PROCEEDINGS OF SCIENCE

Alleviating the $\sigma 8$ tension via Soft Cosmology and Modified Gravity

Charalampos Tzerefos[a,b] **and Emmanuel N. Saridakis**[a,c,d]

[a] *National Observatory of Athens, Lofos Nymfon, 11852 Athens, Greece*

[b] *Department of Physics, National & Kapodistrian University of Athens, Zografou Campus GR 157 73, Athens, Greece*

[c] *CAS Key Laboratory for Researches in Galaxies and Cosmology, Department of Astronomy, University of Science and Technology of China, Hefei, Anhui 230026, P.R. China*

[d] *Departamento de Matemáticas, Universidad Católica del Norte, Avda. Angamos 0610, Casilla 1280 Antofagasta, Chile*

E-mail: chtzeref@phys.uoa.gr, msaridak@noa.gr

We examine the possibility of "soft cosmology", namely small deviations from the usual cosmological framework due to the effective appearance of soft-matter properties in the Universe sectors. One effect of such a scenario would be that dark energy and/or dark matter exhibit a different equation-of-state parameter at large scales (which determine the universe expansion) and at intermediate scales (which determine the sub-horizon clustering and the large-scale structure formation). These properties could help alleviate issues of the standard cosmological paradigm, such as the $\sigma 8$ tension. In this work, we shall demonstrate how an $f(R)$ modified theory of gravity could naturally facilitate such properties for the dark Universe sectors.

Corfu Summer Institute 2022 "School and Workshops on Elementary Particle Physics and Gravity", 28 August - 1 October, 2022
Corfu, Greece

PoS(CORFU2022)210

1. Introduction

We shall first briefly present the standard cosmological perturbation theory analysis for general relativity (henceforth GR) before proceeding to $f(R)$ theories. At the background level, it is assumed that the Universe is flat, homogeneous and isotropic, thus the spacetime metric has the simple form $ds^2 = a^2(\tau)(-d\tau^2 + \delta_{ij}dx^i dx^j)$, which is the well known Friedmann - Lemaître - Robertson -Walker (FLRW) metric. Additionally, the content of the Universe is described by the energy- momentum tensor of a perfect fluid $T^\mu_\nu = diag(-\rho, p, p, p)$, which can be written as $T^{\mu(tot)}_\nu = \sum_i T^{\mu(i)}_\nu$, where $T^{\mu(i)}_\nu$ represents the energy- momentum tensor of each (effective) fluid species eg matter, radiation or dark energy. The Einstein field equations are:

$$G^\mu_\nu = 8\pi G\, T^{\mu(tot)}_\nu. \tag{1}$$

By virtue of the Bianchi identity $\nabla_\mu G^\mu_\nu = 0$, the total energy-momentum tensor is conserved, as well as each tensor for each individual species: $\nabla_\mu T^{\mu(tot)}_\nu = \sum_i \nabla_\mu T^{\mu(i)}_\nu = 0 \to \nabla_\mu T^{\mu(i)}_\nu = 0$, provided that the different species do not interact with one another. This yields the (background) continuity equation: $\rho^{(i)\prime} = -3\mathcal{H}(\rho^{(i)} + p^{(i)})$, where $' \equiv d/d\tau$ and $\mathcal{H} = a'/a$ the conformal Hubble parameter. For the FLRW metric, equation (1) gives the well known Friedmann equations:

$$\mathcal{H}^2 = \frac{8\pi G a^2}{3}\bar{\rho}^{(tot)} \tag{2}$$

$$\mathcal{H}' = -\frac{4\pi G a^2}{3}(\bar{\rho}^{(tot)} + 3\bar{p}^{(tot)}) \tag{3}$$

One can allow linear scalar perturbations around the background metric, which in the so-called Newtonian gauge read as $ds^2 = a^2(\tau)\left[-(1+2\Psi)d\tau^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right]$. In a similar manner, one can consider scalar perturbations around the ideal fluid background energy-momentum tensor of each species, which can be written as:

$$\begin{aligned}
T^{0(l)}_0 &= -(\bar{\rho}^{(l)} + \delta\rho^{(l)})\\
T^{0(l)}_i &= (\bar{\rho}^{(l)} + \bar{p}^{(l)})\upsilon^{(l)}_i\,,\ \upsilon^{(l)}_i \equiv a u^{(l)}_i \text{ and } \upsilon^{(l)}_i = -\upsilon^{(l)}_{,i}\\
T^{i(l)}_j &= \bar{p}^{(l)}(\delta^i_j + \Pi^{i(l)}_j),
\end{aligned}$$

with $\Pi^{i(l)}_j$ being the anisotropic stress tensor.From the conservation of each $T^{\mu(l)}_\nu$ and by going to Fourier space we get the following general equations [2]:

$$\delta^{(l)\prime} = -3\mathcal{H}\left(c^{(l)2}_{eff} - w^{(l)}_{eff}\delta^{(l)}\right) - (1 + w^{(l)}_{eff})k\upsilon^{(l)} + 3(1 + w^{(l)}_{eff})\Phi' \tag{4}$$

$$\upsilon^{(l)\prime} = -\mathcal{H}\left[1 - 3w^{(l)}_{eff} - \frac{w^{(l)\prime}_{eff}}{\mathcal{H}(1 + w^{(l)}_{eff})}\right]\upsilon^{(l)} + k\left[\Psi + \frac{(c^{(l)}_{eff})^2}{1 + w^{(l)}_{eff}} - \frac{2\Pi^{(l)} w^{(l)}_{eff}}{3(1 + w^{(l)}_{eff})}\right], \tag{5}$$

where $\delta^{(l)} \equiv \delta\rho^{(l)}/\bar{\rho}^{(l)}$ is the relative density perturbation, $w^{(l)}_{eff} \equiv \bar{p}^{(l)}/\bar{\rho}^{(l)}$ is the parameter of state and $\left(c^{(l)}\right)^2_{eff} \equiv \delta P^{(l)}/\delta\rho^{(l)}$ is the effective sound speed square of the l-th species respectively.

The evolution of Φ and Ψ is governed by the perturbed Einstein equations, which in Fourier space are [2]:

$$3\mathcal{H}(\Phi' + \mathcal{H}\Psi) + k^2\Phi = -4\pi G a^2\, \delta\rho^{tot} \tag{6}$$

$$k(\Phi' + \mathcal{H}\Psi) = 4\pi G a^2 \sum_l (\bar{\rho}^{(l)} + \bar{p}^{(l)})v^{(l)} \tag{7}$$

$$\Phi'' + \mathcal{H}(\Phi' + 2\Psi') + (\mathcal{H}^2 + 2\mathcal{H}')\Phi - k^2(\Phi - \Psi)/3 = 4\pi G a^2\, \delta p^{tot} \tag{8}$$

$$k^2(\Phi - \Psi) = 8\pi G a^2 \sum_l \bar{p}^{(l)}\Pi^{(l)} \tag{9}$$

By combining the (4) and (5) one can get the following general equation:

$$\begin{aligned}
&-\delta^{(l)\prime\prime} - \mathcal{H}\delta^{(l)\prime}\left(1 - 6w^{(l)}_{eff}\right) + \left(\frac{\delta P^{(l)}}{\bar{\rho}^{(l)}} - w^{(l)}_{eff}\delta^{(l)}\right)\left[3\mathcal{H}^2(3w^{(l)}_{eff} - 1) - 3\mathcal{H}'\right] \\
&-3\mathcal{H}\left(\frac{\delta P^{(l)}}{\bar{\rho}^{(l)}}\right)' + 3\mathcal{H}w^{(l)\prime}_{eff}\delta^{(l)} = -3(1 + w^{(l)}_{eff})\left[\Phi'' + \Phi'\mathcal{H}\left(1 - 3w^{(l)}_{eff} + \frac{w^{(l)\prime}_{eff}}{\mathcal{H}(1 + w^{(l)}_{eff})}\right)\right] + \\
&+k^2\left[(1 + w^{(l)}_{eff})\Psi + \frac{\delta P^{(l)}}{\bar{\rho}^{(l)}} - \frac{2\Pi^{(l)}w^{(l)}_{eff}}{3}\right]
\end{aligned} \tag{10}$$

It is important to point out that equation (10) doesn't depend on the underlying gravitational theory, but rather only on the conservation of the energy-momentum tensor of the l-th sector. In the framework of ΛCDM cosmology l can represent the matter and radiation fluids as well as the dark energy "fluid", the cosmological constant Λ.

2. The need for "soft cosmology"

As it was first pointed out in [3], in the aforementioned standard framework there is a rather strong assumption, namely that the Universe sectors are *simple*, which means that one can apply the physics of usual *hard* matter. To elaborate, the underlying assumption is that the laws that determine the Universe behavior at large scales can be induced by the laws that determine the interactions between its individual constituents. Specifically regarding the hydrodynamic description of the Universe sectors, the use of fluid energy densities and pressures relies one the assumption that we can define fundamental "particles" of the corresponding sector, the collective flow of which gives rise to ρ_i and p_i, while all physics below the particle scale has been integrated out.

This process can in principle miss essential physics mainly through the following two ways: First, the integrated-out physics below the "particle" scale could impact differently the physics above the "particle" scale, depending on this coarse-graining scale. Secondly, the assumption that the Universe sectors and interactions are simple and more or less scale-independent though successfull to a very good degree, still it could miss some details a priori (see [3] for more details).

3. "Soft cosmology" and its perks

In condensed matter physics, it is well known that there is a large variety of "soft" matter forms, which are characterized by complexity, simultaneous co-existence of phases, entropy dominance, extreme sensitivity, viscoelasticity, etc, properties that arise effectively at *intermediate* scales due to *scale-dependent* effective interactions that are not present at the fundamental scales [4, 5].

These characteristic properties could help alleviate the cosmological tensions of standard $\Lambda_{\rm CDM}$ cosmology [6]. One can see a specific illustration of this as follows: As was first done in [3], as a first approximation we can introduce the effective "softness parameter" s_{de} for the dark energy sector. At cosmological, large scales (ls) dark energy can have the usual equation of state parameter (henceforth EoS), namely w_{de-ls}, but at intermediate scales (is) it has

$$w_{de-is} = s_{de} \cdot w_{de-ls}, \tag{11}$$

and standard cosmology is recovered for $s_{de} = 1$. The background evolution remains unchanged. According to (11) at intermediate scales the dark energy EoS w_{de-is} can be different, either constant $w_{de-is} = w_2$ or time-varying. Therefore, the $c_{eff}^{(de)}$ will change and due to the Poisson equation and (4) and (5) we will acquire a different evolution for the matter overdensity δ_m. As a result, the $f\sigma_8 \equiv f(a)\sigma(a)$ value, with $f(a) = d\ln\delta_m(a)/d\ln a$ and $\sigma(a) = \sigma_8\delta_m(a)/\delta_m(1)$, will be different than the corresponding one of standard cosmology with the above dark energy EoS. It is evident that we have a straightforward way to alleviate the σ_8 tension since we can suitably adjust w_{de-is} in order to obtain slightly lower $f\sigma_8$. A specific demonstration is presented in Fig. 1, where we depict the $f\sigma_8$ as a function of z. The dashed curve is for ΛCDM. The solid curve is for soft dark energy with $s_{dm} = 1.1$, i.e. with $w_{de-is} = -1.1$, and $c_{eff}^{(de)} = 0.1$, while dark matter is the standard one (i.e. not soft) with $w_{dm} = 0$. Note that in principle s_{de} can be varying too and one could introduce its parametrization, or one could additionally have more complicated situations in which w_{de-is} and w_{de-ls} have different parametrizations. Since our purpose is primarily to illustrate the utility of this approach, we considered the simplest case of (11). For a for more complete analysis of the possible values of these parameters based on cosmological data one can consult [6].

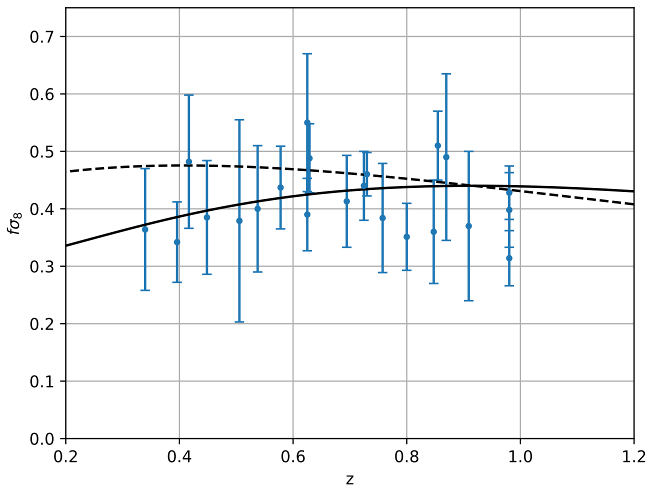

Figure 1: *The $f\sigma_8$ as a function of z. The dashed curve is for ΛCDM. The solid curve is for soft dark energy with $s_{de} = 1.1$, i.e. with $w_{de-ls} = -1$ and $w_{de-is} = -1.1$, and $c_{\rm eff}^{(de)} = 0.1$, while dark matter is standard (i.e. not soft) with $w_{dm} = 0$.*

4. Basics of $f(R)$ gravity

At this point we saw some of the advantages of considering the soft cosmology paradigm, but how can such a situation come about? We shall show that $f(R)$ gravity could give rise to such phenomena, so we shall briefly introduce them in this section.

Under the framework of $f(R)$ gravity theories the action describing gravity is the following:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\, f(R) + \int d^4x\sqrt{-g}\mathcal{L}_m, \tag{12}$$

where $f(R)$ is a general function of the Ricci scalar R, $\mathcal{L}_m$ corresponds to the (total) matter content of the Universe and $T^{(m)}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_m}{\delta g^{\mu\nu}}$. Varying the action (12) with respect to $g_{\mu\nu}$ yields the following field equation [7]:

$$FR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)F = 8\pi G T^{(m)}_{\mu\nu}, \tag{13}$$

where we set $F \equiv df(R)/dR$. It is interesting to rewrite equation (13) as the standard Einstein equation (1) and interpreting the higher order curvature terms as an effective fluid, whose energy-momentum tensor is $T^{eff}_{\mu\nu}$, as done for instance in [8, 9]:

$$G^\mu_\nu = 8\pi G\left(T^{(m)\mu}_\nu + T^{(eff)\mu}_\nu\right), \tag{14}$$

$$T^{(eff)\mu}_\nu \equiv (1-F)R^\mu_\nu + \frac{1}{2}\delta^\mu_\nu(f-R) - (\delta^\mu_\nu\Box - \nabla^\mu\nabla_\nu)F \tag{15}$$

Once again the Bianchi identity $\nabla_\mu G^\mu_\nu = 0$ ensures that from equation (14) it follows:

$$\nabla_\mu T^{(eff)\mu}_\nu = 0 \tag{16}$$

Thanks to this construction, both the background and perturbed field equations have the same form as in the case of general relativity that we described in the previous section. For the background FLRW metric the equations (2) and (3) still hold with:

$$\bar{\rho}_{eff} \equiv -T^{(eff)0}_0 = \frac{1}{8\pi G a^2}\left(3\mathcal{H}^2 - \frac{1}{2}a^2 f + 3F\mathcal{H}' - 3\mathcal{H}F'\right) \tag{17}$$

$$\bar{p}_{eff} \equiv \frac{T^{(eff)i}_i}{3} = \frac{1}{8\pi G a^2}\left(-2\mathcal{H}' - \mathcal{H}^2 + \frac{1}{2}a^2 f - F\mathcal{H}' - 2F\mathcal{H}^2 + F'' + \mathcal{H}F'\right) \tag{18}$$

After performing the same (scalar) perturbations as in the previous section, the set of perturbed field equations is the same as (6), (7), (8) and (9) with:

$$\delta\rho_{eff} \equiv -\delta T_0^{(eff)\,0} = -\frac{1}{8\pi G a^2}\{(1-F)\big[-6\mathcal{H}'\Psi + k^2\Psi - 3\mathcal{H}(\Phi'+\Psi') - 3\Phi''\big]$$
$$-3\mathcal{H}'\delta F + a^2\delta f/2 - k^2\Psi + 2k^2\Phi + 6(\mathcal{H}'+\mathcal{H}^2)\Psi + 3\Phi'' + 3\mathcal{H}(\Psi'+3\Phi')$$
$$+k^2\delta F + 3\mathcal{H}\delta F' - 3F'(\Phi'+2\mathcal{H}\Psi)\} \tag{19}$$

$$\delta p_{eff} \equiv \frac{\delta T_i^{(eff)\,i}}{3} = \frac{1}{8\pi G a^2}\{(1-F)\big[-k^2\Phi - \Phi'' - 3\mathcal{H}(5\Phi'+\Psi') - (2\mathcal{H}'+4\mathcal{H}^2)\Psi - k^2(\Phi-\Psi)/3\big]$$
$$-(\mathcal{H}'+2\mathcal{H}^2)\delta F + a^2\delta f/2 + 3\Phi'' + k^2(2\Phi-\Psi) + 3\mathcal{H}(\Psi'+3\Phi') + 6(\mathcal{H}'+\mathcal{H})\Psi$$
$$+\delta F'' + 2k^2\delta F/3 + \mathcal{H}\delta F' - F'(2\Phi'+2\mathcal{H}\Psi+\Psi') - 3\Psi F''\} \tag{20}$$

$$(\bar\rho_{eff}+\bar p_{eff})\upsilon^{eff}_{,i} \equiv -\delta T_i^{(eff)\,0} = \frac{1}{8\pi G}\Big[2(1-F)(\Phi'+\mathcal{H}\Psi)_{,i} + \delta F'_{,i} + F'\Psi_{,i} - \mathcal{H}\delta F_{,i}\Big] \tag{21}$$

$$\Pi^{eff}_{ij}\,\bar p_{eff} \equiv \delta T_j^{(eff)\,i} = \frac{1}{8\pi G a^2}[(1-F)(\Phi-\Psi)_{,ij} + \delta F_{,ij}],\ i \neq j \tag{22}$$

Additionally, the equations (4) and (5) hold for our effective fluid, as does the equation (10).

5. "Soft" properties from $f(R)$ gravity

In this section we are going to deduce from first principles the w_{eff} of the dark energy effective fluid, which we introduced in the previous section. Since we aim to reveal the "soft" properties of this fluid, we can't just use the simple expression $w_{eff} = \bar p_{eff}/\bar\rho_{eff}$ from equations (17) and (18). Instead, we shall infer w_{eff} by constructing an equation like (10) via algebraic manipulations of the field equation (6) with $\delta\rho_{tot} = \delta\rho_m + \delta\rho_{eff}$ where $\delta\rho_{eff}$ is given by (19).

First, we construct the quantities $\delta_{eff} \equiv \delta\rho_{eff}/\bar\rho_{eff}$, δ'_{eff} and δ''_{eff}. Next, we add to both sides of the equation (6) every perturbation term in (10) that doesn't have a w_{eff} factor, which means the terms $\delta''_{eff} + \mathcal{H}\delta'_{eff} + 3\frac{\delta p_{eff}}{\bar\rho}(\mathcal{H}'+\mathcal{H}^2) + 3\mathcal{H}\left(\frac{\delta p_{eff}}{\bar\rho}\right)'$. After using the equations (17) and (19), we get a very large number of terms. Luckily, we are only interested in the terms which contain a factor of $k^2\Psi$, since our aim is to identify those terms with w_{eff} using the corresponding term of equation (10). Therefore, if we only consider the aforementioned terms and factorise them appropriately, we finally obtain the following equation:

$$\delta''_{eff} + \mathcal{H}\delta'_{eff} + 3\frac{\delta p_{eff}}{\bar\rho}(\mathcal{H}'+\mathcal{H}^2) + 3\mathcal{H}\left(\frac{\delta p_{eff}}{\bar\rho}\right)' = k^2\frac{\Psi}{\rho_{eff}}\Bigg\{\mathcal{H}^2 + \mathcal{H}' + \frac{\rho_{eff}}{2} + 2A^2$$
$$+k^2\left\{2\frac{F_{,R}}{a^2}\left[-4\mathcal{H}^2 - 4\mathcal{H}' - \frac{\rho_{eff}}{2} - 2A^2 + 6\mathcal{H}A - B\right] - 2\frac{F'_{,R}}{a^2}(5\mathcal{H}+2A) + 2\frac{F''_{,R}}{a^2}\right\} +$$
$$+F\left(3\mathcal{H}^2 - 2\mathcal{H}' + 2B + 2\mathcal{H}A\right) - F'2\mathcal{H} + 2\frac{F_{,R}}{a^2}\{12\mathcal{H}^4 + 3(\mathcal{H}')^2 + 21\mathcal{H}^2\mathcal{H}' + 9\mathcal{H}\mathcal{H}'' - 3\mathcal{H}'''$$
$$+\frac{\rho_{eff}}{2}\left(3\mathcal{H}' + 6\mathcal{H}^2\right) + 2\frac{F'_{,R}}{a^2}\left[15\mathcal{H}^3 - 3\mathcal{H}\mathcal{H}' - 3\mathcal{H}'' - \frac{3}{2}\mathcal{H}\rho_{eff} + A\left(12\mathcal{H}^2 - 6\mathcal{H}A\right) - 3\mathcal{H}B\right]$$
$$-18\frac{F''_{,R}}{a^2}\mathcal{H}A\Bigg\} + \text{other terms} \equiv (1+w_{eff})k^2\Psi + \text{other terms},$$

where $A \equiv \rho'_{eff}/\rho_{eff}$, $B \equiv \rho''_{eff}/\rho_{eff}$ and ρ_{eff} and its derivatives are given by equation (17). Our final result can be written as:

$$
\begin{aligned}
w_{eff} = -1+\frac{1}{\rho_{eff}}\Bigg\{&\mathcal{H}^2+\mathcal{H}'+\frac{\rho_{eff}}{2}+2A^2+k^2\left\{2\frac{F_{,R}}{a^2}\left[-4\mathcal{H}^2-4\mathcal{H}'-\frac{\rho_{eff}}{2}-2A^2+6\mathcal{H}A-B\right]\right.\\
&-2\frac{F'_{,R}}{a^2}(5\mathcal{H}+2A)+2\frac{F''_{,R}}{a^2}+F\left(3\mathcal{H}^2-2\mathcal{H}'+2B+2\mathcal{H}A\right)-F'2\mathcal{H}+\\
&+2\frac{F_{,R}}{a^2}\left\{12\mathcal{H}^4+3(\mathcal{H}')^2+21\mathcal{H}^2\mathcal{H}'+9\mathcal{H}\mathcal{H}''-3\mathcal{H}'''+\frac{\rho_{eff}}{2}\left(3\mathcal{H}'+6\mathcal{H}^2\right)\right\}\\
&\left.+2\frac{F'_{,R}}{a^2}\left[15\mathcal{H}^3-3\mathcal{H}\mathcal{H}'-3\mathcal{H}''-\frac{3}{2}\mathcal{H}\rho_{eff}+A\left(12\mathcal{H}^2-6\mathcal{H}A\right)-3\mathcal{H}B\right]-18\frac{F''_{,R}}{a^2}\mathcal{H}A\right\}
\end{aligned}
\tag{23}
$$

As one can readily observe, indeed our final EoS exhibits the desired scale dependance behaviour.

6. Conclusions

We briefly examined the possibility of “soft" cosmology, namely small deviations from the usual framework due to the effective appearance of soft properties in the Universe sectors. This framework can alleviate the persistent cosmological tensions. We demonstrated that modified theories of gravity (MG) and $f(R)$ in particular can facilitate such a paradigm, since they naturally give rise to equation of state parameters (EoS) which exhibit scale dependence. A potential refinement of these results could provide a more quantitatively useful bridge between parameters of MG and these phenomenological "soft" parameters, like the ones we introduced in (11). In order to incorporate complexity and estimate the scale-dependent behavior of the EoS's from first principles we should revise and extend the cosmological perturbation theory.

Acknowledgments

C.T. acknowledges financial support from the A. G. Leventis Foundation as well as the contribution of the COST Actions CA18108 “Quantum Gravity Phenomenology in the multi-messenger approach” and CA21136 “Addressing observational tensions in cosmology with systematics and fundamental physics (CosmoVerse)”.

References

[1] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003) [astro-ph/0207347].

[2] A.R. Liddle and D.H. Lyth: Cosmological Inflation and Large-Scale Structure (Cambridge University Press 2000)

[3] E. N. Saridakis, Phys. Lett. B **822** (2021), 136649 doi:10.1016/j.physletb.2021.136649 [arXiv:2105.08646 [astro-ph.CO]]

[4] R. A. L. Jones, *Soft Condensed Matter*, Oxford University Press, Oxford (2002).

[5] L. M. C. Sagis, Rev. Mod. Phys. 83, 1367-1403 (2011).

[6] E. N. Saridakis, W. Yang, S. Pan, F. K. Anagnostopoulos and S. Basilakos, Nucl. Phys. B **986**, 116042 (2023) doi:10.1016/j.nuclphysb.2022.116042 [arXiv:2112.08330 [astro-ph]].

[7] A. De Felice and S. Tsujikawa, Living Rev. Rel. **13**, 3 (2010) doi:10.12942/lrr-2010-3 [arXiv:1002.4928 [gr-qc]]

[8] R. Arjona, W. Cardona and S. Nesseris, Phys. Rev. D **99**, no.4, 043516 (2019) doi:10.1103/PhysRevD.99.043516 [arXiv:1811.02469 [astro-ph.CO]]

[9] S. Tsujikawa, Phys. Rev. D76, 023514 (2007), 0705.1032. [arXiv:0705.1032 [astro-ph]]