PROCEEDINGS OF SCIENCE

Quantum vacuum, a cosmic chameleon

Cristian Moreno-Pulido[a,*] **and Joan Solà Peracaula**[a]

[a] *Department de Física Quàntica i Astrofísica, and Institute of Cosmos Sciences, Universitat de Barcelona, Av. Diagonal 647, 08028, Barcelona, Catalonia, Spain*

E-mail: cristian.moreno@fqa.ub.edu, sola@fqa.ub.edu

The ΛCDM cosmology, which includes a Cosmological Constant (CC), has been the dominant paradigm for the past 25 years. However, the appearance of tensions in different cosmological parameters and the persistence of traditional theoretical problems associated with the CC challenge the validity and viability of the model, leading to the search for new physics. Recent studies of vacuum energy in quantum field theory in a FLRW spacetime predict that this new physics may be related to a slow running of the vacuum energy density with the Hubble function. The running can be described by the difference of two close values near the present, $\delta\rho_{\rm vac} \sim \mathcal{O}(H^2)$, which is a characteristic of the traditional Running Vacuum Models. Higher powers of H may be relevant in the early universe and could naturally drive a mechanism for inflation. On the other hand, the equation of state (EoS) of the running vacuum is no longer predicted to be fixed at exactly -1. Instead, it is expected to evolve, mimicking the dominant component at the time. Thus, during inflation, it starts at $w_{\rm vac} - 1$, during the radiation-dominated epoch, $w_{\rm vac} = 1/3$, during the matter-dominated epoch, $w_{\rm vac} = 0$, and near the present, it behaves as either quintessence or phantom. The additional features revealed by these calculations may lead to a consistent model capable of alleviating the current σ_8 and H_0 tensions of modern cosmology and shedding light on the problem of the CC.

Corfu Summer Institute 2022 "School and Workshops on Elementary Particle Physics and Gravity",
28 August - 1 October, 2022
Corfu, Greece

*Speaker

PoS(CORFU2022)211

1. Introduction

The Cosmological Constant (CC), Λ, was added by Einstein to his own field equations in 1917 [1] in order to describe a model of a static Universe. However, it was later dismissed by observations and stability problems a few years after its introduction. Nevertheless, in the 1990s, the CC turned out to be a crucial ingredient in solving the cosmological puzzle once again. However, its modern role in the current framework of cosmology, the ΛCDM, is different from what Einstein had intended. After the observational confirmation of the accelerated expansion of the Universe in 1998 from Supernovae SnIa luminosity distances [2], the inclusion of Λ in the field equations became the best way to mathematically model Dark Energy (DE), the mysterious entity responsible for this accelerated expansion. Since then, the ΛCDM has been our best model for explaining the overall cosmological observations and the main features of the Universe at the largest scales. However, some unignorable flaws are present both at the theoretical and phenomenological level.

The phenomenological problems with the ΛCDM model are related to high discrepancies in the values of certain cosmological parameters as reported by different collaborations. These discrepancies are exacerbated to the level of tensions in the case of two important parameters [3–5]. Firstly, the so-called Hubble tension is related to H_0, the Hubble function at the present time that measures the current pace of cosmological expansion. CMB observations [6] report a low value of this parameter compared with local geometrical estimations based on the distance ladder method [7], differing by as much as 5σ in some particular scenarios. Secondly, the σ_8 tension afflicts the root mean square fluctuations in matter density at the $8h^{-1}$ Mpc scale, which is related to the structure formation. Conflicts arise between its value obtained from measurements of the amplitude of the power spectrum of density perturbations inferred using CMB data and those directly measured from redshift space distortions and weak-lensing data [15, 16]. The tension in this case is less severe, of the order of 2-3σ discrepancy, but still worrisome. While we will not discuss these tensions more profoundly in this work, it is true that their existence has motivated a search for new models beyond the vanilla ΛCDM. Many of these models have flourished with the aim of solving the tensions from different perspectives.

On the other hand, the ΛCDM faces many remarkable theoretical conundrums. In particular, let us center the topic on those related to DE and its very conception in quantum field theory (QFT). We are specifically referring to the CC problem: our inability to match theoretical predictions for the value of Λ (or the associated vacuum energy density (VED) $\rho_{\rm vac} = \Lambda/(8\pi G_N)$, where G_N is the gravitational constant) with experiments. The mismatch is usually said to stem from the naive predictions of QFT, which associate the ZPE of a matter field of mass m, proportional to m^4, with the value of the VED inferred from cosmological observations, $\rho_{\rm vac}^{\rm obs} \sim 10^{-47}$ GeV^4. For instance, if we take $m \sim 0.5$ MeV, the mass of the electron, the ratio $\rho_{\rm vac}^{\rm obs}/m^4$ is of the order of 10^{-34}. Far from being just an artifact created by the simplicity of a CC model, it seems to be a pathology common to all known forms of DE.

The Cosmological principle opens the window to the possibility of the VED depending on time through a dynamical variable $\xi(t)$, so that $\rho_{\rm vac}(\xi)$, and choosing the CC to be a constant of nature that does not evolve with the expansion is purely due to Occam's razor. However, once we aim to look for models that go beyond the traditional ΛCDM, the possibility of promoting the VED to be a function of time is tantalizing. In this work, however, we will not focus on generic parametrizations of the VED. Instead, we will follow the line started in recent papers [8–11] (see also [12] for a review) in which the running of the vacuum is a consequence of the renormalization of quantum matter fields in a FLRW background. In the absence of a successful quantum theory of gravity, a semiclassical approach to the problem seems to provide us with some interesting results that will be reviewed in Sect. 2. In Sect. 3, we describe the evolution of the Equation of State (EoS) of vacuum, as predicted by the QFT computations of Sect. 2. Finally, we summarize the main ideas presented here in the conclusions.

2. Renormalized vacuum energy and pressure

Traditional approaches to the computation of the VED in the context of QFT have failed in the task of performing a suitable prediction capable of matching the current tiny value stated by observations. For instance, dimensional regularization combined with a Minimal Subtraction scheme for a simple case of a free scalar field leads to an unfathomable expression for VED which reads as

$$\rho_{\rm vac} = \rho_\Lambda(\mu) + \frac{m^4}{64\pi^2}\left(\ln\frac{m^2}{\mu^2} + C_{\rm vac}\right)\,. \tag{1}$$

The former equation comes from the joint renormalized contribution of the CC in the Einstein-Hilbert action, represented by $\rho_\Lambda(\mu)$, and the one-loop contribution to the ZPE. The term $C_{\rm vac}$ is a constant that appears after performing the usual counterterm procedure for renormalizing the result, and μ is the 't Hooft scale. The fact that the left-hand side of eq. (1) has a contribution of order $\rho_{\rm vac} \sim 10^{-47}$ GeV^4, while on the right-hand side the quartic power of a massive field is present, means that we have to adjust the remaining term ρ_Λ at an unprecedented precision. For instance, if we follow the example of the mass of the electron presented before, adjusting $\rho_\Lambda(\mu)$ requires a precision of 35 decimal places. It is even worse if we take heavier fields such as the top quark, which would mean to fix the value of $\rho_\Lambda(\mu)$ with precision for the first 56 decimal places, in order for the sum on the right-hand side of eq. (1) to match the value on the left-hand side. Of course, this fine-tuning is totally unacceptable, and it can become even more involved if we add higher-order loop contributions and include the nontrivial effects of spacetime curvature.

Let's approach the problem from a more humble perspective. Rather than computing the cosmological value of the VED from first principles, we will present a dynamical formulation of the VED obtained by a rigorous computation in QFT. We will work within a Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime, with flat hypersurfaces of constant cosmic time. To avoid some complications, let us consider, for simplicity, a free scalar field of mass m nonminimally coupled to curvature (with coupling ξ) and in the absence of an effective potential. The action is given by

$$S[\phi] = -\int d^n x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}\left(m^2+\xi R\right)\right)\,. \tag{2}$$

The zero-point energy (ZPE) associated with the quantum fluctuations of the field is, of course, UV divergent and requires a renormalization process. The renormalization procedure involves the use of an extension of the well-known method of adiabatic regularization presented in the textbooks of QFT in curved spacetime [13, 14]. The method consists of expressing physical observables and other intermediate quantities in the so-called adiabatic orders, which are organized depending on the number of time derivatives present in the different terms of the expansion. The novelty with respect to the traditional approach lies in the arbitrariness present in asymptotic series. This fact permits us to introduce an arbitrary renormalization scale M replacing the mass of the scalar field in the 0th adiabatic order terms, doing the renormalization process off-shell. We will not describe the whole procedure here[1], but let us mention that the renormalized version of the vacuum expected value of the Energy-Momentum tensor (EMT) associated with the quantum fluctuations of the field, $\delta\phi$, takes the following form:

$$\left\langle T_{\mu\nu}^{\delta\phi}\right\rangle_{\rm Ren}(M) \equiv \left\langle T_{\mu\nu}^{\delta\phi}\right\rangle(m) - \left\langle T_{\mu\nu}^{\delta\phi}\right\rangle^{(0-4)}(M)\,. \tag{3}$$

We have performed a subtraction from the on-shell quantity. The cancellation produced in the first four adiabatic orders is sufficient to cancel the UV divergences associated with the EMT in $n = 4$ spacetime dimensions. The structure of the EMT is formed by powers of the Hubble function and its derivatives. As an example, let us explicitly write the ZPE (the 00 component of (3)) up to the 4th adiabatic order:

$$\begin{aligned}\langle T_{00}^{\rm vac}\rangle_{\rm Ren}^{(0-4)}(M) &= \frac{a^2}{128\pi^2}\left(-M^4 + 4m^2M^2 - 3m^4 + 2m^4\ln\frac{m^2}{M^2}\right)\\ -\left(\xi-\frac{1}{6}\right)\frac{3a^2H^2}{16\pi^2}&\left(m^2 - M^2 - m^2\ln\frac{m^2}{M^2}\right) + \left(\xi-\frac{1}{6}\right)^2\frac{9a^2\left(6H^2\dot{H} + 2\ddot{H}H - \dot{H}^2\right)}{16\pi^2}\ln\frac{m^2}{M^2}\,.\end{aligned} \tag{4}$$

Here, $H \equiv \dot{a}/a$ and the dot denotes the derivative with respect to cosmic time. The absence of odd adiabatic orders is justified by general covariance, as tensors of odd adiabatic order are not present in Einstein's equations. The next term in the adiabatic expansion (4) contains terms of order $\mathcal{O}(H^6/m^2)$. It is obvious that the Energy-Momentum Tensor (EMT) associated with the vacuum state should contain the contribution of the Zero-Point Energy (ZPE), together with the geometrical contribution associated with the parameter ρ_Λ in the Einstein-Hilbert action[2],

$$\langle T_{\mu\nu}^{\rm vac}\rangle_{\rm Ren}(M) = \left\langle T_{\mu\nu}^{\delta\phi}\right\rangle_{\rm Ren}(M) - \rho_\Lambda g_{\mu\nu}\,. \tag{5}$$

Einstein's equations read $\mathcal{M}_{\rm Pl}^2(M)G_{\mu\nu} + \alpha(M)\,{}^{(1)}H_{\mu\nu} = \langle T_{\mu\nu}^{\rm vac}\rangle_{\rm Ren}(M)$, where $\mathcal{M}_{\rm Pl}(M)$ is the (running) Planck's mass, ${}^{(1)}H_{\mu\nu}$ is a higher derivative tensor, containing terms of 4th adiabatic order, and α is a running coupling. When the vacuum is treated as a perfect fluid, the former equation leads to the following expressions for the VED and pressure of the vacuum fluid:

$$\begin{aligned}\rho_{\rm vac}(M,H) &= \frac{\langle T_{00}^{\rm vac}\rangle_{\rm Ren}(M)}{a^2} = \rho_\Lambda(M) + \frac{\left\langle T_{00}^{\delta\phi}\right\rangle_{\rm Ren}(M)}{a^2}\,,\\ P_{\rm vac}(M,H) &= \frac{\langle T_{11}^{\rm vac}\rangle_{\rm Ren}(M)}{a^2} = -\rho_\Lambda(M) + \frac{\left\langle T_{11}^{\delta\phi}\right\rangle_{\rm Ren}(M)}{a^2}\,.\end{aligned} \tag{6}$$

We have made explicit the dependence on H in the former expression in order to emphasize that, beyond the running with the renormalization scale, there is an explicit dependence on H and its

[1]The reader interested in the rigorous derivations may find all the details in the preceding papers [8, 9, 11].

[2]It is tempting to define $\rho_\Lambda \equiv \Lambda/(8\pi G_N)$, where Λ is the cosmological constant. However, this would be inaccurate since the CC is associated with the entire VED, $\rho_{\rm vac} = \Lambda/(8\pi G_N)$, from which ρ_Λ is just a contribution.

PoS(CORFU2022)211

derivatives due to performing the computation in a FLRW background. Remarkably, we can use the expressions in equation (6) to compute the equation of state (EoS) of the quantum vacuum, which we present in Section 3. The next step in the renormalization procedure is choosing an appropriate renormalization scale, which is similar to renormalization group approaches in ordinary gauge theories where the scale is set to a characteristic energy of the process. In the cosmological case, fixing $M = H$ is natural since H represents the characteristic energy scale of the expanding background. Therefore, $\rho_{\rm vac}(H, H)$ (which we will simply denote as $\rho_{\rm vac}(H)$) represents the physical value of the VED, depending explicitly and implicitly (through setting the renormalization scale to the same value as the Hubble function) on H. By subtracting the current VED (evaluated at H_0) from its value at an arbitrary point in time, we can write the running law of the VED as

$$\rho_{\rm vac}(H) = \rho^0_{\rm vac} + \frac{3m^2\left(\xi - \frac{1}{6}\right)}{16\pi^2}\left[H^2\left(-1 + \ln\frac{m^2}{H^2}\right) - H_0^2\left(-1 + \ln\frac{m^2}{H_0^2}\right)\right] + \mathcal{O}(H^4)\,, \tag{7}$$

where $\rho^0_{\rm vac} \equiv \rho_{\rm vac}(H_0)$. The former expression can be cast in a more compact way as

$$\rho_{\rm vac}(H) \approx \rho^0_{\rm vac} + \frac{3\nu_{\rm eff}(H)}{8\pi}m^2_{\rm Pl}(H^2 - H_0^2)\,. \tag{8}$$

In (8) we have neglected the terms $\mathcal{O}(H^4)$ as they are unimportant for describing the postinflationary history. The effective running parameter $\nu_{\rm eff}(H)$ is a mildly evolving function of H during the cosmological expansion, but for the late time universe it suffices to approximate its value by a constant

$$\nu_{\rm eff} \equiv \frac{1}{2\pi}\left(\xi - \frac{1}{6}\right)\frac{m^2}{m^2_{\rm Pl}}\ln\frac{m^2}{H_0^2}\,. \tag{9}$$

The former expression (8) is a perfectly smooth function of the VED and the Hubble function, typical of the Running Vacuum Models (RVM). We show a plot from the evolution of the VED and the other energy densities in the late-time Universe in Fig. 1. Additionally, we observe the absence of dangerous m^4 terms in (7), which would produce a gargantuan difference between observations and predictions as commented in the introduction. Equation (7) or (8) do not represent the computation of the VED from first principles at a particular time; instead, its meaning is to specify the evolution of the VED with the expansion history. The appearance of $\rho^0_{\rm vac}$ reinforces the analogy with gauge theories, as it represents the connection of the running law with observations. That is, $\rho^0_{\rm vac}$ is a mandatory experimental input in order to obtain the value of the VED at any other point in time. This can also be seen from the perspective of the beta function of the running vacuum, prior to setting $M = H$:

$$\beta_{\rho_{\rm vac}} = M\frac{\partial\rho_{\rm vac}}{\partial M} = \left(\xi - \frac{1}{6}\right)\frac{3H^2}{8\pi^2}(M^2 - m^2) + \left(\xi - \frac{1}{6}\right)^2\frac{9\left(\dot{H}^2 - 2H\ddot{H} - 6H^2\dot{H}\right)}{8\pi^2}\,. \tag{10}$$

As we observe, the β function is free from the aforementioned pernicious terms proportional to m^4. The renormalization group formalism let us to derive the renormalization group equations for all the couplings. In particular, the gravitational constant acquires a running evolution [26] with the Hubble function after replacing $M = H$, whose low energy regime reads

$$G(H) \equiv \frac{1}{\mathcal{M}^2_{\rm Pl}(H)} = \frac{G_N}{1 - \epsilon\ln\frac{H^2}{H_0^2}}\,. \tag{11}$$

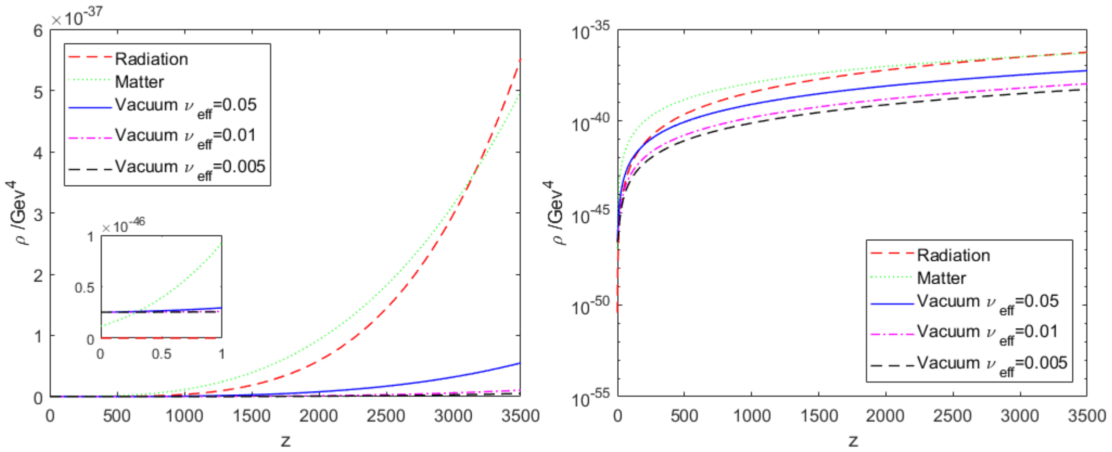

Figure 1: In the left plot, the evolution of the energy densities (matter, vacuum and radiation) in the FLRW framework is shown for different values of ν_{eff}. The small inner window provides more details on the low energy regime near our time. In the right plot, a logarithmic axis is used to magnify the differences. The VED exhibits very mild dynamics up to the radiation dominated epoch.

2.1 Higher powers

In eq. (4) we showed the expression for the ZPE up to 4th adiabatic order. While the lower order terms are the only relevant ones at the low energy regime, i.e. near the present time, higher order terms may play a critical role in the early universe, particularly in the inflationary epoch. The key idea is that these terms may be able to significantly amplify the magnitude of the VED. Therefore, there exists the possibility of a mechanism of inflation that relies on the natural capability of the quantum vacuum to be enhanced at the primordial era, rather than on ad hoc inflaton fields in the classical action. A realization of inflation in this context requires the VED to depend on an even power of the Hubble function beyond H^2, such as H^4, H^6, and so on. We call this mechanism RVM-inflation, and it occurs through a short period where H ≈const. During this time, vacuum energy is entirely dominated by the higher powers of the adiabatic expansion. For instance, we can study RVM-inflation triggered by the effects of the $O(H^6/m^2)$ terms of the ZPE that were omitted previously[3]. It is sufficient to say that during the inflationary process, the VED is given by

$$\rho_{\text{vac}}^{\text{inf}} \sim \frac{\left\langle T_{00}^{\delta\phi} \right\rangle_{\text{Ren}}^{\text{6th}} (m)}{a^2} = \frac{C_{\text{inf}}}{m^2} H^6 \,, \tag{12}$$

where C_{inf} is a constant that depends on the nonminimal coupling and the mass of the field, see [9] for the explicit expression of C_{inf} and [11] for a generalization of the calculation in the presence of several free massive quantized fields, including both fermions and (non-minimally coupled) scalars in an arbitrary number. The mass of the involved fields may reach the characteristic scale of Grand Unified Theories, presumably $m \sim M_X \sim 10^{16}$ GeV. Solving the Friedmann equations yields

$$H(\hat{a}) = H_I \left(1 + \hat{a}^8\right)^{-1/4} \,, \tag{13}$$

[3] A similar mechanism relying on terms proportional to $O(H^4)$ is also possible and they can be motivated from a stringy approach, see [17].

and the energy densities

$$\rho_{\rm vac}(\hat{a}) = \rho_I \left(1+\hat{a}^8\right)^{-3/2}, \qquad \rho_{\rm r}(\hat{a}) = \hat{a}^8 \rho_{\rm vac}(\hat{a})\,. \tag{14}$$

Here, $\hat{a} \equiv a/a_*$, where a_* determines the end of inflation, i.e., the transition point from vacuum dominance to the radiation-dominated epoch (RDE), estimated to be around $a_* \sim 10^{-29}$ [27]. At the beginning of inflation, H evolves very little around the gigantic value $H_I \sim C_{\rm inf}^{-1/4} \mathcal{M}_{\rm Pl}^{1/2} m^{1/2}$. Similarly, $\rho_{\rm vac}$ remains close to $\rho_I \sim C_{\rm inf}^{-3/2} \mathcal{M}_{\rm Pl}^3 m$, while radiation energy density is negligible, i.e., $\rho_{\rm r} \approx 0$.

After inflation, when $\hat{a} \gg 1$, we enter the standard FLRW radiation epoch where the radiation energy density takes its usual form $\rho_{\rm r} \propto a^{-4}$. The primordial vacuum energy density decreases quickly, and the higher powers present in the adiabatic expression of the vacuum energy density become irrelevant, without affecting critically the primordial Big Bang Nucleosynthesis (BBN) physics. The VED is then described by equation (8).

Additionally, it is worth noting that the equation of state (EoS) of the quantum vacuum remains essentially -1 during the inflationary period. In other words, during inflation the relation between the vacuum's pressure and VED follows the traditional form $P_{\rm vac} = -\rho_{\rm vac}$, as can be seen after a lengthy computation starting from (6). The detailed calculation reveals that the sum $P_{\rm vac} + \rho_{\rm vac}$ depends solely on terms proportional to the derivatives of the Hubble function, which implies $P_{\rm vac} + \rho_{\rm vac} = 0$ during this short period when H is constant. However, after the inflationary process, this is no longer the case, and a dynamical evolution of the vacuum's EoS begins to take place.

3. Equation of state of the quantum vacuum in a FLRW universe

Quantum effects trigger a dynamical behavior of the vacuum's EoS, $w_{\rm vac}$, during the subsequent FLRW expansion after inflation. Rather than being stuck at $w_{\rm vac} = -1$, it changes with the cosmological eras in a way that can be computed exactly through the QFT calculations presented before, particularly from the expressions of $\rho_{\rm vac}$ and $P_{\rm vac}$ in equation (6). After some calculations (see the references [10, 11] for more details), an accurate expression in terms of the redshift emerges as

$$w_{\rm vac}(z) = -1 + \frac{\nu_{\rm eff}\left(\Omega_{\rm m}^0(1+z)^3 + \frac{4}{3}\Omega_{\rm r}^0\,(1+z)^4\right)}{\Omega_{\rm vac}^0 + \nu_{\rm eff}\left(-1+\Omega_{\rm m}^0\,(1+z)^3 + \Omega_{\rm r}^0\,(1+z)^4 + \Omega_{\rm vac}^0\right)}\,, \tag{15}$$

where $\Omega_\Lambda^0 \sim 0.7$, $\Omega_{\rm m}^0 \sim 0.3$, and $\Omega_{\rm r}^0 \sim 10^{-4}$ are the current energy fractions of vacuum, matter, and radiation in the Universe. The above formula depends on the coefficient $\nu_{\rm eff}$ defined earlier, which may be fitted to the current-era cosmological data in the last few years [18–22], showing that $\nu_{\rm eff} \sim 10^{-2} - 10^{-3}$ and is preferred by the data to be positive. See also [25] for a phenomenological analysis of modified gravity models and the RVM, where a BBN bound constrains $|\nu_{\rm eff}| \sim 10^{-3} - 10^{-2}$, in good accordance with the fitting values obtained from the previous cited papers. From eq. (15) we can distinguish three different regimes. Denoting by $z_{\rm eq} = \Omega_{\rm m}^0/\Omega_{\rm r}^0 - 1 \approx 3300$ the equality point

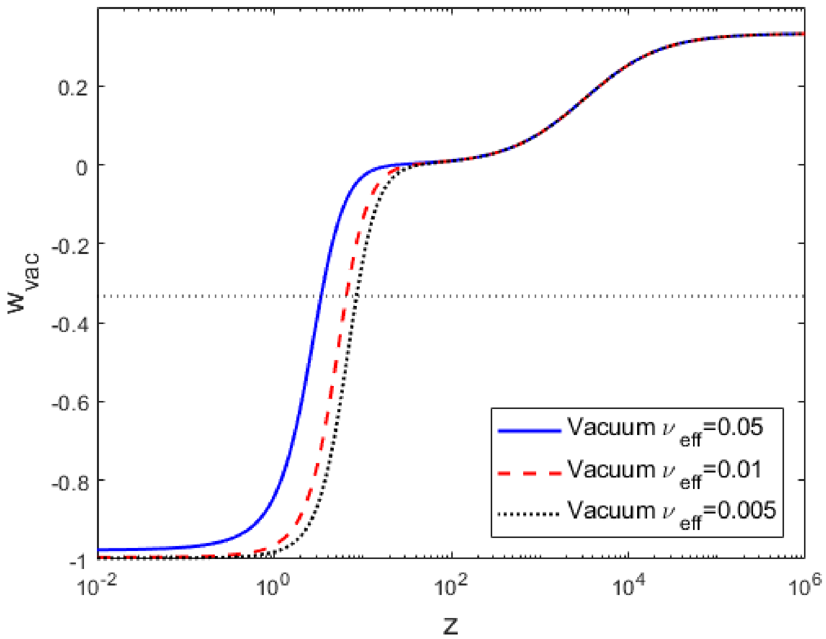

Figure 2: The evolution of the equation of state of vacuum in the low energy regime near our time for different values of $\nu_{\rm eff}$. We distinguish three different plateaus: the RDE, where $w_{\rm vac} \approx 1/3$, the MDE, where $w_{\rm vac} \approx 0$, and the current era, where $w_{\rm vac} \gtrsim -1$, behaving as quintessence for $\nu_{\rm eff} > 0$.

between matter and radiation, we find

$$w_{\rm vac}(z) = \begin{cases} \frac{1}{3} \quad \text{for } z \gg z_{\rm eq} \text{ with } \Omega_{\rm r}^0(1+z) \gg \Omega_{\rm m}^0, & \text{radiation behavior } (\nu_{\rm eff} \neq 0), \\ 0 \quad \text{for } \mathcal{O}(1) < z \ll z_{\rm eq} \text{ with } \Omega_{\rm m}^0 \gg \Omega_{\rm r}^0(1+z), & \text{dust behavior } (\nu_{\rm eff} \neq 0), \\ -1 + \nu_{\rm eff}\frac{\Omega_{\rm m}^0}{\Omega_{\rm vac}^0}(1+z)^3 \quad \text{for } \; -1 < z < \mathcal{O}(1)\,, & \text{quintessence behavior } (\nu_{\rm eff} > 0)\,. \end{cases} \tag{16}$$

Vacuum seems to imitate the dominant component at the time. The reader may visualize these different regimes in Fig. 2. Namely, the running vacuum follows the EoS of radiation in the RDE, the EoS of pressureless Cold Dark Matter during the matter dominated epoch (MDE) and behaves as quintessence (for $\nu_{\rm eff} > 0$), approaching again to -1 in a future de Sitter era.

This shapeshifting ability of the vacuum may help to alleviate or cure the σ_8 and H_0 tension [3–5]. In [18, 19] it was shown that, if we assume an interaction between cold dark matter and vacuum which is suddenly activated at a recent redshift $z_{\rm thr} \sim \mathcal{O}(1)$, that we call 'threshold', then the resulting dynamics of the vacuum fluid can be extremely helpful for solving the σ_8. This situation may be a reflection of the QFT prediction: the interaction between matter-radiation models the transmutation from $w_{\rm vac} \approx 0$ in the matter dominated epoch to $w \approx -1$ near our epoch. In that sense, it is only up to a very recent redshift that the vacuum behaves as a dynamical DE, the agent responsible for the acceleration of the Universe, while before it mimics matter. At the same time, in these works [18, 19] it was shown that an evolving gravitational constant may also help to alleviate the H_0 tension. Here, the renormalization group equations dictate that we should expect a smooth logarithmic evolution of G along the cosmic history. This fact together with the chameleonic behavior of vacuum's EoS may combine constructively to mitigate the σ_8 and H_0 tensions at a time.

Conclusions

In this work we have listed some new features of the quantum vacuum that emerge after taking into account the quantum fluctuations of matter. For the sake of simplicity, the original computations presented here were performed for the quantum fluctuations of a single free scalar field nonminimally coupled to gravity [8–10], but have recently been extended to other fermion fields [11].

The running vacuum models (RVM) are a family of models in which the running vacuum is parametrized as powers of the Hubble function, $\rho_{\rm vac}(H)$. These models have been around for some years, sustained by semi-qualitative renormalization group arguments. In [8, 9, 11], the functional dependence of $\rho_{\rm vac}(H)$ was rigorously justified, giving strong theoretical grounds for the RVM for the first time. What was shown in these papers and revised here is that the value of the vacuum energy density, $\rho^0_{\rm vac} \equiv \rho_{\rm vac}(H_0)$, is not a constant of nature anymore. Instead, it is just the current value of a VED, which evolves slowly with time. As a consequence, there is no cosmological constant after all, but rather a slowly varying $\Lambda = 8\pi\rho_{\rm vac}(H)G(H)$. The dynamics are quite moderate, creating the illusion that Λ as defined before is just a constant for most of the recent history, and not being in severe conflict with the ΛCDM model, at least in the late-time Universe. The increase/decrease of the VED between two nearby moments in the recent history can be related to the change in their respective Hubble functions as $\delta\rho_{\rm vac} \propto \nu_{\rm eff}\delta H^2$, with $|\nu_{\rm eff}| \ll 1$. The former parameter is proportional to the β-function of the running VED, and theoretical predictions point out to a small value of this parameter, although it is ultimately fitted by observations. Interestingly, the β-function of the VED is free from the undesired $\sim m^4$ term, meaning that the running is not afflicted by this enormous contribution. Additionally, the renormalization group formalism implies that the gravitiational constant is a running coupling [26] and acquires a dynamical evolution, which is expected to be logarithmic, $G = G(\ln(H))$.

While in the recent era the dynamics of the field are moderate and driven by the quadratic power of the Hubble function, the higher orders terms (higher powers and higher order derivatives of the Hubble tension) in the adiabatic series can play a capital role in the early Universe. More specifically, they can enhance naturally the magnitude of the VED and may describe a simple mechanism for inflation. In the present work we have shown an example of this mechanism by considering the terms $O(H^6/m^2)$, however a careful study is still necessary.

Another astonishing prediction of QFT calculations is that vacuum's EoS is not equal to -1 along the whole cosmological history [10, 11], but a function of the redshift, $w_{\rm vac}(z)$. What is even more surprising is that it tends to mimic the dynamical component at the time: it behaves as a true cosmological constant during inflation with $w^{\rm inf}_{\rm vac} \approx -1$, imitates radiation at the RDE with $w^{\rm RDE}_{\rm vac} \approx 1/3$ and matter during the MDE with $w^{\rm MDE}_{\rm vac} \approx 0$. Finally, it mimics quintessence at the present time, $w_{\rm vac}(z) \gtrsim -1$. In the remote future, it is expected to imitate again a cosmological constant with $w_{\rm vac} = -1$.

It is important to remark that these features have been obtained after a rigorous computation in the framework of QFT in curved spacetime as general predictions for the quantum vacuum, rather than being obtained by ad hoc scalar fields in the classical action. This has been the case in many works in cosmology [23], where cosmic scalar fields are used to motivate a quintessential behavior of DE to solve some of its pathologies [24].

We also mentioned the capability of some related models [18, 19] to partially solve the tensions on σ_8 or H_0. We speculate about the possibility that a cosmological model incorporating the different features presented here may be a good candidate for resolving the cosmological tensions. This can be elucidated soon in the light of the new precise and numerous cosmological data.

To sum up, it follows from the QFT calculations that the running vacuum is a time-evolving entity whose equation of state is also dynamic, and changes significantly over cosmic evolution. Remarkably, in the late Universe, it plays the role of (dynamical) dark energy and could provide a reasonable explanation for cosmic acceleration.

Acknowledgements

JSP and CMP are funded by projects PID2019-105614GB-C21 and FPA2016-76005-C2-1-P (MINECO,Spain), 2021-SGR-249 (Generalitat de Catalunya) and CEX2019-000918-M (ICCUB). The work of JSP is also partially supported by the COST Association Action (European Cooperation in Science and Technology) CA18108 "Quantum Gravity Phenomenology in the Multimessenger Approach (QG-MM)". This work is also based upon work from COST Action CA21136 - Addressing observational tensions in cosmology with systematics and fundamental physics (CosmoVerse).

References

[1] A. Einstein, *Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie*, Sitzungsber. Königl. Preuss. Akad. Wiss. phys.-math. Klasse VI (1917) 142.

[2] A.G. Riess et al., Astron. J. **116** (1998) 1009; S. Perlmutter et al., ApJ **517** (1999) 565.

[3] L. Perivolaropoulos and F. Skara, New Astron. Rev. **95** (2022), 101659.

[4] E. Di Valentino et al., Astropart. Phys. **131** (2021) 102604; Astropart. Phys. **131** (2021) 102605.

[5] E. Abdalla *et al.*, JHEAp **34** (2022), 49-211.

[6] N. Aghanim *et al.* [Planck], Astron. Astrophys. **641** (2020), A6 [erratum: Astron. Astrophys. **652** (2021), C4].

[7] A. G. Riess *et al.*, Astrophys. J. Lett. **934** (2022) no.1, L7.

[8] C.Moreno-Pulido and J. Solà Peracaula, Eur. Phys. J. C **80** (2020) no.8, 692.

[9] C.Moreno-Pulido and J. Solà Peracaula, Eur. Phys. J. C **82** (2022) no.6, 551.

[10] C.Moreno-Pulido and J. Solà Peracaula, Eur. Phys. J. C **82** (2022) no.12, 1137.

[11] C. Moreno-Pulido, J. Sola Peracaula and S. Cheraghchi, arXiv:2301.05205.

[12] J. Solà Peracaula, Phil. Trans. Roy. Soc. Lond. A**380** (2022) 20210182.

[13] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space*, Cambridge U. Press (1982).

[14] L.E. Parker and D.J. Toms, *Quantum Field Theory in Curved Spacetime: quantized fields and gravity*, Cambridge U. Press (2009).

[15] H. Gil-Marín *et al.*, Mon. Not. Roy. Astron. Soc. **465** (2017) no.2, 1757-1788.

[16] A. H. Wright *et al.*, Astron. Astrophys. **640** (2020), L14.

[17] N. E. Mavromatos and J. Solà Peracaula, Eur. Phys. J. Spec. Top. **230** (2021) 2077; Eur.Phys.J.Plus **136** (2021) 1152.

[18] J. Solà Peracaula, A. Gómez-Valent, J. de Cruz Pérez and C. Moreno-Pulido, EPL 134 (2021) 19001.

[19] J. Solà Peracaula, A. Gómez-Valent, J. de Cruz Pérez and C. Moreno-Pulido, arXiv:2304.11157.

[20] J. Solà, A. Gómez-Valent and J. de Cruz Pérez, Astrophys. J. **836** (2017) **43**; Astrophys. J. Lett. **811** (2015) L14.

[21] J. Solà Peracaula, J. de Cruz Pérez and A. Gómez-Valent, EPL **121** (2018) 39001; MNRAS **478** (2018) 4357.

[22] A. Gómez-Valent and J. Solà Peracaula, MNRAS **478** (2018) 126; EPL **120** (2017) 39001.

[23] P.J.E. Peebles, B. Ratra, Rev.Mod.Phys. **75** (2003) 559.

[24] P.J. Steinhardt, Phil. Trans. Roy. Soc. Lond. A**361** (2003) 2497.

[25] P. Asimakis, S. Basilakos, N. E. Mavromatos and E.N. Saridakis, Phys.Rev. D**105** (2022) 8.

[26] A.Ferreiro and J.Navarro-Salas, Phys. Lett. B**792** (2019) 81; Phys. Rev. D**102** (2020) 045021.

[27] J.Solà Peracaula and H.Yu, Gen. Rel. Grav. **52** (2020) 17.