth were wonderful dinner companions. I only have two pictures of him, where I suspect he is describing our work to a sceptic:

5. Acknowledgements

I enjoyed collaborating with Graham Ross, Ian Jack and Marty Einhorn on the work presented here. I thank KITP, UC Santa Barbara for hospitality while part of this work was performed, the Leverhulme Trust for kindness and the award of an Emeritus Fellowship, and the organisers for the opportunity to speak. Financial support was provided by the Leverhulme Trust.

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