

Seeking de-Sitter Vacua in the String Landscape

George K. Leontaris^{a,*} and Pramod Shukla^{b,c}

^a*Physics Department, University of Ioannina
University Campus, Ioannina 45110, Greece*

^b*Department of Physics, University of Allahabad,
University Road, Old Katra, Prayagraj 211002, India*

^c*Department of Physics, Unified Academic Campus, Bose Institute,
EN 80, Sector V, Bidhannagar, Kolkata 700091, India*

E-mail: leonta@uoi.gr, pshukla@jchose.ac.in

In this report, we present a concise review on the various moduli stabilisation schemes proposed in the context of type IIB superstring compactifications using Calabi-Yau orientifolds. We discuss the details of the known schemes by classifying them into two categories; the first one includes non-perturbative superpotential contributions leading to the well-known class of models like KKLT, Racetrack and LVS, while the second one includes only perturbative corrections arising from the series of α' - and string-loop effects, leading to a new scheme - known as perturbative LVS. In addition, we motivate and briefly discuss about the global embedding requirements for all these moduli stabilisation schemes, and present some details for the perturbative LVS scheme by focusing on a concrete CY orientifold setting. In this context, the resulting 4D effective supergravity model indeed does not receive any non-perturbative superpotential effects due to the lack of necessary divisor topologies, and the Kähler moduli stabilisation is performed entirely by perturbative effects, leading to AdS minimum with exponentially large VEV for the overall volume of the internal CY manifold. It is further shown how this class of AdS solutions can be uplifted to de-Sitter solutions by using a couple of prescriptions such as D -term uplifting and the T -brane uplifting.

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*Speaker

1. Preface

Realising de-Sitter (dS) solutions in string (inspired) models has been in the centre of attraction since more than two decades, and the tireless efforts made in this program have resulted in a huge amount of literature. In fact, this task of searching for stable dS solutions has been found to be notoriously difficult, especially due to the presence of several no-go results arising from the theoretical constraints while working with the minimal set of ingredients at hand; e.g., the Maldacena-Nuñez no-go theorem [1]. Likewise, a series of no-go theorems obstructing the dS realisation has been observed/proposed in the context of type IIA superstring compactifications using Calabi Yau orientifolds [2–24]. However, it has been reported at plenty of occasions that considering the possible loopholes in the minimal dS no-go scenarios, one can construct (relatively less simple) models with more ingredients which may realise (stable) dS vacua [25–47]. Nevertheless, the simple type IIA no-go results with minimal ingredients have been always useful as a guiding tool to narrow down the suitable region in the flux landscape. Moreover, these no-go scenarios have also played a central role in the recent revival of the swampland conjectures [48, 49] and the proposal of the Quintessence alternative ¹ [51–73].

This review will mainly focus on the physics of de-Sitter vacua and the stabilisation of the moduli fields in effective field theory models of string origin. To briefly explain the importance of such a project, in the beginning we should like to make a small digression and present a few of many possible underlying motivations.

In cosmology, observational data already known more than two decades ago [74], have undoubtedly confirmed that there is a continuous accelerated expansion of the universe that began before its reheating. At first glance, this phenomenon contradicts our intuition about what could happen long after the big bang when the attractive force of gravity prevails. According to the widely accepted interpretation, the observed accelerated expansion indicates that the universe is entering an era dominated by dark energy that permeates its entire space. In the context of Einstein’s general relativity, the density of dark energy can be attributed to a positive value of the cosmological constant, $\Lambda > 0$, with a current value of $\Lambda \sim 10^{-122} M_P^4$, where M_P is the four-dimensional Planck mass (for latest update see [75, 76] and relevant referencenes therein). It is perhaps worth noting that the order of magnitude of Λ coincides with that of the mass square differences of the light neutrinos $\Delta m_{\nu_{ij}}^2 \lesssim 10^{-30} M_P^2$ which might indicate some connection between Planck scale and EW physics [77].

Dark energy has an important role in the theory of General Relativity and Einstein’s equations in particular. To understand its effect, let’s recall that in the action of a scalar field within a simple theory context, only potential differences matter. In General Relativity, however, adding a cosmological constant in the Einstein action, a $\sqrt{-g}\Lambda$ term is induced in the Lagrangian, i.e., proportional to $\sqrt{-g}$ which describes the geometry of space-time. Hence, any change in the potential energy modifies the vacuum energy.

In the context of effective field theories, the simplest scenario describing the basic features of cosmological observations consists of a scalar field acquiring a potential $V(\phi)$ where there are two viable possibilities: (i). the potential is a slow varying function of ϕ whose present value is equal to a slowly varying cosmological constant $V(\phi_{today}) = \Lambda$, and (ii). the potential has a local

¹For a detailed review on the subject, see [42, 50].

minimum with a positive vacuum energy equal to Λ . In the first case the potential has a gentle (almost zero) slope, whilst in the second one the scalar potential has a stable or metastatic dS-type vacuum. Provided that some additional requirements are met in the second case, the $V(\phi)$ potential could be suitable for a successful slow-roll inflationary scenario with the scalar field ϕ playing the role of the inflaton field. Recall that cosmic inflation is the theory of the exponential expansion of space in the early universe and explains the origin of the large-scale structure of Cosmos (see for example [75] and for string inflation [50, 78]).

Based on the preceding observations and remarks one can draw the logical inference that a variety of fundamental open questions involving vast range of scales are intertwined. Thence, it would be desirable to contemplate an effective theory with ultra-violet (UV) completion where Planck-scale Physics are naturally integrated. Currently, the most successful and robust candidate towards a UV completion is string theory which unifies all fundamental forces including gravity, however, it predicts six extra dimensions which must be compactified to a minuscule size. Thus the ten-dimensional space is written as

$$\mathcal{R}^{3,1} \times M_6 ,$$

where $\mathcal{R}^{3,1}$ the four-dimensional Minkowski space and M_6 is associated with the extra compactified dimensions which escape our detection. In the context constructing four-dimensional (semi)realistic models using superstring compactifications, the so-called Calabi-Yau (CY) threefolds have been found to be directly useful, and studying the properties of such geometries has been among the most important tasks in string model building. In fact, "Which CY threefold is better or more suitable for constructing realistic models?" can be considered to be one of the longstanding questions one still seeks an answer for. Nevertheless, one can always make a classification of the known CY threefolds to single out "some" classes of examples which can be more useful for certain requirements as compared to the other ones. In this context, apart from studying the global properties of the CY threefolds, the study of co-dimension one topologies also plays a crucial role due to a variety of reasons, e.g. the various scalar potential contributions which are used for moduli stabilisation purpose are governed by such divisor topologies, and a pheno-motivated classification has been recently presented in [79].

While effective models derived from string theory naturally satisfy the criterion of UV completion, a number of problems must be remedied before they can be considered as successful low energy theory candidates. Here we focus on two of the most important issues, the moduli fields and the vacuum of the theory which must be of de-Sitter type to ensure a positive Λ . A generic problem in string derived models is the appearance of several scalar (moduli) fields in the massless spectrum. In compactifications on a Calabi-Yau manifold, for example, deformations respecting the Ricci flatness conditions give rise to moduli fields that do not acquire a potential at the classical tree-level and do not affect the four-dimensional action. Massless fields in effective theory, however, might have unwanted consequences. If they couple to the gravitational field, they could mediate long-range forces, yet not observed. Furthermore, specific classes of moduli fields control the volume and the shape of the extra dimensions. At the level of the four-dimensional effective field theory such moduli fields control the magnitude of the gauge and Yukawa couplings and other parameters such as masses, the hierarchy of scales, etc. Therefore, in order to construct a realistic model in the framework of superstring-derived field theory, it is important to create a potential with

dS-type vacuum energy, and to ensure a positive mass squared $m^2 > 0$ for the various moduli fields. This has been called the moduli stabilisation problem. Seeking a solution to these issues one faces a number of additional challenges. As already pointed out, compactification happens on appropriate Calabi-Yau (CY) manifolds and some choice of quantum fluxes such as the electromagnetic ones F_3, H_3 in type IIB string theory. However, there exists a large number of choices of CY manifolds and fluxes which eventually determine the properties of the effective quantum field theory and, in particular, the scalar potential thereof. As a result, string theory is characterised by an enormous number of vacuum ground states which constitute the so-called “string landscape”. However, the last few years there is considerable activity on whether all possible consistent quantum field theories do belong to the string landscape (see for example [20] and references therein). According to recent conjectures it is quite possible that not all consistent quantum theories emerge as low-energy limits of string compactifications and a vast majority of them lies outside of the *landscape*, the so called *swampland* [71, 80–82]. Whatever the theory might be, according to the cosmological data accepted by the majority of the scientific community today, the universe is of dS type. For all these reasons, currently, the fundamental problem of moduli stabilisation and the quest for dS vacua are a subject of intensive research activity in string theory. Despite the continuous efforts over the last couple of decades, both issues remain open to this day. Possible solutions, if they exist at all ², are sought beyond the classical level and are based on quantum corrections which modify the Kähler potential and the superpotential. During the last few decades, a broad spectrum of perturbative and non-perturbative contributions have been implemented to confront these issues. However, it is reasonable to anticipate that an elegant and viable solution should be achieved only with a minimum number of well defined ingredients. From this perspective, it would be of particular interest to investigate whether a successful outcome can emerge by incorporating only perturbative quantum corrections.

Building on these lines, the central task for having a genuine dS solution may be divided into three steps; (i). ensuring the existence by evading any possible no-go restrictions which could come on the way in a given setting, (ii). ensuring the stability, e.g. via checking the absence of tachyons (and possible flat directions which may have the potential of decompactifying the effective supergravity model), and (iii). arguing for the viability of the dS solution along with the underlying effective supergravity construction against any possible (un-)known sub-leading corrections or any other new obstacle which the underlying model may face in course of an improved understanding of the current state-of-the-art. These aspects become more relevant especially for the cases in which very little is known about the series of infinitely many corrections inducing the scalar potential terms responsible for performing moduli stabilisation, and subsequently this presents a challenge to ensure whether the dS vacua which pass the tests in the first two steps are really genuine. For instance, one may take the issues like scale separation and field excursions in moduli space [83–99], tadpole conjecture [100–103] etc.

The structure of present review is as follows: we start with collecting the necessary ingredients for type IIB model building using CY orientifolds, and discuss the stabilisation of the complex-structure moduli and axio-dilaton using the background fluxes in section 2. In section 3 we present a brief summary of the various (non-)perturbative corrections known in the literature, which can

²The recent literature is vast. For comprehensive analysis and related work on these issues see the reviews [20, 71].

generically be useful for breaking the so-called no-scale-structure in order to facilitate the Kähler moduli stabilisation. We present a detailed and classified revisit of the known moduli stabilisation schemes such as KKLT, Racetrack, LVS and perturbative LVS in section 4. Followed by the same, in section 5 we motivate the need of global embedding of these schemes and present a concrete CY orientifolds which fulfils the requirements of realising perturbative LVS, and subsequently uplifting the AdS solution into dS one via including additional corrections from the D -term effects and the so-called T -brane uplifting. Finally we summarise the report with conclusions and future directions in section 6.

2. Ingredients for type IIB superstring model building

To begin with, in this section, the basic geometric setup and the moduli field content is briefly reviewed within the type IIB framework with six internal dimensions compactified on a Calabi-Yau threefold, denoted hereafter with X . The moduli fields associated with deformations of X relevant to this work are the dilaton field ϕ , the Kähler moduli T_i , and the complex structure (CS) moduli, U^α . We start with a brief discussion on the Kähler- and complex-structure moduli and then proceed with the rest of the bosonic spectrum of type IIB.

2.1 CY compactifications and moduli fields

In this section, first we will give a brief review about the origin of moduli fields where we mainly follow the analysis of [104] (see also [105]). First we recall that a Kähler manifold M is associated with a (closed) Kähler form

$$J = g_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}, \quad dJ = 0.$$

A Calabi-Yau (CY) manifold is a compact Kähler manifold of vanishing first Chern class, and in order to dimensionally reduce a 10-dimensional superstring theory to 4-dimensions one needs CY threefolds. For F-theory compactifications to 4-dimensions, the compactifying CY manifold is of complex dimension four, while having the three Kähler manifold as the base of the fibration which is not necessarily a CY. Once the manifold M and a Kähler form are determined it can be proven that there is a unique Ricci-flat Kähler metric g on M (i.e. $R_{i\bar{j}}(g) = 0$) such that the associated Kähler form J is in the same cohomology class as the given one. In view of this we may consider the parameter space of Calabi-Yau manifolds to be the parameter space of Ricci-flat Kähler metrics³. Thus, requiring g_{ij} and $g_{ij} + \delta g_{ij}$ to be Ricci flat metrics, so $\nabla^i \delta g_{ij} = 0$ and this implies that g_{ij} must satisfy a particular type of differential equation (the Lichnerowicz equation) [106, 107],

$$\nabla^k \nabla_k \delta g_{ij} + 2R_{ij}^{mn} \delta g_{mn} = 0.$$

Due to the properties of the Kähler manifold, the mixed $\delta g_{i\bar{j}}$ and pure $\delta g_{ij}, \delta g_{i\bar{j}}$ type zero modes satisfy independently the Lichnerowicz equation. Now, variation of the Kähler structure $\delta g_{i\bar{j}}$ (mixed type), gives $h^{1,1}$ parameters⁴, which counts the number of Kähler moduli in the 4D

³i.e., deformations of the background metric $g + \delta g$ and the NS-NS two-form B_2 variations that do not change the Calabi-Yau condition $R(g) = R(g + \delta g) = 0$.

⁴The hodge numbers $h^{r,s}$ determine the dimension of Dolbeault cohomology $H^{r,s} = \frac{\{\omega^{r,s} | \bar{\partial} \omega^{r,s} = 0\}}{\{\alpha^{r,s} | \alpha^{r,s} = \bar{\partial} \beta^{r,s-1}\}}$.

effective supergravity theory. On the other hand, pure type metric variations g_{ij} , $g_{\bar{i}\bar{j}}$ give rise to complex structure (CS) parameters counted by $h^{2,1}$ of the CY threefold, implying a nowhere vanishing holomorphic harmonic 3-form Ω_3 induced via:

$$\Omega_{ijk} g^{k\bar{l}} \delta g_{\bar{l}m} dz^i \wedge dz^j \wedge d\bar{z}^{\bar{m}},$$

which is a basic constituent of the Calabi-Yau space. For the case of a CY_3 manifold with $SU(3)$ holonomy, $h^{3,0} = h^{0,3} = 1$, hence Ω is unique. Since more than three decades, a huge amount of effort has been continuously made for constructing and classifying the CY geometries [108–116] resulting in two broadly defined classes:

- First, the class of CY threefolds realised as multi-hypersurfaces in the product of projective spaces are popularly known as CICYs (Complete Intersection CY threefolds), and this class of CYs involves a total of 7890 geometries [108–110]. These CICYs have been used not only for constructing local MSMS-like models [117–119] but also for addressing other phenomenological issues such as moduli stabilisation, inflation etc. [120–127].
- The second class of CY threefolds corresponds to those which are realised as hypersurfaces in toric varieties, and are known as THCYs (Toric Hypersurface CY threefolds) [112]. Explicit examples of these CYs can be easily constructed using the four-dimensional reflexive polytopes of the Kreuzer-Skarke (KS) database [115]. Interestingly, these CY geometries have been studied in great detail since more than a decade ago [111, 113, 114] and have been used for explicit “global embedding” of many of the earlier proposed models useful for moduli stabilisation, dS realisation, inflationary embedding etc. see e.g. [33, 35, 43, 44, 128–142]. This has led to the development of several efficient/powerful tools/packages such as “Package for analyzing lattice polytopes” (PALP) [143, 144], `cohomCalc` [145, 146] and `CYTools`.

Let us finally mention that, there exists a very huge number of CY threefolds that can be possible candidates for compactifying the internal six-dimensions of the superstring theory based models, and each of these compactifications can generically result in models with a large number of Kähler- and complex-structure moduli, whose stabilisation is typically a very tricky task. In this regard, an estimated upper bound for the number of topologically inequivalent Calabi-Yau hypersurfaces in toric varieties, arising from the Kreuzer-Skarke (KS) database [115], is found to be $N_{CY}^{max} \simeq 10^{428}$ [142]. We will get back to the discussion on constructing explicit global models with some concrete moduli stabilisation and dS realisation in one of the upcoming sections.

2.2 IIB bosonic spectrum and the flux superpotential

The type IIB superstring compactification using CY threefolds results in $\mathcal{N} = 2$ supergravities in four-dimensions, and in order to further reduce the amount of supersymmetry one instead uses the orientifolds of the CY threefolds. It turns out that the massless states in the type IIB spectrum are in one-to-one correspondence with the harmonic forms which can be either even or odd under the holomorphic involution σ acting on the internal CY threefold. Subsequently, the dimensions of the equivariant cohomology groups $H_{\pm}^{p,q}(X)$ counts the number of fields in the low energy spectrum. We fix our notations by denoting the bases of even/odd 2-forms as (μ_i, ν_a) while the dual 4-forms are denoted as $(\tilde{\mu}^i, \tilde{\nu}^a)$ where $i = 1, \dots, h_+^{1,1}(X)$, $a = 1, \dots, h_-^{1,1}(X)$. Configurations

with $h_{-}^{1,1}(X) \neq 0$ have been found to be quite interesting, e.g. see [127, 128, 131, 135, 147–151]. Also, we denote the bases for the even and odd cohomologies of 3-forms $H_{\pm}^3(X)$ respectively as the symplectic pairs (a_K, b^J) and $(\mathcal{A}_{\Lambda}, \mathcal{B}^{\Delta})$. Subsequently we fix the normalisation in the various cohomology bases by the following relations:

$$\begin{aligned} \int_X \mu_i \wedge \tilde{\mu}^j &= \delta_i^j, & \int_X \nu_a \wedge \tilde{\nu}^b &= \delta_a^b, & \int_X \mu_i \wedge \mu_j \wedge \mu_k &= k_{ijk}, \\ \int_X \mu_i \wedge \nu_a \wedge \nu_b &= \hat{k}_{iab}, & \int_X a_K \wedge b^J &= \delta_K^J, & \int_X \mathcal{A}_{\Lambda} \wedge \mathcal{B}^{\Delta} &= \delta_{\Lambda}^{\Delta}. \end{aligned} \quad (1)$$

Note that, there are two types of orientifold choices; one involving the O3/O7-planes in the fixed point set of the involution corresponding to $K = 1, \dots, h_+^{2,1}$ and $\Lambda = 0, \dots, h_-^{2,1}$, while the other one with O5/O9-planes such that $K = 0, \dots, h_+^{2,1}$ and $\Lambda = 1, \dots, h_-^{2,1}$.

Given that various fields/fluxes can be expanded in suitable bases of the equivariant cohomologies, the involutively-odd holomorphic 3-form Ω , which generically depends on the complex-structure moduli U^{α} counted by $h_-^{2,1}(X)$, can be written in terms of the period vectors in the following form:

$$\Omega \equiv \mathcal{X}^{\Lambda} \mathcal{A}_{\Lambda} - \mathcal{F}_{\Lambda} \mathcal{B}^{\Lambda}, \quad (2)$$

where $\mathcal{F} = (\mathcal{X}^0)^2 f(U^{\alpha})$ is a generic pre-potential, with $U^{\alpha} = \frac{\delta_{\Lambda}^{\alpha} \mathcal{X}^{\Lambda}}{\mathcal{X}^0}$ and with $f(U^{\alpha})$ being some function dependent on the complex-structure moduli [152]. Similarly, one can consider the Kähler form J expanded as $J = t^i \mu_i$, where t^i denotes 2-cycle volume moduli. In addition to the complex-structure and Kähler moduli, there exist other fields in the closed string spectrum of effective 4D supergravity theory rising from the type IIB superstring compactifications using CY threefolds. The latter is obtained by combining left and right moving open strings with Neveu-Schwarz (NS) and Ramond (R) boundary conditions. The following four combinations are possible:

$$(NS_+, NS_+), (R_-, R_-), (NS_+, R_-), (R_-, NS_+).$$

From the (NS_+, NS_+) sector we obtain the graviton $g_{\mu\nu}$, the dilaton ϕ and the Kalb-Ramond field B_2 which is a rank-2 antisymmetric tensor. The (R_-, R_-) sector gives rise to the p -form potentials C_p , $p = 0, 2, 4$. The C_0 potential and the dilaton field, in particular, define the usual axio-dilaton combination:

$$S = C_0 + i e^{-\phi} \equiv C_0 + \frac{i}{g_s}, \quad (3)$$

where $g_s = e^{\phi}$ corresponds to the string coupling after dilaton ϕ receives a VEV. The other NS-NS and RR p -form potentials can be expanded as:

$$B_2 = b^a \nu_a, \quad C_2 = c^a \nu_a, \quad C_4 = \rho_i \tilde{\mu}^i + \dots, \quad (4)$$

while b^a , c^a and ρ_i are various axions, and \dots denotes pieces from quantities (such as a dual-pair of spacetime 1-forms, and the 2-form dual to the scalar field ρ_i) which are not directly relevant for our purpose.

The dynamics of the $\mathcal{N} = 1$ type IIB 4D effective supergravity theory can be described by using the following chiral variables $(U^{\alpha}, S, G^a, T_i)$ defined as [153]:

$$\begin{aligned} U^{\alpha} &= v^{\alpha} - i u^{\alpha}, & S &= C_0 + i e^{-\phi} = C_0 + i s, & G^a &= c^a + S b^a, \\ T_i &= \left(\rho_i + \hat{k}_{iab} c^a b^b + \frac{1}{2} C_0 \hat{k}_{iab} b^a b^b \right) - i \left(\tau_i - \frac{s}{2} \hat{k}_{iab} b^a b^b \right), \end{aligned} \quad (5)$$

where $\tau_i = \frac{1}{2} k_{ijk} t^j t^k$ is an Einstein frame 4-cycle volume. Notice also that $\text{Im}(T_i) = -\tau_i$ as we follow the conventions of [138]. Let us also mention that throughout this review article, we will limit our discussion to the compactifying CY threefolds with holomorphic involutions such that $H_{-}^{1,1}(X)$ is trivial, which means that all the fluxes/moduli/axions counted by $a \in h_{-}^{1,1}$ indices are absent.

The tree-level Kähler potential \mathcal{K}_0 can be expressed in the following manner which depends logarithmically on the various moduli fields,

$$\mathcal{K}_0 = -\log\left(-i \int_X \Omega \wedge \bar{\Omega}\right) - \log(-i(S - \bar{S})) - 2 \log(\mathcal{V}), \quad (6)$$

where the internal volume \mathcal{V} of the CY threefold can be given in terms of the two-cycle volume moduli as follows:

$$\mathcal{V} = \frac{1}{6} k_{ijk} t^i t^j t^k, \quad (7)$$

where k_{ijk} denotes the classical triple intersection numbers on X .

From the p -form potentials and the RR-field one gets the field strengths $F_p = dC_{p-1}$, and $H_3 = dB_2$. A particular combination defined as $G_3 = F_3 + S H_3$ appears in type IIB action and the flux-induced superpotential of the moduli fields. The latter is a holomorphic function which depends on the axio-dilaton modulus S , and the CS moduli U^α given by the Gukov-Vafa-Witten (GVW) formula [154]:

$$W_0 = \int_X G_3 \wedge \Omega(U^\alpha). \quad (8)$$

Let us also mention here the fact that the T -dual completion of the GVW superpotential suggests the presence of several other fluxes in a generalised flux framework [8, 21, 153, 155–165] which also includes (non-) geometric fluxes, in addition to the standard S-dual pair of (F_3, H_3) fluxes. In fact, most of the well-known dS no-go scenarios which have been initially proposed in the (geometric) type IIA setting, have been T -dualised to some non-geometric type IIB settings in [21], which may be helpful in narrowing down the vast flux landscape while hunting for dS vacua. However, let us also admit that the naive thought of enriching the model via incorporation of various (non-) geometric fluxes for creating better model building possibilities may not be always helpful, as one does not clearly know how many and which types of fluxes can be consistently turned-on, in the lights of satisfying the the full set of constraints, e.g. those induced from the various Bianchi identities and the tadpole conditions [156, 157, 166, 167].

From the above general picture we may infer that due to the existence of many CY-manifolds leading to many moduli, axions and fluxes in the model, it is possible to have a large number of possibly realistic models arising from CY orientifold compactifications. Moreover, there are also many choices of fluxes that in principle could satisfy the required restrictions (such as tadpole cancellation etc). These possibilities entail an enormous number of ‘‘String Vacua’’ which constitute the so called ‘‘String Landscape’’. A long standing question therefore is whether there are any stable dS solutions in the vast String Landscape. We certainly know, however, that even if the answer is ‘yes’, dS vacua are certainly scarce. If this is the case indeed, it would be interesting to search for unique observational or experimental signatures associated with dS vacua.

Given the above, a reasonable sequence of tasks would be to ensure all moduli/axionic fields getting non-tachyonic masses in a dynamical fashion, resulting in a dS vacuum in some string (inspired) setting. If possible this should be: (i). based only on perturbative corrections, (ii). can be argued to be controllable, and (iii). can be generically present in effective underlying models. If these requirements are fulfilled one may also examine the cosmological implications of the effective model, such as inflation.

The effective potential is computed from the Kähler- and super-potential expressions using the standard supergravity formula,

$$V_{\text{eff}} = e^{\mathcal{K}} \left(\sum_{\alpha, \beta} (\mathcal{D}_\alpha W) \mathcal{K}^{\alpha\bar{\beta}} (\mathcal{D}_{\bar{\beta}} \bar{W}) - 3|W|^2 \right), \quad (9)$$

where $\mathcal{D}_\alpha = \partial_\alpha + \mathcal{K}_\alpha$ denotes the covariant derivatives with the index α running over all moduli fields. Now using (6) and (8) through the standard two-step procedure, one initially fixes the axio-dilaton S and the CS moduli U^α via the supersymmetric (flatness) conditions imposed to preserve the supersymmetry,

$$\mathcal{D}_S W_0 = 0, \quad \mathcal{D}_{U^\alpha} W_0 = 0. \quad (10)$$

However, the Kähler moduli do not appear in the perturbative GVW superpotential. Furthermore, at the classical level, i.e., when non-perturbative (in W)- or any perturbative (in \mathcal{K})- corrections are not present, the potential (9) vanishes identically $V_{\text{eff}} \equiv 0$, (by virtue of the no-scale structure), hence, the Kähler moduli cannot be stabilised at this level. In a second step, one considers the sub-leading (quantum) corrections which are used to stabilise the Kähler moduli. At this stage we tacitly assume that the masses of the dilaton S the CS moduli U^α have been fixed at much larger values than those of the Kähler moduli T_k to be fixed by sub-leading effects, so that this two-step procedure makes sense. In the next section, we are going to consider various possible corrections suitable for breaking the no-scale structure, and hence for fixing all the moduli.

3. Breaking the no-scale structure via various corrections

The GVW superpotential (8) induced through the background fluxes F_3 and H_3 can generically help in stabilising the complex-structure moduli along with the axio-dilaton, however all the Kähler moduli remain unfixed due to the so-called no-scale structure in the underlying 4D effective supergravity theory. In order to dynamically stabilise these moduli the subleading (quantum) corrections play an important role through the generation of additional pieces in the scalar potential. Such corrections can be either perturbative or non-perturbative in nature, and can be further classified in terms of two (possibly intertwined) series of terms: (i) via an expansion with respect to the string tension (where the leading order perturbative corrections have been found to be proportional to α'^3) and (ii) an expansion where the string coupling g_s acts like expansion parameter. Before proceeding to the discussion on their role in moduli stabilisation, let us elaborate a bit more on the form of these corrections which can be leading order in regards to the breaking the no-scale structure.

3.1 Non-perturbative corrections

One class of non-perturbative corrections can arise from the E3-brane instanton effects or gaugino condensation through D7-brane, wrapping some suitable 4-cycles inside the compactifying

CY threefold. Such a superpotential term can be expressed as [168] (see [169] also for zero-mode analysis):

$$W_{\text{np}} = \sum_k A_k e^{-i a_k T_k}, \quad (11)$$

where the coefficients A_k may generically depend on complex-structure moduli and the axio-dilaton (or any other moduli, e.g. open string deformations), but in most cases they are considered constants under the assumption that these moduli (unlike the Kähler moduli) do not have no-scale structure and subsequently can be stabilised by the leading order flux superpotential. Further, the coefficients a_k on the exponents are equal to $a_k = 2\pi$ for E3-instanton contributions and $a_k = 2\pi/N$ for gaugino condensation on stacks of $D7$ -branes, N being the rank of the corresponding gauge group. Such non-perturbative superpotential contributions have been proven to be the central ingredients for various popular Kähler moduli stabilisation schemes, e.g. KKLT [25], racetrack [170, 171] and LARGE volume scenarios (LVS) [172].

In addition to these, there are non-perturbative contributions appearing as exponential corrections on top of the usual E3-instanton or gaugino-condensation effects leading to the following schematic form of the superpotential [132],

$$W_{\text{np}}^{\text{poly}} = \sum_k A_k e^{-i a_k (T_k + \sum_w A_w e^{-i a_w T_w})}, \quad (12)$$

where A_w 's are again some parameters similar to A_k 's while T_w corresponds to the complexified four-cycle volume of the so-called "Wilson divisor" which are essential for generating this effect [132]. Such effects are known as poly-instanton corrections to the superpotential which can generate sub-leading contributions for Kähler moduli stabilisation and also help in driving inflation [133, 173–176].

Moreover, there are non-perturbative effects to the Kähler potential which can also generate contribution to the scalar potential, and have the potential of stabilising (many of) the moduli, e.g. the so-called worldsheet- and D1-instanton corrections, e.g. see [123, 177, 178]. Such corrections have received a lot of attention recently, in the context of realising the so-called perturbatively flat flux vacua [126, 179–182].

After enumerating the possible non-perturbative corrections which can generically induce scalar potential terms useful for moduli stabilisation, let us now review the possible leading order perturbative effects.

3.2 Perturbative α'^3 -corrections

3.2.1 BBHL corrections

Let us start by recalling that the tree level Kähler potential (6) -as far as the Kähler moduli are concerned- exhibits a no-scale structure⁵. It is well known, however, that the prepotential for the Kähler deformations receives corrections on the worldsheet which have been computed long time

⁵The no-scale is ensured since $\mathcal{K}_i \mathcal{K}^{i\bar{j}} \mathcal{K}_{\bar{j}} = 3$ for $\mathcal{K} = \mathcal{K}_0$, and the Kähler moduli part does not mix with the pieces involving the moduli U^α and S . The no-scale structure of the $\mathcal{N} = 1$ model is established if also the superpotential does not depend on T_i , hence, when non-perturbative terms are ignored, i.e., $W = W_0$.

ago [183]. In the Kähler potential these α' -corrections are captured by a shift in the volume \mathcal{V} of equation (6), given in reference [184], where it was shown that they arise from a term proportional to α'^3 in the ten-dimensional action. In fact, the equation of motion for the dilaton to order α'^3 is derived from the action [184]

$$S = \int d^{10}x \sqrt{-g^{10}} e^{2\phi} (R + 4(\partial\phi)^2 + \alpha'^3 J_0),$$

with solution $\phi = \phi_0 + \zeta(3)Q/16$, and Q defined through the 6-d Euler integrand $\chi = \int_{CY_3} d^6x \sqrt{g} Q$. Upon compactification, α'^3 -corrections are incorporated in the redefinition of 4D dilaton:

$$e^{-2\phi_4} = e^{-\frac{1}{2}\phi_{10}} (\mathcal{V} + \hat{\xi}/2),$$

where \mathcal{V} is the CY volume in the Einstein frame and the coefficient $\hat{\xi}$ appearing as a shift in the overall volume via a redefinition of the dilaton expressed in terms of the Euler characteristic of the manifold given as below,

$$\mathcal{V} = \frac{1}{3!} \kappa_{ijk} t^i t^j t^k, \quad \hat{\xi} = -\frac{\zeta(3) \chi(X)}{2(2\pi)^3 g_s^{3/2}}.$$

Note that the Einstein-frame volumes are related with their respective string-frame quantities as $\mathcal{V} = g_s^{-3/2} \mathcal{V}_s$ and $t^i = g_s^{-1/2} t_s^i$, where \mathcal{V}_s denotes the string-frame CY volume while t_s^i denotes the string-frame 2-cycle volume moduli. Subsequently, the α' -correction is incorporated into the Kähler potential through the shift ⁶:

$$\mathcal{V} \rightarrow \mathcal{Y}_0 = \mathcal{V} + \frac{\hat{\xi}}{2} \equiv \mathcal{V} + \frac{\xi}{2} \left(\frac{S - \bar{S}}{2i} \right)^{3/2} \equiv \mathcal{V} + \frac{\xi}{2g_s^{3/2}}.$$

As a consequence, the α' -corrected Kähler potential acquires the following form:

$$\mathcal{K}_{\alpha'} = -\log(-i \int_X \Omega \wedge \bar{\Omega}) - \log(-i(S - \bar{S})) - 2 \log(\mathcal{Y}_0). \quad (13)$$

Provided that the supersymmetric conditions (10) are implemented, when α' corrections are taken into account break the no-scale invariance and generate a non-vanishing scalar potential, $V_{\text{eff}} \neq 0$. In the large volume limit scenario (LVS) [138, 172, 185–187], for example, the leading term of the scalar potential is proportional to $V_{\text{eff}}^{\text{LVS}} \propto \frac{\hat{\xi}}{\mathcal{V}^3} |W_0|^2$. Furthermore with the inclusion of non-perturbative corrections in the superpotential as in (12) and an uplifting term to the scalar potential emerging from appropriate contributions either from $\overline{D3}$ branes or D-terms, it is possible to stabilise the Kähler moduli and uplift the vacuum to a de-Sitter one. We will get back to this aspect in some detail later on.

3.2.2 Higher derivative F^4 -corrections

Apart from the BBHL correction which appears at the two-derivative level through the Kähler potential, there are additional higher derivative $\mathcal{O}(F^4)$ effects [188]. These corrections are also

⁶ ξ appears into the prepotential $\mathcal{F} = \frac{i}{3!} k_{abc} \frac{X^a X^b X^c}{X^0} + \xi (X^0)^2$ [104].

induced at $(\alpha')^3$ order like BBHL, and can be argued to be generic for a given CY orientifold model⁷. The scalar potential can be expressed in the following simple form:

$$V_{F^4} = -\frac{g_s^2 \lambda |W_0|^4}{4 g_s^{3/2} \mathcal{V}^4} \Pi_i t^i, \quad (14)$$

where the t^i 's are the volumes of the 2-cycles for the generic CY manifold (X), λ is an unknown combinatorial factor whose value is expected to be around 10^{-3} [190], and the Π_i 's are topological numbers, also called *second Chern numbers*, defined as:

$$\Pi_i = \int_{D_i} c_2(X). \quad (15)$$

In recent years, these corrections have received a significant amount of attention in the context of LVS moduli stabilisation and inflation, e.g. see [137, 139, 191].

3.3 Perturbative string-loop corrections

The known string-loop corrections can be classified into two parts; first, those which are of the so-called ‘‘KK-type’’ and ‘‘winding-type’’ as proposed in [192–195], while the second class consists of those which are of ‘‘logarithmic-type’’ as proposed in [37, 39]. Let us present the relevant pieces of information about those classes one-by-one.

3.3.1 KK-type and winding-type corrections

The initial computations of the string-loop corrections to the Kähler potential have been done for the toroidal models, e.g. see [192–195]. Subsequently, using some guided route, those have been eventually conjectured for generic CY orientifolds as well [186]. These so-called KK- and winding-type string-loop corrections have been conjectured to take the following form in the Einstein-frame [186]:

$$\delta\mathcal{K}_{g_s}^{\text{KK}} = g_s \sum_i \frac{C_i^{\text{KK}} t_{\perp}^i}{\mathcal{V}}, \quad \delta\mathcal{K}_{g_s}^{\text{W}} = \sum_i \frac{C_i^{\text{W}}}{\mathcal{V} t_{\cap}^i}, \quad (16)$$

where the quantities C_i^{KK} and C_i^{W} are some generic functions depending on the complex-structure moduli (and the other possible moduli as well, e.g. the open string moduli). The 2-cycle volume moduli t_{\perp}^i denote the transverse distance among stacks of non-intersecting $D7$ -branes and $O7$ -planes while t_{\cap}^i denotes the size of the non-contractible 1-cycles lying on the intersection loci of the two $D7/O7$ stacks. A couple of concrete global realisations of these Ansätze have been presented in explicit CY orientifold models in [136, 137]. The scalar potential terms arising from the corrections in (16) can be given as [186]:

$$\begin{aligned} \delta V_{g_s}^{\text{KK}} &= \frac{g_s^3 |W_0|^2}{2 \mathcal{V}^2} \sum_{ij} C_i^{\text{KK}} C_j^{\text{KK}} \mathcal{K}_{ij}^0, \\ \delta V_{g_s}^{\text{W}} &= -g_s \frac{|W_0|^2}{\mathcal{V}^2} \delta\mathcal{K}_{g_s}^{\text{W}} = -g_s \frac{|W_0|^2}{\mathcal{V}^3} \sum_i \frac{C_i^{\text{W}}}{t_{\cap}^i}. \end{aligned} \quad (17)$$

⁷In the context of CY threefolds having divisors of specific topologies, such corrections may be (partially) absent; for example see [79, 189].

Here, \mathcal{K}_{ij}^0 is the tree-level Kähler metric:

$$\mathcal{K}_{ij}^0 = \frac{1}{16\mathcal{V}^2} (2t^i t^j - 4\mathcal{V} k^{ij}) , \quad (18)$$

with $k^{ij} = (k_{ij})^{-1} = (k_{ijk} t^k)^{-1}$. It has been found that the scalar potential is protected against the leading-order pieces of the KK-type corrections due to the so-called ‘‘extended no-scale structure’’ [186, 196], and subsequently the leading order string-loop pieces in the scalar potential are sub-leading as compared to those of the BBHL’s α'^3 effects despite the later being leading order in the Kähler potential.

The initial arguments of [192–195] have been such that the so-called winding loop corrections can arise whenever two divisors wrapped by O -planes or D -branes intersect each other, admitting a non-contractible one-cycle at the intersection locus while the KK-corrections can arise instead from the exchange of KK modes between non-intersecting D -branes/ O -planes. However, building on the field theoretic approach of [196], in a recent revisit [197], it has been found that the winding-type effects can appear more generically as to what is expected from [192–195].

3.3.2 Logarithmic corrections

Apart from the KK-type and winding-type loop corrections discussed so far, there are additional one-loop effects which can appear as logarithmic corrections in the Kähler potential [37, 39]. These effects can serve as an alternative way to stabilise the Kähler moduli using the perturbative effects only [37, 39]⁸. In the context of the suggested geometric configuration of intersecting $D7$ -brane stacks, such corrections emerge when higher-order curvature terms in the ten-dimensional effective action are also taken into account. The leading correction term of this type is proportional to the fourth power of curvature R^4 . Including this term, the type IIB action takes the following form:

$$\mathcal{S} \supset \frac{1}{(2\pi)^7 \alpha'^4} \int_{M_{10}} e^{-2\phi_{10}} R_{(10)} - \frac{6}{(2\pi)^7 \alpha'} \int_{M_{10}} \left(-2\zeta(3) e^{-2\phi_{10}} - 4\zeta(2) \right) R_{(10)}^4 \wedge e^2, \quad (19)$$

where $R_{(D)}$ is the D -dimensional Ricci scalar, $\phi_{(D)}$ the D -dimensional dilaton, and $R_{(10)}^4 \wedge e^2$ is a short hand expression in differential forms (see [39], as well as [198] and [199]). Compactifying six dimensions on a CY threefold (X) while taking into account the tree-level and one-loop generated EH terms, the ten-dimensional action reduces to

$$\mathcal{S}_{\text{grav}} = \frac{1}{(2\pi)^7 \alpha'^4} \int_{M_4 \times X} e^{-2\phi_{10}} R_{(10)} + \frac{\chi}{(2\pi)^4 \alpha'} \int_{M_4} \left(2\zeta(3) e^{-2\phi_{10}} + 4\zeta(2) \right) R_{(4)}, \quad (20)$$

Here, $M_{10} = M_4 \times X$, and χ is the Euler characteristic which defined as

$$\chi = \frac{3}{4\pi^3} \int_X R \wedge R \wedge R .$$

One readily observes that χ contains three powers of the Ricci scalar R , thus it follows that the effective EH term $R_{(4)}$ emerging from the corresponding one $R_{(10)}^4$, is possible only in four

⁸In this subsection, for completeness, a few computational details are included. The reader not interested in these details can skip this section.

spacetime dimensions. Besides this, $R^4_{(10)}$ can be considered as a vertex localised at the specific points in the bulk space where $\chi \neq 0$. As such, these vertices can emit gravitons and Kaluza-Klein (KK) excitations within the compactified six-dimensional space. The one-loop corrections are extracted from the graviton scattering amplitude involving one Kaluza-Klein (KK)-excitation in the $(-1, -1)$ -ghost picture as well as two massless gravitons in the $(0, 0)$ picture as it is depicted in Figure 1. Performing the computations in the orbifold limit \mathbb{T}^6/Z_N [39, 199] we can express the final result as follows:

$$\langle V^2_{(0,0)} V_{(-1,-1)} \rangle = -C_R \frac{1}{N^2} \sum_{f,k} e^{i\gamma^k q \cdot x_f} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \int \prod_{i=1,2,3} \frac{d^2z_i}{\tau_2} \sum'_{(h,g)} e^{\alpha' q^2 F_{(h,g)}(\tau, z_i)} .$$

On the right-hand side of the above equation C_R is a constant related to the tensor structure and the N is the integer related to the orbifold lattice Z_N , while the symbol q appearing in the two exponents refers to the KK-momentum. The first sum over (f, k) is associated with the orbifold's fixed points x_f and the representation γ^k of the orbifold's group action. The second sum is over the pairs (h, g) of the orbifold sectors and the prime means that we exclude the untwisted sector $(0, 0)$. In addition, the function $F_{(h,g)}$ appearing in the exponent, indicates the contribution of the twisted sector $(f, g) = (l, m) \frac{v}{N}$. Notice that the integration over the world-sheet torus modulus $\tau = \tau_1 + i\tau_2$ should be restricted in the fundamental domain of the modular group. In order to express the

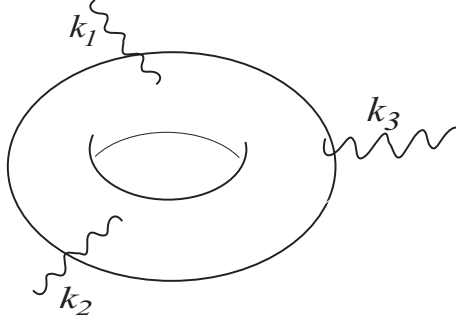


Figure 1: Non-zero contribution from 1-loop; 3-graviton scattering amplitude of 2 massless and 1 KK.

localisation property associated with the aforementioned vertex it is convenient to introduce the definition of the localisation width, denoted hereafter with w [39, 199],

$$w^2 \approx \alpha' F_{(h,g)}(\tau, z_a) \Big|_{\min} \sim \frac{l_s^2}{N} . \quad (21)$$

Here, as usual, $l_s = 2\pi\sqrt{\alpha'}$ stands for the fundamental string length. Then, computing the amplitude it is found that the coefficient of the induced EH action $\mathcal{R}_{(4)}$ exhibits a Gaussian profile

$$C_R \frac{N}{w^6} e^{-w^2 q_\perp^2 / 2} , \quad (22)$$

where q_\perp is the internal momentum along the directions transverse to the D7-brane stack. Then, adopting the interpretation of N as the Euler characteristic $N \rightarrow \chi$, the one loop correction (expressed in terms of the width w and χ) obtains the following form

$$\frac{4\zeta(2)}{(2\pi)^7 \alpha' \chi} \int_{M_4 \times \chi} \frac{e^{-y^2 / (2w^2)}}{w^6} \mathcal{R}_{(4)} , \quad (23)$$

Within the type IIB framework, the geometric configuration may also include $D7$ branes and orientifold $O7$ planes. In this context there can be an exchange of KK-modes between the localised gravity vertex and the various stacks of $D7$ -branes. Since the latter reside in four internal dimensions, KK-excitations transmitted towards each one of them propagate in a 2-dimensional bulk transverse to $D7$. For a visual representation this is depicted in Figure 2. In this way, logarithmic corrections are generated. The magnitude of each one of them depends on the size of the two dimensional transverse space. Using (20) and (23) while replacing $N \rightarrow \chi$, we can compute the corresponding contribution which takes the form (see the relevant works for the details [39, 199])

$$\mathcal{A}_S = -C_R g_s^2 \mathcal{T} \frac{2\pi}{\sin \frac{2\pi}{N}} \left\{ -\frac{\gamma}{2} + \ln \left(\frac{R_\perp \sqrt{2}}{w} \right) + O \left(\frac{w^2}{R_\perp^2} \right) \right\}, \quad (24)$$

where \mathcal{T} represents the $D7$ -tension and R_\perp stands for the size of the transverse space. We observe that the amplitude has a logarithmic dependence on R_\perp . Summing up the two contributions (22) and (24) one finds:

$$\frac{4\zeta(2)}{(2\pi)^3} \chi \int_{M_4} \left(1 - \sum_k e^{2\phi} \mathcal{T}_k \ln(R_\perp^k/w) \right) R_{(4)}. \quad (25)$$

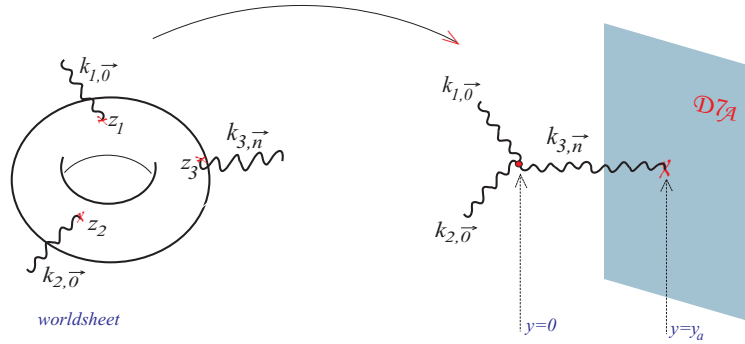


Figure 2: Non-zero contribution from 1-loop; 3-graviton scattering amplitude of 2 massless and 1 KK propagating in 2-dimensions towards a $D7$ brane.

For the implementation of the prescribed plan in the subsequent analysis, a geometric configuration of three intersecting $D7$ brane stacks with quantised 3-form fluxes proposed in [37] can be considered. Each $D7$ brane-stack spans four compact dimensions while it is localised at the remaining two. Table 1 shows how the setup of the $D7$ stacks is arranged in the internal space. Secondly, a novel Einstein-Hilbert (EH) term stemming from R^4 terms in the 10D action, can be taken into account which acts as a graviton vertex in the 6D bulk. In fact the graviton excitations transmitted towards the $D7_i$ -brane stacks can induce loop-corrections which generate the minima of the F-term scalar potential. Subsequently it turns out that the logarithmic one-loop corrections [39] discussed in the this section take the general form $C_{\log} = \sigma + \sum_k \eta_k \log(\tau_k)$. Here, τ_k are the Kähler (volume) moduli defined in the previous section, $\sigma \propto \chi$ and the coefficients η_k depend on the specific geometric setup and the tension of the branes, e.g. the $D7$ -brane tension $\mathcal{T}_i = g_s T_0$ and R_\perp^i describing the $D7$ -transverse 2-dimensions. We will detail these aspects more in the upcoming moduli stabilisation section.

$D7_s$	4d Minkowski				6 Compact Dimensions					
	0	1	2	3	4	5	6	7	8	9
$D7_1$		*	*	*	*	*	*	*	.	.
$D7_2$		*	*	*	*	*	.	.	*	*
$D7_3$		*	*	*	.	.	*	*	*	*

Table 1: $D7$ -brane configuration of the model under consideration. $D7_1$ stack resides in ‘4’‘5’‘6’‘7’ internal dimensions and intersects with $D7_3$ along ‘6’‘7’.

4. Moduli stabilisation schemes in type IIB models

4.1 Models with non-perturbative effects

As we discussed, the effective theory model is described by $\mathcal{N} = 1$ supergravity with the tree-level superpotential given by (8), which is a function of the axio-dilaton S and the complex-structure moduli U^α but does not depend on the Kähler moduli T_i . The Kähler moduli can appear through various (non-)perturbative corrections. In this case the superpotential takes the form (e.g. see [200]):

$$W = W_0 + \sum_k A_k e^{-i a_k T_k}. \quad (26)$$

with W_0 given by (8), and the coefficients A_k may be considered as some constant parameters while $a_k = 2\pi$ for E3-instanton contributions and $a_k = 2\pi/N$ for gaugino condensation on $D7$ branes.

Using the generic superpotential (26) along with the BBHL corrected Kähler potential (13), a master formula for the scalar potential has been presented in [138]. This takes the following form,

$$V = V_{O(\alpha'^3)} + V_{np1} + V_{np2}, \quad (27)$$

where considering $W_0 = |W_0| e^{i\theta_0}$ and $A_i = |A_i| e^{i\phi_i}$ we get:

$$\begin{aligned} V_{O(\alpha'^3)} &= e^{\mathcal{K}} \frac{3\hat{\xi}(\mathcal{V}^2 + 7\mathcal{V}\hat{\xi} + \hat{\xi}^2)}{(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})^2} |W_0|^2, \\ V_{np1} &= e^{\mathcal{K}} \sum_{i=1}^n 2 |W_0| |A_i| e^{-a_i \tau_i} \cos(a_i \rho_i + \theta_0 - \phi_i) \\ &\quad \times \left[\frac{(4\mathcal{V}^2 + \mathcal{V}\hat{\xi} + 4\hat{\xi}^2)}{(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})} (a_i \tau_i) + \frac{3\hat{\xi}(\mathcal{V}^2 + 7\mathcal{V}\hat{\xi} + \hat{\xi}^2)}{(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})^2} \right], \\ V_{np2} &= e^{\mathcal{K}} \sum_{i=1}^n \sum_{j=1}^n |A_i| |A_j| e^{-(a_i \tau_i + a_j \tau_j)} \cos(a_i \rho_i - a_j \rho_j - \phi_i + \phi_j) \\ &\quad \times \left[-4 \left(\mathcal{V} + \frac{\hat{\xi}}{2} \right) (k_{ijk} t^k) a_i a_j + \frac{4\mathcal{V} - \hat{\xi}}{(\mathcal{V} - \hat{\xi})} (a_i \tau_i) (a_j \tau_j) \right. \\ &\quad \left. + \frac{(4\mathcal{V}^2 + \mathcal{V}\hat{\xi} + 4\hat{\xi}^2)}{(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})} (a_i \tau_i + a_j \tau_j) + \frac{3\hat{\xi}(\mathcal{V}^2 + 7\mathcal{V}\hat{\xi} + \hat{\xi}^2)}{(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})^2} \right]. \end{aligned} \quad (28)$$

Here we assume that the complex-structure moduli and the axio-dilaton are supersymmetrically stabilised by the leading order effects. Notice that the BBHL term [184] vanishes for $\hat{\xi} = 0$,

reproducing the standard no-scale structure, and for very large volume $\mathcal{V} \gg \hat{\xi}$, this term takes the standard form which plays a crucial rôle in LVS models [172]:

$$V_{O(\alpha'^3)} \simeq \frac{e^{K_{cs}}}{2s\mathcal{V}^2} \times \frac{3\hat{\xi}|W_0|^2}{4\mathcal{V}}. \quad (29)$$

Moreover the master formula (28) can be directly used for moduli stabilisation purposes, and can be considered to be having the following advantages:

- Moduli stabilisation is more naturally performed in terms of the 2-cycle moduli t^i which helps in bypassing the “ t^i -to- τ_i ” conversion needed for defining the chiral coordinates to be used for the Kähler metric computations.
- Given that the master formula (28) completely determines the scalar potential by merely specifying some topological quantities (such as k_{ijk} and χ of the CY), and the number n regarding the non-perturbative contributions to W , it can elegantly reproduce known moduli stabilisation schemes by choosing parameters like $h_+^{1,1}$, n and $\hat{\xi}$ as can be seen from Table 2.

Model	$h_+^{1,1}$	n	$\hat{\xi}$
1-modulus KKLT [25]	1	1	$\hat{\xi} = 0$
1-modulus α' -uplift [28, 30, 201]	1	1	$\hat{\xi} > 0$
2-moduli KKLT [170, 171]	2	2	$\hat{\xi} = 0$
2-moduli α' -uplift [32]	2	2	$\hat{\xi} > 0$
2-moduli Swiss cheese LVS [33, 131, 134, 172]	2	1	$\hat{\xi} > 0$
3-moduli Swiss cheese LVS [35, 202]	3	2	$\hat{\xi} > 0$
3-moduli fibred LVS [203]	3	2	$\hat{\xi} > 0$

Table 2: A set of models for which the scalar potentials can be read-off from the master formula [138].

- A general analysis for multi-axion models can be performed using the master formula.

4.1.1 KKLT-like scenarios

In order to get the so-called KKLT potential [25], one simply needs to consider $h_+^{1,1} = n = 1$ and $\hat{\xi} = 0$ for which case, the 3 contributions to the general potential given in (28) become:

$$\begin{aligned} V_{O(\alpha'^3)} &= 0, & V_{np1} &= 4e^{\mathcal{K}} |W_0| |A_1| a_1 \tau_1 e^{-a_1 \tau_1} \cos(a_1 \rho_1 + \theta_0 - \phi_1), \\ V_{np2} &= 4e^{\mathcal{K}} |A_1|^2 e^{-2a_1 \tau_1} \left(-\mathcal{V} a_1^2 k_{111} t_1 + (a_1 \tau_1)^2 + a_1 \tau_1 \right), \end{aligned} \quad (30)$$

where:

$$\mathcal{V} = \frac{1}{6} k_{111} t_1^3, \quad \tau_1 = \frac{1}{2} k_{111} t_1^2, \quad e^{\mathcal{K}} = \frac{e^{K_{cs}}}{2s\mathcal{V}^2}. \quad (31)$$

This subsequently leads to the standard KKLT scalar potential,

$$V_{\text{KKLT}} = \frac{9e^{K_{cs}} a_1 k_{111} |A_1|}{s\tau_1^2} e^{-a_1 \tau_1} \left[|W_0| \cos(a_1 \rho_1 + \theta_0 - \phi_1) + \frac{|A_1|}{3} e^{-a_1 \tau_1} (a_1 \tau_1 + 3) \right], \quad (32)$$

which admits a supersymmetric AdS minimum.

4.1.2 Racetrack scenarios

Another moduli stabilisation scheme, known as ‘‘racetrack’’ [170, 171] can be considered using the master formula for the choice of parameters $h_+^{1,1} = n = 2$ and $\hat{\xi} = 0$ resulting in:

$$\begin{aligned} V = & \frac{e^{K_{cs}}}{2s\mathcal{V}^2} \left[\sum_{i=1}^2 4a_i\tau_i |W_0| |A_i| \cos(a_i\rho_i + \theta_0 - \phi_i) e^{-a_i\tau_i} \right. \\ & + \sum_{\substack{i=1 \\ i \neq j}}^2 \frac{4}{3} |A_i|^2 a_i^2 \tau_i^2 e^{-2a_i\tau_i} \left(1 + 2\sqrt{\frac{k_{iii}}{k_{jjj}}} \left(\frac{\tau_j}{\tau_i}\right)^{3/2} + \frac{3}{a_i\tau_i} \right) \\ & \left. + 8a_1a_2 |A_1| |A_2| e^{-(a_1\tau_1 + a_2\tau_2)} \cos(a_1\rho_1 - a_2\rho_2 - \phi_1 + \phi_2) \left(\tau_1\tau_2 + \frac{a_1\tau_1 + a_2\tau_2}{2a_1a_2} \right) \right]. \end{aligned}$$

where, like the swiss-cheese CY realised as degree-18 hypersurface in $\mathbb{CP}^4[1, 1, 1, 6, 9]$, we assumed that only $k_{111} \neq 0$ and $k_{222} \neq 0$ leading to the overall volume being given as,

$$\mathcal{V} = \frac{1}{6} \left(k_{111} t_1^3 + k_{222} t_2^3 \right) = \frac{\sqrt{2}}{3\sqrt{k_{222}}} \left(\tau_2^{3/2} - \sqrt{\frac{k_{222}}{k_{111}}} \tau_1^{3/2} \right), \quad (33)$$

where the minus sign in the last piece is dictated by the Kähler cone condition $t_1 < 0$, for the choice of basis having the exceptional divisor with the corresponding volume τ_1 .

4.1.3 LARGE volume scenarios (LVS)

In order to arrive at the standard LVS scalar potential [33, 131, 134, 172] using the master formula, one needs to consider the parameters as: $h_+^{1,1} = 2$, $n = 1$ and $\hat{\xi} > 0$ which results in the following pieces,

$$\begin{aligned} V_{O(a^3)} &= e^{\mathcal{K}} \frac{3\hat{\xi}(\mathcal{V}^2 + 7\hat{\xi}\mathcal{V} + \hat{\xi}^2)}{(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})^2} |W_0|^2, \quad (34) \\ V_{np1} &= 2e^{\mathcal{K}} |W_0| |A_1| e^{-a_1\tau_1} \cos(a_1\rho_1 + \theta_0 - \phi_1) \\ &\quad \times \left[\frac{(4\mathcal{V}^2 + \mathcal{V}\hat{\xi} + 4\hat{\xi}^2)}{(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})} (a_1\tau_1) + \frac{3\hat{\xi}(\mathcal{V}^2 + 7\hat{\xi}\mathcal{V} + \hat{\xi}^2)}{(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})^2} \right], \\ V_{np2} &= 4e^{\mathcal{K}} |A_1|^2 e^{-2a_1\tau_1} \left[-\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right) a_1^2 k_{111} t_1 + \frac{4\mathcal{V} - \hat{\xi}}{4(\mathcal{V} - \hat{\xi})} (a_1\tau_1)^2 \right. \\ &\quad \left. + \frac{(4\mathcal{V}^2 + \mathcal{V}\hat{\xi} + 4\hat{\xi}^2)}{2(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})} (a_1\tau_1) + \frac{3\hat{\xi}(\mathcal{V}^2 + 7\hat{\xi}\mathcal{V} + \hat{\xi}^2)}{4(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})^2} \right]. \end{aligned}$$

In the large volume limit, the leading order pieces can be given as:

$$\begin{aligned} V \simeq & \frac{e^{K_{cs}}}{2s} \left[\frac{3\hat{\xi}|W_0|^2}{4\mathcal{V}^3} - \frac{4a_1^2 |A_1|^2 k_{111} t_1}{\mathcal{V}} e^{-2a_1\tau_1} \right. \\ & \left. + \frac{4a_1\tau_1 |W_0| |A_1|}{\mathcal{V}^2} e^{-a_1\tau_1} \cos(a_1\rho_1 + \theta_0 - \phi_1) \right]. \quad (35) \end{aligned}$$

Again considering the swiss-cheese type CY one gets:

$$V \simeq \frac{e^{K_{cs}}}{2s} \left(\frac{\beta_{\alpha'}}{\mathcal{V}^3} + \beta_{np1} \frac{\tau_1}{\mathcal{V}^2} e^{-a_1\tau_1} \cos(a_1\rho_1 + \theta_0 - \phi_1) + \beta_{np2} \frac{\sqrt{\tau_1}}{\mathcal{V}} e^{-2a_1\tau_1} \right), \quad (36)$$

with:

$$\beta_{\alpha'} = \frac{3\hat{\xi}|W_0|^2}{4}, \quad \beta_{\text{np1}} = 4a_1|W_0||A_1|, \quad \beta_{\text{np2}} = 4a_1^2|A_1|^2\sqrt{2k_{111}}. \quad (37)$$

Notice that (36) matches the form of the potential of standard Swiss cheese LVS models with 2 Kähler moduli [172] which results in stable non-supersymmetric AdS solutions.

4.2 Models without non-perturbative effects

Although it is hard to argue about the superiority of using non-perturbative effects over the perturbative effects and vice-versa, however let us make the following points:

- Non-perturbative effects are quite model specific as we will discuss in the motivation of the global embedding and construction of explicit CY orientifolds in one of the upcoming sections. On contrary, one may argue that perturbative effects may be relatively more generically present. One may take the example of BBHL which is non-vanishing for non-zero $\chi(X)$ while for E3-instanton, gaugino condensation or poly-instanton effects one needs ‘special’ types of 4-cycle in the compactifying CY threefold.
- It maybe fair to argue that one has better understanding of perturbative effects, e.g. in the sense of their 10D origin, as compared to the non-perturbative effects.
- In the sense of lacking iterative behaviour in terms of some expansion parameter, the non-perturbative effects may lead to the issue of the overall control of the model, unless, either they are argued to be absent by specific local construction or if present, are suitably engineered in a useful manner, e.g. in LVS.

Based on these reasons it is always good to seek for alternatives to the standard type IIB moduli stabilisation schemes like KKLT, racetrack and LVS where non-perturbative effects are used for stabilising the Kähler moduli. On these lines, in the remaining part of this section, we will discuss a couple of alternative schemes of purely perturbative nature.

4.2.1 Non-geometric scenarios

In the meantime, the so-called non-geometric flux compactification scenarios have also received some significant amount of attention for string-inspired model building [10, 156, 161, 204–213]. The origin of such (non-geometric) fluxes is argued via a successive application of T-duality on the NS-NS 3-form H_3 -flux which appears as [155],

$$H_{ijk} \longrightarrow \omega_{ij}{}^k \longrightarrow Q^{jk}{}_i \longrightarrow R^{ijk}. \quad (38)$$

Here ω denotes the so-called geometric (or metric) flux while Q/R denote non-geometric fluxes. In the presence of such fluxes, the GVW flux superpotential can receive terms in which the Kähler moduli can generically appear, e.g. taking the following form [158, 160, 214–217],

$$W_{\text{ng}} = \int_X [F + SH + \omega_a G^a + \hat{Q}^i T_i]_3 \wedge \Omega(U^\alpha), \quad (39)$$

where $[\dots]_3$ denotes a combination of 3-forms involving the F_3, H_3 and the non-geometric Q -flux which can be expressed in a more suitable symplectic/cohomology notation as compared to

using the real six-dimensional indices. The appearance of various such fluxes in the generalised superpotential (39) introduces a new set of parameters in the effective scalar potential which can subsequently lead to facilitating the moduli stabilisation process by fixing all moduli at tree level. This can be considered as one of the most attractive features of including non-geometric fluxes, which sometimes also creates the possibility of realising dS vacua in non-geometric setting [8, 208–212, 218].

In addition to the set of fluxes $\{H, \omega, Q, R\}$, the modular completion arguments for the 4D effective type IIB supergravity theory demand to introduce a new type of non-geometric P -flux which forms a S-dual pair with the non-geometric Q -flux, similar to the S-dual pair of (F_3, H_3) fluxes [158, 160, 214, 219–222]. Subsequently, a modular completed version of the so-called generalised flux superpotential (39) takes the following form, e.g. see [158, 160, 214, 215, 223],

$$W_{\text{ng}} = \int_X \left[(F + SH) + \omega_a G^a + \left(\hat{Q}^i + S \hat{P}^i \right) T_i - \frac{1}{2} \hat{P}^i \hat{k}_{iab} G^a G^b \right]_3 \wedge \Omega(U^\alpha), \quad (40)$$

where as we defined earlier $[\dots]_3$ denotes a combination of 3-forms involving all the fluxes expressed in a symplectic/cohomology notation [156]. Most of the non-geometric flux models studied in the literature have been initiated with some toroidal setting [8, 158, 207, 209, 210, 214, 219], e.g. using orientifolds of $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ and $\mathbb{T}^6/(\mathbb{Z}_4)$ etc. However, the 10D origin of the 4D effective type IIB scalar potentials have been explored in a series of works [216, 217, 223–230]. In addition, applications towards moduli stabilisation, searching dS vacua as well as building inflationary models using these fluxes have been initiated in [24, 211, 215, 218, 231–233].

4.2.2 Perturbative LVS

In the context of superstring compactifications consisting of some specific intersecting $D7$ -brane stacks, a new localised Einstein-Hilbert term can be generated from the dimensional reduction of the R^4 terms in the effective ten-dimensional action. As it has been shown in [39] within this configuration logarithmic corrections appear due to local tadpoles induced by the localised gravity kinetic terms. In the presence of such logarithmic one-loop corrections proposed in [39], the corrected Kähler potential can be expressed in the following form,

$$\mathcal{K} = -\log \left(-i \int_X \Omega \wedge \bar{\Omega} \right) - \log(-i(S - \bar{S})) - 2 \log \mathcal{Y}, \quad (41)$$

where both the α'^3 and the logarithmic corrections are incorporated in the new function \mathcal{Y} such that

$$\mathcal{Y} \equiv \mathcal{Y}_0 + \mathcal{Y}_1, \quad (42)$$

where,

$$\begin{aligned} \mathcal{Y}_0 &= \mathcal{V} + \frac{\xi}{2} e^{-\frac{3}{2}\phi} = \mathcal{V} + \frac{\xi}{2} \left(\frac{S - \bar{S}}{2i} \right)^{3/2}, \\ \mathcal{Y}_1 &= e^{\frac{1}{2}\phi} f(\mathcal{V}) = \left(\frac{S - \bar{S}}{2i} \right)^{-1/2} f(\mathcal{V}). \end{aligned} \quad (43)$$

This leads to the following form of the Kähler metric [139],

$$\mathcal{K}_{S\bar{S}} = \mathcal{P}_1, \quad \mathcal{K}_{T_i\bar{S}} = t^i \mathcal{P}_2 = \mathcal{K}_{S\bar{T}_i}, \quad \mathcal{K}_{T_i\bar{T}_j} = (t^i t^j) \mathcal{P}_3 - k^{ij} \mathcal{P}_4, \quad (44)$$

and its inverse

$$\mathcal{K}^{S\bar{S}} = \tilde{\mathcal{P}}_1, \quad \mathcal{K}^{T_i\bar{S}} = k_i \tilde{\mathcal{P}}_2 = \mathcal{K}^{S\bar{T}_i}, \quad \mathcal{K}^{T_i\bar{T}_j} = (k_i k_j) \tilde{\mathcal{P}}_3 - k_{ij} \tilde{\mathcal{P}}_4, \quad (45)$$

where the functions $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3$ and \mathcal{P}_4 are collected as below,

$$\begin{aligned} \mathcal{P}_1 &= \frac{1}{8s^2 \mathcal{Y}^2} \left(\mathcal{V}(\mathcal{Y} + \mathcal{V}) - 4\hat{\xi}(\mathcal{Y} - \mathcal{V}) + 4\hat{\xi}^2 \right), \\ \mathcal{P}_2 &= -\frac{1}{8s \mathcal{Y}^2} \left(\frac{3}{2} \hat{\xi} - s^{-\frac{1}{2}} f + s^{-\frac{1}{2}} (\mathcal{V} + 2\hat{\xi}) \frac{\partial f}{\partial \mathcal{V}} \right), \\ \mathcal{P}_3 &= \frac{1}{8 \mathcal{Y}^2} \left(1 + s^{-\frac{1}{2}} \frac{\partial f}{\partial \mathcal{V}} - \mathcal{Y} s^{-\frac{1}{2}} \frac{\partial^2 f}{\partial \mathcal{V}^2} \right), \\ \mathcal{P}_4 &= \frac{1}{4 \mathcal{Y}} \left(1 + s^{-\frac{1}{2}} \frac{\partial f}{\partial \mathcal{V}} \right), \end{aligned} \quad (46)$$

while the functions $\tilde{\mathcal{P}}_1, \tilde{\mathcal{P}}_2, \tilde{\mathcal{P}}_3$ and $\tilde{\mathcal{P}}_4$ are given as,

$$\begin{aligned} \tilde{\mathcal{P}}_1 &= \frac{\mathcal{P}_4 - 6\mathcal{P}_3 \mathcal{V}}{\mathcal{P}_1 \mathcal{P}_4 + 6\mathcal{P}_2^2 \mathcal{V} - 6\mathcal{P}_1 \mathcal{P}_3 \mathcal{V}}, \\ \tilde{\mathcal{P}}_2 &= \frac{\mathcal{P}_2}{\mathcal{P}_1 \mathcal{P}_4 + 6\mathcal{P}_2^2 \mathcal{V} - 6\mathcal{P}_1 \mathcal{P}_3 \mathcal{V}}, \\ \tilde{\mathcal{P}}_3 &= \frac{\mathcal{P}_2^2 - \mathcal{P}_1 \mathcal{P}_3}{\mathcal{P}_4 (\mathcal{P}_1 \mathcal{P}_4 + 6\mathcal{P}_2^2 \mathcal{V} - 6\mathcal{P}_1 \mathcal{P}_3 \mathcal{V})}, \\ \tilde{\mathcal{P}}_4 &= (\mathcal{P}_4)^{-1}. \end{aligned} \quad (47)$$

Now assuming that the complex-structure moduli and the axio-dilaton are fixed by the leading order effects, the scalar potential for the Kähler moduli can be generically given by the following master formula,

$$V_{\alpha' + \log g_s} = e^{\mathcal{K}} |W|^2 \left[\frac{3 \mathcal{V}}{2 \mathcal{Y}^2} \left(1 + \frac{\partial \mathcal{Y}_1}{\partial \mathcal{V}} \right)^2 (6 \mathcal{V} \tilde{\mathcal{P}}_3 - \tilde{\mathcal{P}}_4) - 3 \right]. \quad (48)$$

This master formula can result in quite complicated terms for generic function $f(\mathcal{V})$. However, assuming that $\eta_1 = \eta_2 = \eta_3 = \eta$ exploiting the underlying exchange symmetries within the model, we take the following form for $f(\mathcal{V})$ [39],

$$f(\mathcal{V}) = \sigma + \eta \log(\mathcal{V}), \quad (49)$$

where coefficients η and σ are expressed in terms of $\xi \propto \chi$, and the following relation holds [139],

$$\sigma = -\eta = \frac{\zeta(2)}{\zeta(3)} \xi. \quad (50)$$

Subsequently, one gets the following ratio which turns out to be of particular interest:

$$\frac{\hat{\xi}}{\hat{\eta}} = -\frac{\zeta(3)}{\zeta(2)} \frac{1}{g_s^2}, \quad (51)$$

with

$$\hat{\xi} = \xi g_s^{-3/2}; \quad \hat{\eta} = g_s^{1/2} \eta. \quad (52)$$

Now, the (F-term) potential in the large volume limit results in the following leading order pieces,

$$V_{\alpha'+\log g_s}^{(1)} \equiv V_F = \frac{3\kappa\hat{\xi}}{4\mathcal{V}^3} |W_0|^2 - \frac{3\kappa(2\hat{\eta} - \hat{\eta} \ln \mathcal{V})}{2\mathcal{V}^3} |W_0|^2 \approx C \frac{\hat{\xi} + 2\hat{\eta} \log \mathcal{V}}{\mathcal{V}^3}, \quad (53)$$

where $C = 3g_s e^{K_{cs}} |W_0|^2 / (32\pi) > 0$ follows from $\kappa = g_s e^{K_{cs}} / (8\pi)$, and the last approximation is due to the fact that in the LSV(-like) scenario $\log \mathcal{V} \gg 1$.

A few properties of the potential (53) are worth mentioning: a minimum can be ensured as long as $\hat{\eta} < 0$ (while $\hat{\xi} > 0$). This confirms that the logarithmic correction plays a decisive role. Furthermore, stabilisation of the volume modulus occurs at **large** values and in the *weak string-coupling regime*. Nevertheless, as long as we consider only the F-term potential, the minimum is of AdS-type. Hence, new contributions are required to **uplift** the vacuum to dS. This can be uplifted either with a $\overline{D3}$ contribution [25] or with a positive D-term (see also [26]) which originates from the the universal $U(1)$ factors of the intersecting D7-branes [39]. This term has a volume dependence of the form $V_D = \frac{Q}{\mathcal{V}^2}$ where Q a positive constant, so the effective potential can be written

$$V_{\text{eff}} = C \left(\frac{\hat{\xi} + 2\hat{\eta} \log \mathcal{V}}{\mathcal{V}^3} + \frac{3d}{\mathcal{V}^2} \right) \quad (54)$$

where $d = Q/(3C) > 0$. This type of potential has been studied extensively in references [37, 39] and is outlined here in brief. Minimising the effective potential it is found that the volume \mathcal{V} at the extrema is given by:

$$\mathcal{V}_{\text{extrema}} = \frac{\eta}{d} W_{0/-1} \left(\frac{d}{\eta} e^{\frac{1}{3} - \frac{\xi}{2\eta}} \right) \equiv -\frac{|\eta|}{d} W_{0/-1} \left(-\frac{d}{|\eta|} e^{\frac{1}{3} + \frac{\xi}{2|\eta|}} \right), \quad (55)$$

where W_0, W_{-1} represent the two branches of the double-valued Lambert-W function. The potential acquires a minimum as long as the coefficient η takes negative values, $\eta < 0$. The volume at the minimum (denoted with \mathcal{V}_{min}) is associated with the W_0 branch, whilst there is also a local maximum located at \mathcal{V}_{max} associated with the W_{-1} branch. Moreover, real values for $W_{0/-1}(z)$ are ensured when $z > -\frac{1}{e}$ which implies the upper bound on $d < |\eta| e^{-\frac{4}{3} - \frac{\xi}{2|\eta|}}$. The requirement for dS vacua, $V_{\text{eff}} > 0$, puts more stringent constraints [37] which can be solved numerically for each set of values of ξ and η . For typical values of $-\chi \sim [100-200]$ and $g_s \sim 0.1$ the values of $\xi \lesssim \mathcal{O}(10)$. As an example, in Figure 3 the effective potential (54) is depicted for $\xi = 8, \eta = -1/2$ and several values of the parameter d .

5. Global embedding of perturbative LVS

Although the compactifying CY threefolds can be considered to part of the so-called hidden sector ingredients for the phenomenological model building, however the underlying geometries

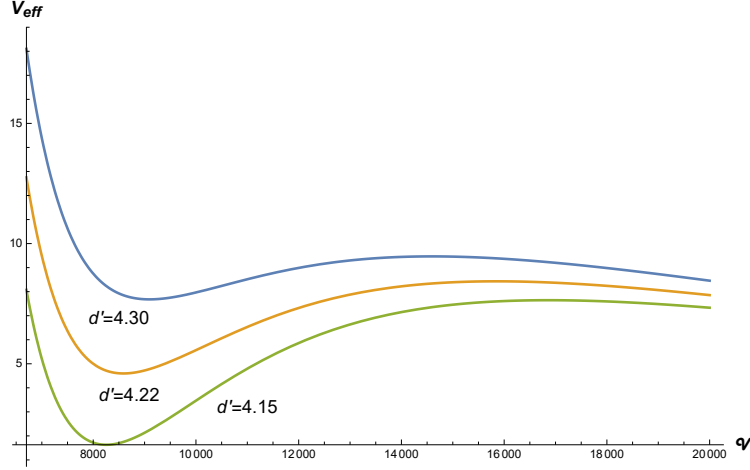


Figure 3: The potential (54) in arbitrary units vs \mathcal{V} for $\xi = 8, \eta = -1/2$ and three values of the parameter d' where $d' = 10^4 d$. The dS minima are restricted in the range $d' \sim [4.15 - 4.42]$. For $d' \lesssim 4.15$ the minimum of the potential is of AdS type. For values $d' \gtrsim 4.42$ the minimum is lost and the potential becomes a monotonic function of \mathcal{V} .

are omnipresent in the 4D effective models, and their imprints can be traced in physical observables such as masses and couplings which in the end depend on the moduli VEVs attained through an appropriate scheme such as KKLT and LVS. In addition, let us recall that while doing the pheno model building one (implicitly) assumes many things and it is always a challenging task to find concrete “global” setting in which all those requirements are fulfilled. In this context, it has been found that apart from knowing the topological data for the compactifying CY threefold (such as hodge numbers which counts the field content in the spectrum), the study of divisor topologies is a very crucial step as it can help in understanding the various classes of scalar potential terms which can be induced at sub-leading order, and hence maybe useful or concerning for a given model⁹. In this section, we will briefly enumerate (some of) such requirements appearing in a couple of well known models and discuss the challenges one has to face in this regard.

5.1 Why global embedding ?

As a motivation for global model building, let us review some of the known corrections to the 4D type IIB scalar potential and their possible connections with the internal geometries of the compactifying CY threefold.

1. In order to break the no-scale structure and facilitate the Kähler moduli stabilisation, the use of non-perturbative effects in the superpotential have been witnessed, leading to interesting scenarios like KKLT [25], Racetrack [170, 171] and LVS [172]. For facilitating these corrections one needs to find the some suitable class of divisors (D) satisfying the Witten’s unit arithmetic genus condition [168]:

$$\chi_0(D) = h^{0,0}(D) - h^{0,1}(D) + h^{2,0}(D) = 1. \quad (56)$$

⁹For example, see [79] regarding a recent classification of the divisor topologies of CY threefolds arising from the KS dataset, and their relevance for pheno-model building.

For this reason, the so-called rigid divisors with topological data $\{h^{0,0}(D) = 1, h^{0,1}(D) = 0, h^{2,0}(D) = 0\}$ are natural candidate to serve this purpose. In cases otherwise, there can be ‘rigidification’ of non-rigid divisors as proposed in [32, 234, 235] in order to make the divisor suitable for wrapping E3-instantons or D7-branes inducing the gaugino condensation effects. In passing, let us also mention that the sub-leading poly-instanton effects to the superpotential are induced through a so-called ‘Wilson divisor’ [132], which are some complex two-surfaces realised as \mathbb{P}^1 -fibrations over tori.

2. The CY threefolds with the so-called Swiss-Cheese structure are found to be central ingredients for the LVS scheme of moduli stabilisation [172]. These divisors are known as “diagonal” del-Pezzo divisors and they can be shrunk to a point-like singularity by squeezing along a single direction [44, 92, 236]. In fact, a del-Pezzo surface dP_n is obtained by blowing up \mathbb{P}^2 at n generic points, and a *diagonal* del-Pezzo is a divisor on the threefold X which is a dP_n surface and can be arbitrarily contractible to a point. To be more specific, a del Pezzo divisor (D_s) satisfies the following topological conditions:

$$\int_X D_s^3 = k_{sss} > 0, \quad \int_X D_s^2 D_i \leq 0 \quad \forall i \neq s. \quad (57)$$

Here $k_{sss} = 9 - n \neq 0$ for a dP_n divisor is the degree of dP_n , and the ‘diagonality’ of D_s is ensured by imposing the following useful condition [92]

$$k_{sss} k_{sij} = k_{ssi} k_{ssj} \quad \forall i, j. \quad (58)$$

This condition can be understood as follows: whenever (58) is satisfied, the volume of the diagonal dP (ddP) divisor D_s turns out to be a complete-square quantity in terms of the 2-cycle volumes because of the following relation:

$$\tau_s = \frac{1}{2} k_{sij} t^i t^j = \frac{1}{2 k_{sss}} k_{ssi} k_{ssj} t^i t^j = \frac{1}{2 k_{sss}} (k_{ssi} t^i)^2, \quad (59)$$

where we sum over i, j but not over s . In the presence of ddP divisors in LVS models, the volume of the CY threefold can be expressed in the so-called *Swiss-Cheese* form:

$$\mathcal{V} = f_{3/2}(\tau_j) - \sum_{i=1}^{n_{\text{small}}} \kappa_i \tau_i^{3/2}; \quad \text{with } j = 1, \dots, N_{\text{large}}. \quad (60)$$

Here n_{small} denotes the Kähler moduli corresponding to the rigid ddP cycles, while the remaining $N_{\text{large}} = h_+^{1,1} - n_{\text{small}}$ Kähler moduli are clubbed in $f_{3/2}$ which is a homogeneous function of degree-3/2 in τ_i moduli; e.g. the CY realised as a degree-18 hypersurface in $WCP^4[1, 1, 1, 6, 9]$ has a ddP and $h^{1,1} = 2$ resulting in $f_{3/2} = \tau_b^{3/2}$.

Using the topological data of the CY threefolds collected in the AGHJN-database [237], an initial classification for the diagonal dP divisors has been presented in [44] as summarised in Table 3, and have resulted in a useful conjecture stated as below [44]:

“The CY threefolds arising from the 4D reflexive polytopes listed in the KS database do not exhibit a ‘diagonal’ dP_n divisor for $1 \leq n \leq 5$, in the sense of satisfying the eqn. (58).”

$h^{1,1}$	Poly*	Geom* (n_{CY})	\mathbb{P}^2 (ddP ₀)	$\mathbb{P}^1 \times \mathbb{P}^1$	ddP _{n} $1 \leq n \leq 5$	ddP ₆	ddP ₇	ddP ₈	n_{LVS} with ddP _{n} ≥ 1
1	5	5	0	0	0	0	0	0	0
2	36	39	9	2	0	2	4	5	22
3	243	305	55	16	0	16	37	34	132
4	1185	2000	304	140	0	97	210	126	750
5	4897	13494	2107	901	0	486	731	374	4104

Table 3: Number of CY geometries with a ‘diagonal’ dP divisor suitable for LVS [44].

3. Another important class of CY threefolds consists of those which are K3-fibered and exhibit some peculiar properties leading to interesting phenomenological applications, including the well-known Fibre inflation models [44, 79, 92, 136, 137, 191, 236, 238]. It is useful to mention a theorem of [239, 240] which states that if the CY intersection polynomial is linear in the homology class \hat{D} corresponding to a divisor D , then the CY threefold has the structure of a K3 or a \mathbb{T}^4 fibration over a \mathbb{P}^1 base, i.e. the self-cubic and all the self-quadratics vanish:

$$\int_{X_3} \hat{D} \wedge \hat{D} \wedge \hat{D} = 0, \quad \int_{X_3} \hat{D} \wedge \hat{D} \wedge \hat{D} = 0, \quad \forall \hat{D}. \quad (61)$$

These conditions are quite restrictive for the volume form, specially when one also has the requirement of a ddP for LVS. For example it leads to the following volume forms:

$$\begin{aligned} h^{1,1} = 2, \quad I_3 = k_{122} D_1 D_2^2, & \implies \mathcal{V} \propto \tau_2 \sqrt{\tau_1}, \\ h^{1,1} = 3 \text{ with a ddP}, \quad I_3 = k_{122} D_1 D_2^2 + k_{sss} D_s^3, & \implies \mathcal{V} \propto \tau_2 \sqrt{\tau_1} - \kappa_s \tau_s^{3/2}, \end{aligned} \quad (62)$$

where the divisor D_1 appearing linearly in the intersection polynomial I_3 is a K3 (and therefore its self-cubics and self-quadratic pieces are trivial), while k_{122} denotes the corresponding triple intersection number. This structure in the volume form has been found to be important from the phenomenological point of view.

4. The string-loop corrections obtained through toroidal computations [193, 195] have been used to extrapolate the main insights for the CY-orientifold-based models resulting in two classes of string-loop effects (known as KK-type and Winding-type) induced for some particular brane settings [186, 195]. For example, KK-type corrections need the presence of non-intersecting stacks of $D7/O7$ and $O3$ planes while Winding-type effects need intersecting stacks of $D7/O7$ configurations which intersect at some non-contractible two-cycles. However some of such requirements have been recently revisited in [197] arguing that Winding-type corrections can appear more generically than what is expected from the original proposal of [193, 195], something which has been anticipated from a field theoretic argument earlier in [196].

For having a global model building of another class of string-loop effect, the so-called log-loop-corrections proposed in [37, 39], one needs three stacks of $D7$ -branes with non-trivial intersection loci as proposed in [139].

5. Let us recall that there are higher derivative F^4 -corrections to the 4D scalar potential, which are beyond the two-derivative contributions induced via the Kähler- and the super-potentials

[188]. Such corrections contribute directly to the scalar potential through a topological quantity defined in terms of the second Chern-class $c_2(X)$ of the CY threefold (X): $\Pi(D) = \int_X c_2(X) \wedge \hat{D}$ where \hat{D} denotes the dual $(1, 1)$ class corresponding to the divisor D of the CY threefold. A classification of such topologies can be found in [189].

6. Realising (an exponentially) small value of the so-called Gukov-Vafa-Witten flux superpotential $|W_0|$ in natural way has been always desired. This is because of the fact that low $|W_0|$ gets correlated with the VEV of the CY volume moduli, and hence becomes important to the issue of control over a series of perturbative and non-perturbative effects. One such recipe has been recently proposed in [179] leading to the so-called Perturbatively Flat Flux Vacua (PFFVs). Several follow ups like [126, 182] have suggested that the models based on the K3-fibred CY threefolds have significantly large number of (physical) PFFVs as compared to those which are based on non-K3-fibred CY threefolds.
7. Another issue which can be addressed only after taking the global model building aspects into account is related to satisfying the so-called tadpole cancellation conditions. In the lights of anti-D3-brane uplifting, the recent demand of constructing models with large tadpole charge [43, 191] and the subsequent challenges [73, 241] have been interesting topics to be explored.

5.2 A concrete global model for perturbative LVS

In this section we present a concrete CY orientifold model which realises (most of) the requirements (if not all) for stabilising Kähler moduli through log-loop effects [37–39, 41, 242–244]. For this purpose, we consider the CY threefold corresponding to the polytope Id: 249 in the CY database of [237] defined by the following Toric data:

Hyp	x_1	x_2	x_3	x_4	x_5	x_6	x_7
4	0	0	1	1	0	0	2
4	0	1	0	0	1	0	2
4	1	0	0	0	0	1	2
Div. topology	$K3$	$K3$	$K3$	$K3$	$K3$	$K3$	

The Hodge numbers are $(h^{2,1}, h^{1,1}) = (115, 3)$, the Euler number is $\chi = -224$ and the SR ideal is:

$$\text{SR} = \{x_1x_6, x_2x_5, x_3x_4x_7\}.$$

Considering the basis of smooth divisors $\{D_1, D_2, D_3\}$ some relevant quantities such as intersection polynomial, the $c_2(X)$ and the Kähler cone are given as,

$$I_3 = 2D_1D_2D_3, \quad \implies \quad \mathcal{V} = 2t^1t^2t^3 = \frac{1}{\sqrt{2}}\sqrt{\tau_1\tau_2\tau_3}, \quad (63)$$

$$c_2(CY) = 5D_3^2 + 12D_1D_2 + 12D_2D_3 + 12D_1D_3,$$

$$\text{KC} : \{t^1 > 0, \quad t^2 > 0, \quad t^3 > 0\},$$

where $\tau_1 = 2t^2t^3$, $\tau_2 = 2t^1t^3$, $\tau_3 = 2t^1t^2$. This shows that there is a toroidal-like exchange symmetry $1 \leftrightarrow 2 \leftrightarrow 3$ under which all the three $K3$ divisors which are part of the basis are

exchanged, and the same happens with the other quantities such as I_3 , \mathcal{V} , τ_i 's and KC as well. In addition, note that the volume \mathcal{V} can also be expressed as:

$$\mathcal{V} = t^1 \tau_1 = t^2 \tau_2 = t^3 \tau_3, \quad (64)$$

which like the toroidal case translates into the fact that the transverse distance for the stacks of $D7$ -branes wrapping the divisor D_1 is given by t^1 and similarly t^2 is the transverse distance for $D7$ -branes wrapping the divisor D_2 and so on. Further, considering the involution $x_7 \rightarrow -x_7$ results in the only fixed point set being given as $\{O7 = D_7\}$ without any $O3$ -planes. So one can satisfy the $D3/D7$ tadpole cancellation conditions by having the brane setting involving 3 stacks of $D7$ -branes wrapping each of the three divisors $\{D_1, D_2, D_3\}$ present in the basis such that,

$$\begin{aligned} \text{D7 tadpole :} \quad & 8 [O_7] = 8 ([D_1] + [D'_1]) + 8 ([D_2] + [D'_2]) + 8 ([D_3] + [D'_3]), \quad (65) \\ \text{D3 tadpole :} \quad & N_{D3} + \frac{N_{\text{flux}}}{2} + N_{\text{gauge}} = 0 + \frac{240}{12} + 8 + 8 + 8 = 44, \end{aligned}$$

which shows some flexibility with turning on background flux as well as the gauge flux. Moreover the divisor intersection analysis shows that all the three $D7$ -brane stacks intersect at \mathbb{T}^2 while each of those intersect the $O7$ -plane on a curve with $h^{0,0} = 1$ and $h^{1,0} = 9$.

Noting the fact that there are no ddP or Wilson divisors present, this setup does not receive non-perturbative superpotential contributions from E3-instanton, gaugino condensation or poly-instanton effects- making this CY to be not suitable for the Kähler moduli stabilisation using the conventional schemes¹⁰. Further one finds that there are no non-intersecting $D7$ -brane stacks and the $O7$ -planes along with no $O3$ -planes present, and so this model does not induce the KK-type string-loop corrections to the Kähler potential. However, as the $D7$ -brane stacks intersect on non-shrinkable two-torus, there are string-loop effects of the winding-type along with the non-trivial higher derivative F^4 -corrections. Subsequently, the leading order pieces in the scalar potential can be collected as below,

$$\begin{aligned} V \simeq & C_1 \left(\frac{\hat{\xi} - 4\hat{\eta} + 2\hat{\eta} \ln \mathcal{V}}{\mathcal{V}^3} \right) + 6 C_1 \left(\frac{3\hat{\eta}\hat{\xi} + 4\hat{\eta}^2 + \hat{\xi}^2 - 2\hat{\eta}\hat{\xi} \ln \mathcal{V} - 2\hat{\eta}^2 \ln \mathcal{V}}{\mathcal{V}^4} \right) \quad (66) \\ & + \frac{C_2}{\mathcal{V}^3} \left(\frac{1}{t^1} + \frac{1}{t^2} + \frac{1}{t^3} \right) + \frac{C_3 (t^1 + t^2 + t^3)}{\mathcal{V}^4} + O(\mathcal{V}^{-5}) + \dots, \end{aligned}$$

where

$$C_1 = \left(\frac{g_s}{8\pi} \right) \frac{3 |W_0|^2}{4}, \quad C_2 = \left(\frac{g_s}{8\pi} \right) |W_0|^2, \quad C_3 = - \left(\frac{g_s}{8\pi} \right)^2 \frac{\lambda |W_0|^4}{g_s^{3/2}}. \quad (67)$$

Now, using $\chi(X) = -224$ and the following useful relations

$$\hat{\xi} = \frac{14 \zeta[3]}{\pi^3 g_s^{3/2}}, \quad \hat{\eta} = -\frac{14 \sqrt{g_s} \zeta[2]}{\pi^3}, \quad \frac{\hat{\xi}}{\hat{\eta}} = -\frac{\zeta[3]}{\zeta[2] g_s^2}, \quad (68)$$

¹⁰However after including all the known/possible perturbative effects arising from the α' as well as g_s series, one can indeed fix all the volume moduli. While we will present some relevant features/insights of this model, we refer the reader to [139] for more details.

it follows that the leading order (first) piece of scalar potential (66) gives a large volume AdS minimum described as:

$$\langle \mathcal{V} \rangle \simeq e^{a/g_s^2+b}, \quad a = \frac{\zeta[3]}{2\zeta[2]} \simeq 0.365381, \quad b = \frac{7}{3}, \quad (69)$$

while the remaining moduli are fixed by the sub-leading winding and F^4 -corrections. For having a numerical estimate, we present the following AdS minimum,

$$\begin{aligned} \chi(X) &= -224, \quad e^{K_{cs}} = 1, \quad W_0 = 1, \quad C_1^W = C_2^W = C_3^W = -1, \quad \lambda = -0.01, \quad (70) \\ g_s &= 0.20, \quad \hat{\xi} = 6.06818, \quad \hat{\eta} = -0.332156, \quad \langle \mathcal{V} \rangle_{\text{approx.}} = 95594.5, \\ \langle t^i \rangle &= 38.553, \quad \langle \mathcal{V} \rangle = 114605.0, \quad \langle V \rangle = -9.29 \cdot 10^{-19}, \end{aligned}$$

where $\langle \mathcal{V} \rangle_{\text{approx.}}$ corresponds to the approximated VEV obtained from (69) in the large volume limit while $\langle \mathcal{V} \rangle$ denotes the value obtained via the 3-field numerical analysis.

5.3 On de-Sitter uplifting

Given that the CY orientifold at hand does not have any $O3$ -planes, the anti- $D3$ uplifting proposal of [43, 191] does not apply to our case. However, having three stacks of $D7$ -branes intersecting at three \mathbb{T}^2 's, there can be various other ways of inducing uplifting term which can result in dS solution.

For example, assuming that the matter fields receive vanishing VEVs along with each one of the $D7$ -brane stack being appropriately magnetised by suitable gauge fluxes to generate a moduli-dependent Fayet-Iliopoulos term leads to the following D -term contributions,

$$V_D \propto \sum_{i=1}^3 \left[\frac{1}{\tau_i} \left(\sum_{j \neq i} q_{ij} \frac{\partial K}{\partial \tau_j} \right)^2 \right] \simeq \sum_{i=1}^3 \frac{d_i}{f_i^{(3)}}, \quad (71)$$

where q_{ij} are some model dependent charges, and $f_i^{(3)}$ is some homogeneous cubic polynomial in generic 4-cycle volume τ_i . Using the underlying exchange symmetry $t^1 \leftrightarrow t^2 \leftrightarrow t^3$ of the F -term scalar potential (66), one can take d_α as $d_1 = d_2 = d_3 \equiv d$ which self consistently leads to $\tau_1 = \tau_2 = \tau_3 \equiv \tau$, i.e. $f_\alpha^{(3)} \sim \tau^3$, and hence can facilitate an uplifting of the AdS vacua à la [37]. One such tachyon-free dS solution is given as below:

$$\begin{aligned} g_s &= 0.2, \quad \hat{\xi} = 6.06818, \quad \hat{\eta} = -0.332156, \quad d = 1.24711 \cdot 10^{-8}, \\ \langle t^\alpha \rangle &= 48.0191 \quad \forall \alpha, \quad \langle \mathcal{V} \rangle = 221447.96, \quad \langle V \rangle = 1.54074 \cdot 10^{-31}, \\ \text{Eigen}(V_{ij}) &= \{5.04286 \cdot 10^{-22}, 5.04286 \cdot 10^{-22}, 5.04286 \cdot 10^{-22}\}. \end{aligned} \quad (72)$$

Alternatively, one can use the so-called T -brane uplifting mechanism in the presence of non-zero gauge flux on the hidden sector $D7$ -branes via inducing a non-vanishing Fayet-Iliopoulos term [34, 35, 44] such that $|\varphi| \simeq \mathcal{V}^{-1/3}$ leading to a positive semidefinite term that can uplift the AdS,

$$V_{\text{up}} \simeq \left(\frac{g_s}{8\pi} \right) \frac{C_{\text{up}} |W_0|^2}{\mathcal{V}^{8/3}} \geq 0, \quad (73)$$

where C_{up} denotes a model dependent coefficient. The numerical estimates for one such stable dS solution is given below:

$$\begin{aligned} g_s &= 0.3, & \hat{\xi} &= 3.3031, & \hat{\eta} &= -0.406806, & C_{up} &= 0.0814039, \\ \langle t^\alpha \rangle &= 19.5862 \quad \forall \alpha, & \langle \mathcal{V} \rangle &= 15027.3, & \langle V \rangle &= 3.74709 \cdot 10^{-30} \\ \text{Eigen}(V_{ij}) &= \{6.81793 \cdot 10^{-18}, 4.68145 \cdot 10^{-19}, 4.68145 \cdot 10^{-19}\}. \end{aligned} \quad (74)$$

6. Summary and conclusions

In this work, we have presented a concise review on the various (conventional) Kähler moduli stabilisation schemes proposed in the literature, along with the possibility of realising de-Sitter vacua in the context of type IIB superstring compactifications using the orientifolds of Calabi-Yau threefolds. However, given the vast nature of the subject it is understood that the presented review covers a subset of topics with some (limited) model building details, and is certainly not an exhaustive collection of all the interesting proposals known in the literature.

First we have presented some basic relevant details about the 4D type IIB effective supergravity arising from a generic CY orientifold compactification, and the interconnected origin of various possible (closed-string) moduli, such as complex-structure moduli, axio-dilaton and the Kähler moduli, which are counted by the topological invariant such as hodge numbers of the internal manifold. This motivates the tight correlation of string model building with the study of the algebraic geometry and topology related to the compactifying threefolds. Given that these moduli appear in the 4D masses and couplings it is necessary to stabilise those in some stable vacuum having VEVs consistent with the model building requirements such as overall control of the effective theory against the various (un-)known corrections. The various attempts made in this program of moduli stabilisation and subsequently finding dS solutions have been presented in the following four steps:

1. In the first step, we discuss the stabilisation of the axio-dilaton and complex-structure moduli via introduction of the conventional S -dual pair of the NS-NS and RR fluxes, namely the H_3 and F_3 fluxes, which can induce superpotential terms depending on these moduli and hence generate a dynamical VEV for each of these moduli. However in 4D type IIB effective theory, the Kähler moduli are protected by the so-called no-scale structure, and turning-on these fluxes does not induce scalar potential contributions for the Kähler moduli.
2. In the second step, we discuss the possible corrections which can break this no-scale structure via generating some Kähler moduli dependent scalar potential terms, which can subsequently facilitate fixing these moduli. We present such corrections in two classes:
 - Non-perturbative effects: In this class we summarise the possible non-perturbative effects arising from the E3-instantons and gaugino condensation effects generated via suitable (rigid) 4-cycles wrapping E3-brane and D7-brane stacks in appropriate setting. Another class of non-perturbative effects known as poly-instanton corrections is also discussed with the necessary superpotential ingredients.
 - Perturbative effects: In this class we summarise a number of interesting corrections such as the so-called BBHL's α'^3 -correction which introduces a shift in the overall volume

of the CY threefold appearing in the expression of the Kähler potential. In addition, we discuss the possible higher derivative F^4 -corrections which can be induced to contribute in the scalar potential beyond the two-derivative effects arising from the Kähler- and the super-potentials.

Moreover, apart from the α' -corrections, we summarise three classes of perturbative string-loop effects: (i). “KK-type”, (ii). “winding-type”, and (iii). “logarithmic-type”. These corrections have received great amount of interest in recent years, especially towards model building with moduli stabilisation, dS realisation, and inflationary embeddings.

3. In the third step, we discuss how the various possible corrections presented in step-2 can be utilised for Kähler moduli stabilisation, leading to several popular schemes which we further classify in two categories:

- Models with non-perturbative effects: This class of schemes utilises the non-perturbative superpotential contributions, such as KKLT, Racetrack and LVS.
- Models without non-perturbative effects: In this class we discuss two possible (Kähler) moduli schemes which do not utilise any non-perturbative effects:
 - (i). The first one corresponds to the non-geometric models in which the superpotential couplings are introduced for all moduli, including the Kähler moduli, even at the tree level, which facilitates the possibility of fixing all moduli via tree-level effects. However such constructions have other unresolved issues such as overall control of the EFT, lack of knowledge about a complete set of Bianchi identities for the so-called cohomology formulation of the generalised fluxes etc.
 - (ii). The second scenarios corresponds to what we call as “perturbative LVS”. This is very similar to the standard LARGE volume scenario which realises an exponentially large VEV for the overall volume of the CY threefold via a competition between the BBHL’s α' -correction in the Kähler potential and a non-perturbative correction in the superpotential. However this perturbative LVS uses string-loop effects of “logarithmic-type” instead of using any non-perturbative correction in the superpotential. Nevertheless the overall volume still receives an exponentially large VEV given as: $\mathcal{V} \simeq e^{a g_s^{-2} + b}$ where $a = 0.37$ and $b = 7/3$, and results in a stable AdS minimum.

4. In the final step, we present the arguments about the need of global embedding for the pheno-inspired models which is a very crucial ingredient of string model building, given that one simply assumes many things while building the models based on phenomenological interests, and it is very important to ensure that those assumptions can be consistently realised in some concrete and explicit CY orientifold setting. We illustrate these arguments for several corrections which are guaranteed only when certain topological constraints are satisfied in a given model. In addition, we present a particular CY orientifold with specific ingredients such as a volume form similar to the toroidal case, brane setting with 3 stacks of D7-branes wrapping K3-surfaces, and each stacks intersecting on 2-tori etc. It turns out that these ingredients are the necessary requirement of generating the log-loop effects a la [39]. In

this context it is important to mention that this type IIB model based on the concrete CY orientifold at hand does not receive non-perturbative effects due to the absence of (diagonal) del-Pezzo divisors and the Wilson divisors. In addition, the chosen brane-setting does not generically allow the KK-type string-loop effects, nevertheless one can fix all the Kähler moduli using the BBHL and log-loop effects, purely perturbative in nature. Subsequently, it turns out that the resulting AdS in perturbative LVS can be uplifted to dS solution by using D -terms effects or T -brane uplifting prescriptions which are well-known in the literature.

For each of these four steps we have devoted separate sections to present the current state-of-the-art in a concise manner. This includes presenting a very simple and compact formulation of the F-term scalar potential taking into account the well-known BBHL's α'^3 -corrections and the one-loop logarithmic corrections, and subsequently incorporating a volume-dependent uplifting term in the effective potential. Using the full package of various possible corrections, it has been shown that metastable de-Sitter vacua can be achieved, with all the (Kähler) moduli being stabilised, except for some axions arising from the four-form potential C_4 . The adopted analytic description in this work is instrumental towards a direct comparison among the various suggested schemes of moduli stabilisation such as KKLT, Racetrack, LVS, perturbative LVS or any other combinations of these where multiple ingredients across these schemes are adopted, e.g. see [40, 245]. The possible extensions of the log-loop models towards inflationary purpose have been an interesting direction to explore, and in this regard some initiatives are already being taken [38, 41, 242, 244, 246–248]. However, for the current work, it is extremely difficult to extend the detailed discussion to a subject as diverse and vast as inflation, and for the time being we leave the interested readers with a couple of nice reviews [50, 78].

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