

## Precision test of the muon-Higgs coupling at a high-energy muon collider

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Muon colliders offer the possibility to go to very high energies with relatively small circular colliders, energies up to 10 or 14 TeV are envisioned. Due to their very clean collider environment they provide a fantastic tool to search for new physics in the electroweak sector, especially through the production of multiple EW vector and Higgs bosons, and they allow to measure the Higgs-muon coupling very precisely. I will elucidate the physics capabilities from these processes and also discuss issues on precision predictions for SM backgrounds at high-energy lepton colliders.

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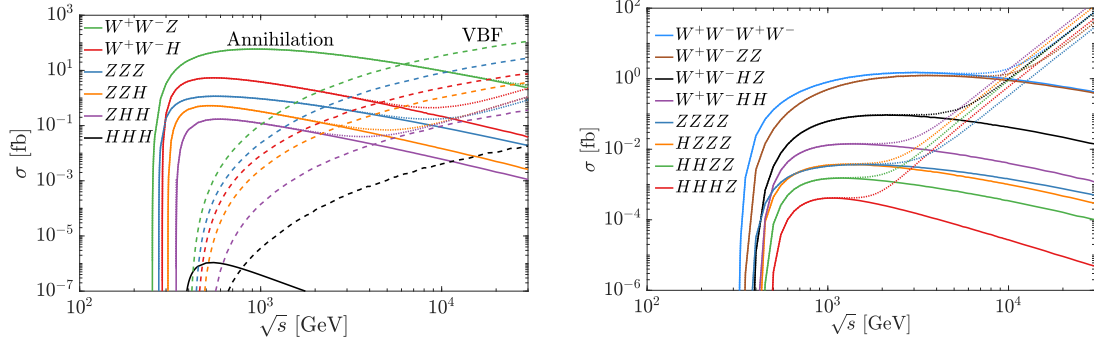
## 1. Introduction

Muon colliders have recently gained a lot of interest again driven by technological progress made on the accelerator side. The muon collider is now not any longer considered as a Higgs factory operating at the Higgs threshold, but at an energy frontier machine in the multi-TeV range which very prominently featured in the US Snowmass Community Summer Study [1–4]. Due to the very clean environment of the pointlike leptonic beams, their search reach for new particles and phenomena can supersede that of high-energy hadron colliders like the FCC-hh (one of these examples is the search for heavy neutrinos, cf. [5]). Muon colliders do have much less synchrotron radiation and much less beam-energy spread than high-energy (i.e. linear)  $e^+e^-$  colliders, as well as much less QED initial-state radiation which leads to different physics reaches between MuC and e.g. CLIC at the same collider energies (cf. e.g. [5]). Here, we focus on the search for new physics in deviations of the muon Yukawa coupling from its Standard Model (SM) value. This proceedings article contains two sections, Sec. 2 on the sensitivity of muon colliders to anomalous muon-Higgs couplings and the discovery potential for higher-dimensional operators in this sector from multi-boson signatures at the MuC, while in Sec. 3 we show that it is possible to provide precision predictions for these signatures in the SM regarding complete electroweak next-to-leading order (NLO) corrections. Finally, we briefly conclude.

## 2. The muon Yukawa coupling sensitivity

After the H(125) particle discovery in 2012, many of its properties have been proven to be SM-like in the past decade, however, the couplings to the second-generation fermions are still very elusive. In the SM, the muon Yukawa coupling is one of the smallest parameters. It is expected, that after the high-luminosity run of the Large Hadron Collider (LHC) this coupling could be measured with a precision of ca. 5 %, however, its sign cannot be determined as it is only accessible from the Higgs decay. A model-independent measurement would be highly desirable, and indeed it is possible in direct production at a muon collider. Here, the muon Yukawa coupling shows a subtle cancellation with production of longitudinal gauge bosons at high energies. As the muon Yukawa coupling is a tiny parameter in the SM, huge deviations beyond the SM are possible, and could deceit a default power counting in SM Effective Field Theory (SMEFT). There are also BSM examples, like extra-dimensional theories, where the muon-Yukawa coupling could follow a completely different renormalization-group running as in the SM, and hence has completely different values at very energies. Here, we investigate the effect of non-standard muon-Higgs couplings on multi-boson final states: here, longitudinal polarizations dominate at high energies and analytic calculations can be approximated by the Goldstone-boson equivalence theorem. We will parameterize new physics effects by EFT operator insertions.

We considered two different EFT representations, a non-linear realization with non-linear sigma model of Goldstone bosons and a linear representation in terms of a SMEFT expansion. As an extreme case we assume a toy model where the higher-dimensional operators completely cancel the SM contribution, leading to a vanishing muon-Higgs coupling, called "matched" case. The



**Figure 1:** Cross sections of the production of three and four bosons as a function of the MuC energy. Full lines are the SM, dotted lines show the increase due to higher-dimensional operators, and dashed lines cross section from the VBF channel.

modification of the muon-Higgs coupling gets the following modifications from its SM value:

$$K_\ell = 1 - \frac{v}{\sqrt{2}} M_\ell^{-1} \sum_{n=1}^{\infty} C_{\ell\phi}^{(n)} \frac{nv^{2n}}{2^{n-1}}, \quad (1)$$

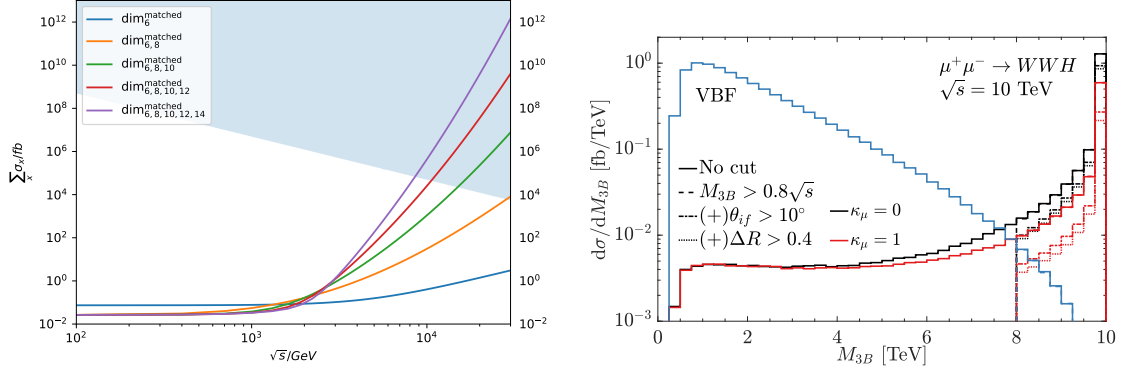
with contributions of dimensionality  $4 + 2n$ . Here,  $M_\ell$  is the muon mass,  $v$  the Higgs vacuum expectation value and  $C_{\ell\phi}^{(n)}$  are the Wilson coefficients of the corresponding higher-dimensional operators. For more details on the model setup cf. [6].

Among the three-boson processes on which we concentrate here, the process  $\mu^+\mu^- \rightarrow ZZH$  shows the largest deviations from the SM cross section, however, has a smaller rate than the process  $\mu^+\mu^- \rightarrow W^+W^-H$ . For this reason, we mainly use the latter process in this study.

In Fig. 1 we show the cross sections for the production of three and four gauge or Higgs bosons for the SM (full lines, prompt production), dashed lines (SM, vector-boson fusion, VBF) and dashed lines with insertion of higher-dimension operators as a function of the MuC energy. The VBF channel is not sensitive to the (anomalous) muon Yukawa coupling and needs to be filtered out in the event selection. This is achieved with a cut-based selection, demanding that the total invariant mass of the 3- or 4-boson systems exceeds 80% of the total energy,  $M_{3/4B} > 0.8 \cdot \sqrt{s}$ , requesting a minimum polar angle of 10 degrees,  $\theta_{if} > 10^\circ$  for each boson (as the VBF process originates from collinear splitting being mostly forward), and a minimal  $R$ -space distance,  $\Delta R_{BB} > 0.4$  as resolution criterion. The separation of signal and VBF background is shown in the right hand side of Fig. 2 for the invariant mass of 3-boson system for the process  $\mu^+\mu^- \rightarrow W^+W^-H$ , where one sees that the first cut above is really efficient.

There are bounds from perturbative partial wave unitarity for the processes of several Higgs bosons on the size of operator coefficients or total cross sections. These are shown in the left plot of Fig. 2 for the matched cases (i.e. vanishing explicit muon-Higgs coupling) for the case of dimension-6 operators only, and then including dimension-8, -10, -12 and -14 operators, respectively. For the most interesting cases of dimension-6 and -8, the considered MuC energies of  $1 < \sqrt{s} < 30$  TeV are still within the bounds.

Our results have been cross-checked with two independent analytic calculations in the Goldstone-boson equivalence approximation as well as by numeric simulation, using the Monte-Carlo event



**Figure 2:** Left plot: Unitarity bound for the matched cases considering Wilson coefficients from different combinations of higher-dimensional operators as a function of the MuC energy. Right plot: differential distribution of the invariant mass of the 3-boson system in  $\mu^+\mu^- \rightarrow W^+W^-H$  at 10 TeV: prompt production vs. VBF, and the effect of the fiducial phase-space cuts.

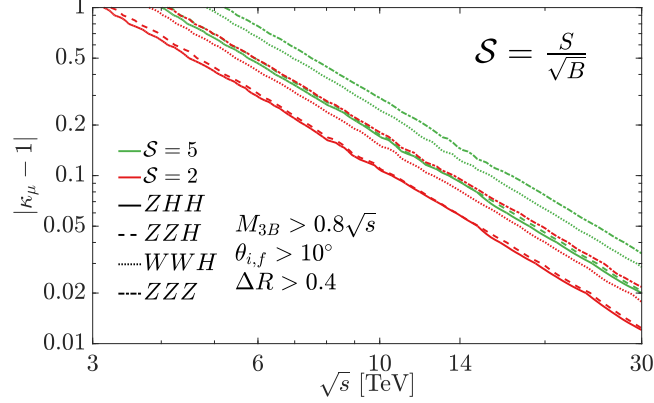
generator WHIZARD [7, 8], using a UFO model implementation [9, 10]. Several different benchmark scenarios have been studied: dimension-6 alone, dimension-8 alone, dimension-6+8 combined as well as the abovementioned "matched" case. MuC energies between 1 and 30 TeV have been considered with the luminosity scaling of  $\mathcal{L} = (\sqrt{s}/10 \text{ TeV})^2 \cdot 10 \text{ ab}^{-1}$ . The number of signal events is defined by the number of events with signal-strength modifier  $\kappa_\mu$  subtracted by the number of corresponding SM events,  $S = N_{\kappa_\mu} - N_{\kappa_\mu=1}$ . The number of background events is given by  $B = N_{\kappa_\mu=1} + N_{\text{VBF}}$  is this SM event number and the number of VBF events in the fiducial phase space volume defined above. Our significance is then defined via  $S = S/\sqrt{B}$ , where the number of events in the SM is always the smallest:  $N_{\kappa_\mu} \geq N_{\kappa_\mu=1}$ . Also note that because the deviations of the cross sections is the same for up- and downwards deviations from the SM value,  $\kappa_\mu = 1 \pm \delta$ , the significance is the same for these two value. The  $5\sigma$  significance for establishing such a deviation from a measurement of these processes is roughly possible for a 20% deviation at a 10 TeV MuC, and goes down to a 2% deviation at a 30 TeV MuC. This translates to an access to new physics scales of several tens of TeV to almost a hundred TeV:

$$\Lambda > 10 \text{ TeV} \cdot \sqrt{\frac{g}{\Delta\kappa_\mu}} \quad . \quad (2)$$

This reach is summarized in Fig. 3.

### 3. Precision predictions for multi-boson processes at the muon collider

To match the experimental precision at colliders, high-precision higher-order theoretical predictions and simulations are indispensable. In the last section, we discussed multi-boson signatures,  $\mu^+\mu^- \rightarrow V^n H^m$  with  $V \in \{W^\pm, Z\}$  and  $n + m \leq 4$ , as discovery probes for new physics in the electroweak sector. Electroweak processes at high energies experience large electroweak corrections in terms of Sudakov logarithms (double and single logarithms of the form  $\log \mu^2/M_V^2$ ) that are typically negative and reduce the cross section, especially in the high- $p_T$  tails of final-state



**Figure 3:**  $2\sigma$  and  $5\sigma$  significance reach for different deviations of the muon-Higgs coupling from its SM value as a function of the muon collider energy. Four different triple-boson processes are shown, within the same fiducial phase-space volume.

particles. These Sudakov logarithms originate from two different sources: (1) virtual electroweak corrections are not cancelled fully with corresponding real corrections, as one only consider real QED radiation, but no  $W/Z$  radiation, and (2) the initial state,  $\mu^+\mu^-$ , and hence also the final state is not an electroweak singlet. Leading EW corrections can be approximated by these Sudakov logarithms (and they also provide means of their resummation), but a complete fixed-order NLO EW calculation is nevertheless necessary to provide high precision in all regions of phase space, and not only the kinematically enhanced ones. In Ref. [11], all relevant processes with two, three and four EW gauge or Higgs bosons have been computed. For brevity, we show here the result only for 3 TeV center-of-mass energy, with the result given in Tab. 1. For the 10 TeV results and more technical details confer the original paper [11]. The NLO EW results have been calculated with massive muons, without a collinearly factorized lepton PDF in the initial state. The calculations rely on the NLO QCD+EW automation in the Monte Carlo framework WHIZARD [7] with virtual matrix elements from RECOLA [12], using WHIZARD’s parallelized phase-space integrator [13]. This built upon an early EW NLO implementation for SUSY particles [14, 15] and NLO QCD applications for high-energy lepton colliders [16, 17]. The results have been validated with results from the literature for electron-positron processes, and the NLO corrections have been compared to the leading Sudakov corrections and have been found to be in good agreement.

#### 4. Conclusions

In this proceedings article we have shown the importance of future high-energy muon colliders for the search of deviations of the muon-Higgs coupling from its SM value. For collider energies beyond 10 TeV,  $5\sigma$  significance can be reached for deviations at the level of signal-digit per cent. Unlike the LHC, where this is accessible only from the decay, the sign of the deviation can be accesses. This search potential is reach in processes of one to three electroweak gauge bosons in association with the 125 GeV Higgs boson. Different processes, e.g.  $WWH$  and  $ZZH$  final state,

$\mu^+\mu^- \rightarrow X, \sqrt{s} = 3 \text{ TeV}$	$\sigma_{\text{LO}}^{\text{incl}}$ [fb]	$\sigma_{\text{NLO}}^{\text{incl}}$ [fb]	$\delta_{\text{EW}}$ [%]
$W^+W^-$	$4.6591(2) \cdot 10^2$	$4.847(7) \cdot 10^2$	+4.0(2)
$ZZ$	$2.5988(1) \cdot 10^1$	$2.656(2) \cdot 10^1$	+2.19(6)
$HZ$	$1.3719(1) \cdot 10^0$	$1.3512(5) \cdot 10^0$	-1.51(4)
$W^+W^-Z$	$3.330(2) \cdot 10^1$	$2.568(8) \cdot 10^1$	-22.9(2)
$W^+W^-H$	$1.1253(5) \cdot 10^0$	$0.895(2) \cdot 10^0$	-20.5(2)
$ZZZ$	$3.598(2) \cdot 10^{-1}$	$2.68(1) \cdot 10^{-1}$	-25.5(3)
$HZZ$	$8.199(4) \cdot 10^{-2}$	$6.60(3) \cdot 10^{-2}$	-19.6(3)
$HHZ$	$3.277(1) \cdot 10^{-2}$	$2.451(5) \cdot 10^{-2}$	-25.2(1)
$HHH$	$2.9699(6) \cdot 10^{-8}$	$0.86(7) \cdot 10^{-8} *$	
$W^+W^-W^+W^-$	$1.484(1) \cdot 10^0$	$0.993(6) \cdot 10^0$	-33.1(4)
$W^+W^-ZZ$	$1.209(1) \cdot 10^0$	$0.699(7) \cdot 10^0$	-42.2(6)
$W^+W^-HZ$	$8.754(8) \cdot 10^{-2}$	$6.05(4) \cdot 10^{-2}$	-30.9(5)
$W^+W^-HH$	$1.058(1) \cdot 10^{-2}$	$0.655(5) \cdot 10^{-2}$	-38.1(4)
$ZZZZ$	$3.114(2) \cdot 10^{-3}$	$1.799(7) \cdot 10^{-3}$	-42.2(2)
$HZZZ$	$2.693(2) \cdot 10^{-3}$	$1.766(6) \cdot 10^{-3}$	-34.4(2)
$HHZZ$	$9.828(7) \cdot 10^{-4}$	$6.24(2) \cdot 10^{-4}$	-36.5(2)
$HHHZ$	$1.568(1) \cdot 10^{-4}$	$1.165(4) \cdot 10^{-4}$	-25.7(2)

**Table 1:** Total inclusive cross sections at LO and NLO EW with corresponding relative corrections  $\delta_{\text{EW}}$ , for two-, three- and four-boson production at  $\sqrt{s} = 3 \text{ TeV}$ .

show different event numbers and different significance to such deviations. Systematic uncertainties can be reduced by taking ratios of these processes.

In the second part, we have provided the electroweak one-loop corrections to the SM processes, which are negative and between 10-40 per cent of the LO cross section: this can be explained with the importance of electroweak Sudakov logarithms in the high-energy regime.

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