

The many tensions with dark-matter based models and implications on the nature of the Universe

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The current standard models of cosmology (SMoC) – specifically Λ CDM and warm dark matter models – served for a few decades as the basis of research in astronomy and cosmology, and have been studied extensively. However, fundamental tensions between observations and theory have emerged. This updated review has two purposes: to explore new tensions that have arisen in recent years, compounding the unresolved tensions from previous studies, and to use the shortcomings of the current theory to guide the development of a successful model.

In any representative volume, more than 90 per cent of all galaxies have thin, extended star-forming ancient disks. But these structures are too fragile to withstand the repeated mergers that dark matter would induce. According to SMoC cosmological simulations, galaxies in the Local Group around the Mpc scale should be distributed in a spheroidal configuration. Observations show that they are instead arranged in thin planes. This poses major questions on the nature and dynamical history of the Local Group. Furthermore, there exist mutually correlated planes of satellite dwarf galaxies located around the Andromeda and Milky Way galaxies. These planes may have been created by the tidal forces generated by a previous encounter between the two galaxies. Also the configuration of the nearby M81 group poses challenges to the SMoC. None of these structures could exist in the presence of dark matter, because dynamical dissipation would cause the galaxies to merge within a Gyr time scale. In addition, the El Gordo galaxy cluster has been observed at a redshift at emission of $z_e = 0.87$ to already have reached a mass of $\approx 2 \times 10^{15} M_\odot$, being impossible in the SMoC. In the light of the growing evidence for large-($>$ few hundred Mpc)-scale inhomogeneities, we have shown that the Hubble Tension is simply caused by the matter bulk flows of the large scale structure. These observations suggest that the Universe is more inhomogeneous and that structures grow more rapidly than what is allowed by the SMoC on the Gpc scale. Novel tensions have emerged from observations of the early Universe. For instance, galaxies of mass $10^9 - 10^{10} M_\odot$ have been detected in the redshift range of $10 \lesssim z_e \lesssim 20$, indicating a faster galaxy formation than predicted in the SMoC. An independent indication for such early galaxy formation comes from the downsizing timescales of early-type galaxies and their associated rapid formation of central super-massive black holes (SMBHs).

We discuss a few candidate models of cosmology from the literature, but most fail on all or a number of the above problems. Given the nature of the tensions, the real Universe needs to be described by a model in which gravitation is effectively stronger than Einsteinian/Newtonian gravitation at accelerations below Milgrom’s acceleration scale. Interestingly, Milgromian dynamics (MOND) coincides with the generalized Poisson equation given by the non-linear p-Laplacian when $p = 3$ with $p = 2$ providing the standard Newtonian Poisson equation. A promising model that solves several of the above tensions is ν HDM. While it embraces MOND, eliminating the need for dark matter, it retains dark energy and consequently the SMoC expansion history. However galaxy formation appears to occur too late in this model, model galaxy clusters reach too large masses, and the mass function of model galaxy clusters is too flat and thus top-heavy in comparison to the observed mass function.

The above mentioned evidence casts doubts on the viability of dark matter and dark energy as fundamental components of the Universe, with severe consequences. Specifically, the Hot Big Bang Theory cannot provide a good fit to the CMB power spectrum without invoking both of these components. Consequently, inflation – introduced to justify why causally-disconnected regions of the CMB should be homogeneous on a flat geometry – would also cease to be needed. The models that have been simulated to-date with these boundary conditions appear to not be able to generate structure rapidly enough to be consistent with the high-redshift JWST observations. Given all the noted anomalies, the classes of models that relax these boundary conditions should be explored.

*Corfu Summer Institute 2022 “Workshop on Tensions in Cosmology”,
7 – 12 September, 2022
Corfu, Greece*

POS(CORFU2022)231

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1. Introduction

The development of the Λ CDM and Λ WDM models of cosmology, based on the required existence of cold or warm dark matter particles (CDM and WDM, respectively), rightly counts as one of the greatest achievements of physics (e.g. [1–4]). Hereinafter, to simplify terminology, we will refer to Λ CDM and Λ WDM (e.g. [5]) jointly as the currently favoured standard model of cosmology, or SMOc (this includes fuzzy dark matter, see Sec. 2).

The SMOc accounts for the observed expansion of the Universe, the cosmic microwave background (CMB) and allows the numerical quantification of the formation of model galaxies, model galaxy clusters as well as the filamentary distribution of matter on Mpc and Gpc scales.

Recent high-resolution simulations use a range of algorithms to compute gravitational potentials, most commonly opting for smooth-particle hydrodynamics or adaptive mesh refinement methods. These simulations also incorporate additional physical processes beyond gravity, including hydrodynamics, star formation, and feedback from both stars and massive black holes. As a result, it is now possible to directly compare simulated galaxy populations with the respective observations. Yet, a growing body of observational constraints – covering scales from open star clusters to the cosmological horizon – have highlighted several discrepancies. Some of these discrepancies are especially troubling. Given these, it is useful to develop and study different cosmological models

in order to improve our appreciation of the current SMOc, while perhaps also finding directions that might lead to fruitful advances away from it. In this process, it is important to develop models that allow the simulation of structures down to galaxies. This requires, like the SMOc, the alternative model to provide equations of motion that can be integrated in a computer to solve for structure formation on all scales. The emphasis here is thus set on computable models of structure formation down to the scales of galaxies or smaller, rather than a more theoretical discussion of abstract models. We emphasise that any theoretical approach away from the current SMOc needs to be guided by the constraints noted here, namely by the non-existence of dark matter particles in galaxies and that the real Universe appears to be structured on all scales that have been probed so far and up to the horizon scale, and needs to be able to have formed galaxies with baryonic masses $\gtrsim 10^9 M_\odot$ by a redshift $10 \lesssim z_e \lesssim 20$ subject to these observations being spectroscopically confirmed (see possible tension pT3 below).

Two such new model universes that can be simulated in a computer are presently being studied in detail. The first is the Angus model of cosmology (AMoC). It rests on a traditional or conservative approach by adopting the same expansion history as the SMOc. The AMoC is based on the Milgromian equations of motion, rather than the Newtonian ones. A less-conventional or neoconservative model¹, the Bohemian model of cosmology (BMoC), is now being studied in Bonn, Prague and Nanjing and will be reported elsewhere.

This contribution can be viewed as an update on the previous assessments of the SMOc ([6–8]), outlining the motivation for studying non-dark-matter based models (Sec. 2). Some such alternatives are discussed in Sec. 3. A particular case, the AMoC, allows the computation of galaxies and larger structures and is briefly described in Sec. 4. The conclusions are available in Sec. 5. Appendix A provides a short introduction to MOND.

2. Motivation

2.1 Standard (dark-matter-based) Models of Cosmology

In the following the postulates and hypotheses underlying the SMOc are discussed with the aim of confronting these with those underlying the AMoC (Sec. 4) and for paving the way for constructing the BMoC.

The current SMOc rests on five fundamental postulates: **(A1)** The locally observed laws of physics (as embodied in the standard model of particle physics, the SMOpp) are valid everywhere. **(A2)** Cosmological redshifts are given by expanding space along the line of sight.² **(A3)** The Equivalence Principle applies and Einsteinian gravitation is universally valid. **(A4)** The Cosmological Principle applies, i.e., the global coordinates of the Universe are characterized by homogeneity and isotropy. Thus, the large scale structure of the Universe is represented by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, $ds^2 = c^2 dt^2 - a_e(t)^2 (dx^2 + dy^2 + dz^2)$, where c is the speed

¹“Neoconservative” in terms of relying on less-exotic physics than the current SMOc, exotic physics encompassing inflation, dark matter particles and dark energy.

²This is a fundamental postulate not further mentioned and underlies all models discussed here. A cosmological redshift does not signify a velocity but the photon wavelength is stretched through the space expansion. A galaxy at a high redshift is stationary apart for its peculiar velocity, but it appears to be receding due to the expansion of space along the line of sight.

of light, dx, dy, dz are co-moving three-dimensional distance intervals and $a_e(t) = (1 + z_e)^{-1}$ is the time-dependent scale factor with $z_e(t)$ being the redshift (e.g. [3]), so that a present-day separation of 1 Mpc spanned 48 kpc at redshift $z_e = 20$. **(A5)** All baryons and leptons have a primordial origin, i.e. they formed in the electroweak epoch (10^{-36} s to 10^{-12} s) before the four fundamental forces separated [9, 10].

Given these five postulates, auxiliary hypotheses need to be invoked to account for some observations:

(AH1) A *cosmic inflationary epoch* during the earliest stages of the Big Bang (e.g. [11]) is needed to ensure for the homogeneity and isotropy of the observed CMB. Inflation was introduced [12] to explain why causally-disconnected regions of the cosmic microwave background (CMB) should be homogeneous, and why the geometry of the Universe is flat. Given that the observed Universe appears to adhere to exactly the same SMOPP³, inflation also ensures causal connection of every part of the Universe. The model universe is implied to be flat and this is supported by the angular distance of the three baryonic oscillation peaks in the observed CMB that serve as a standard ruler [15].⁴ **(AH2)** *Dark matter particles* need to make up about 95 per cent of all of the matter. This is motivated by the gravitational anomalies observed in galaxies that appear as missing mass in the adopted gravitational theory, as well as through the three major acoustic peaks observed in the CMB power spectrum. It is important to note that there is no motivation from the SMOPP for the existence of additional such particles. In order to not interfere with the tight experimental constraints available for the SMOPP, the dark matter needs to be made of particles that are not in the SMOPP and that only interact with SMOPP particles via gravitation and at most through the weak force. Dark matter particles can be cold, warm or fuzzy, depending on their kinetic energy and Compton wavelength set during the matter creation era during inflation. Ultimately, each version needs to account for the observationally deduced dark matter content of dwarf (diameter < 500 pc) galaxies constraining the Compton wavelength to be small such that the dynamics of galaxies is not much affected in comparison to standard CDM particles with a negligible Compton wavelength [17–19]. **(AH3)** With A1–A5 and AH1 & AH2 an expanding model universe is obtained that slows with time through its eigengravity. Observations lead to the result that the real Universe expands more rapidly than thusly expected [20–23]. This discrepancy can only be remedied by invoking, as a third auxiliary hypothesis, that *dark energy* (DE) accounts for about 67 per cent of the total energy content of the Universe. Adding such a constant term is allowed by Einstein’s theory of general relativity. DE, an energy density, leads to a time-increasing rate of expansion, starting about 5 Gyr ago. The expansion history of the SMOc is conveniently given by eq. 36 in [24]. Postulates A1–A5 together with AH2 imply that Newtonian gravitation can be used to model the evolution of structures in an isotropic and homogeneous background expansion described by the Friedmann equations for times later than $z_e = 1100$.

Given the above, six parameters define the current (today, at time $t = t_0$) state of the model universe in terms of its expansion rate, mass components, initial density fluctuations and curvature (see [15]): The Hubble-Lemaitre constant $H_0 \approx 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$, the baryon density parameter $\Omega_{b,0} \approx 0.05$, the dark matter density parameter $\Omega_{c,0} \approx 0.26$ (leading to the matter density

³Variations of the fundamental constants are being searched for, see [13] for a recent review. See also [14] for a suggestion to measure the possible variation of c .

⁴Possible evidence for a small curvature has emerged [16]

parameter $\Omega_{m,0} \approx 0.31$) and the dark energy density parameter $\Omega_{\Lambda,0} \approx 0.69$ which follows from the condition $\Omega_{\text{tot}} = 1$ for a flat universe. The age of this model universe is $t_0 = 13.8$ Gyr. Note that as the SMOc expands the fraction of its energy content that is in dark energy increases, while the total energy content in matter is fixed. The scalar spectral index, $n_s \approx 0.97$, defines the initial power-spectrum of fluctuations stemming from the inflation period, and the power is normalised by the observed abundance of galaxy clusters yielding the the present root-mean-square matter fluctuation averaged over a sphere of radius $8 h^{-1}$ Mpc, $\sigma_8 \approx 0.81$, and the model has no curvature.

2.2 Tensions between the SMOc and observations

The literature on the problems facing the SMOc comprises a large body and the reader is encouraged to explore it and the suggested solutions (e.g. [25–29]). Here only a concise discussion is summarised of some of the tensions between the SMOc and data with the aim of spelling out the minimal challenges that any new cosmological model (e.g. the AMoC, Sec. 4) must address in order to qualify as a tangible alternative model.

The need to introduce AH1 (inflation) and AH3 (dark energy) is met with four fundamental tensions: **(FT1)** Observational and experimental data disfavour the inflationary scenarios at the core of the SMOc. Ljjas, Steinhardt & Loeb (2017, [30]) argue that the theory of inflation needs to be questioned because the simplest models of inflation have been ruled out by Planck data, inflation is unstable and postmodern inflation lies outside of normal science as it cannot be tested (see also [31]). **(FT2)** The “cosmological energy catastrophe” [6] arises from the fact that, in the SMOc, there is currently no accepted standard as to why the existence of dark energy would not violate the principle of energy-momentum conservation. Dark energy is taken to be a constant vacuum energy density. So, as the universe expands, the larger volumes imply that in the SMOc, the Universe tends towards an infinite energy content [2, 32–34] – i.e., cosmological models with dark energy do not conserve energy. In particle physics, dark energy can be interpreted as the energy density of the vacuum. It has been argued (e.g., Zumino 1975 [35]) that if there exist equal numbers of degrees of freedom for bosons and fermions, then the total vacuum energy equals zero. If this symmetry were broken, then a field would have a net sum for the zero-point energy of normal modes. The cut-off scale of its momentum must be set at the Planck scale where the effects of gravity and quantum mechanics become comparable. The resulting prediction from the SMOc exceeds the observed energy density of dark energy by approximately 120 orders of magnitude. This discrepancy is widely regarded as the worst prediction in the history of physics. Hence, the dark energy term defies current understanding of vacuum energy in the context of space expansion and the SMOc (e.g. [36–40]). **(FT3)** The large-scale properties implied by dark matter would exclude any candidate particle (or system)⁵ that originates from the SMOc. There is no evidence for the existence of dark matter particles that is independent of the assumed gravitational theory (A3 above), since all laboratory-based searches have lead to null results [43]. **(FT4)** Another major unsolved problem of the SMOc that is relevant for all alternative models is the baryon asymmetry, because at the moment the physical process of creating more matter than antimatter during inflationary baryo-leptogenesis remains unknown (e.g. [44–46]).

⁵A now disfavored dark matter candidate are massive compact halo objects (MACHOs), which have been ruled out by microlensing surveys in the mass range $0.6 \times 10^{-7} - 15M_{\odot}$ [41] and $0.3 - 30M_{\odot}$ [42].

The above FTs are not a reason to discard the model since they may merely indicate our current lack of physical interpretation of inflation and dark energy. Upholding the SMOc to be the correct model necessitates a continued significant research effort to progress on these tensions (e.g. [47]). The lack of experimental evidence for dark matter particles is also not detrimental, because the dark matter particles may have the property that the non-gravitational interaction cross section with SMOc constituents may be negligible, rendering the dark matter particles undetectable in any experiment. It is therefore permissible to assume the SMOc to be valid.

As any mature theory, the SMOc needs to be tested and compared with advancing observational data. In the past, the SMOc (in the form of the Λ CDM model) has been heralded as being successful in accounting for observational data (e.g. [48, 49]) while serious issues had also been noticed [6–8]. At the present time, the comparison of the SMOc with state-of-the-art observations has led to the recognition of certain tensions:

(T1) Lack of observational evidence for dynamical dissipation by extended particle-based dark matter halos: If dark matter particles do exist, then they will populate massive and extended dark matter halos around the baryonic galaxy components (e.g. [50–52]). This is a strict prediction of the SMOc. Galaxy–galaxy encounters will consequently suffer dynamical dissipation (i.e. Chandrasekhar dynamical friction when one galaxy is significantly minor compared to the other) when the extended dark matter halos begin to touch each other and the galaxies will slow down their relative velocity and fall towards each other to merge. This dissipation can be calculated (e.g. [53, 54]). It leads to every galaxy growing, in the SMOc, through many mergers to its present-day mass. Likewise, the bars of disk galaxies are rigid rotators and as a body suffer dynamical dissipation on the dark matter halos of their galaxies – in each barred galaxy the dark matter halo absorbs the kinetic energy of the rotating bar which consequently slows down. In addition to the lack of evidence in a variety of data for mergers dominating galaxy growth noted previously ([8] and references therein), this dynamical dissipation is excluded by the observed fast rotation speeds of galactic bars which should typically be slow in the presence of the dark matter halos (Roshan et al. 2021, [55]). The calculations falsify the existence of dark matter halos with a confidence of 10σ . That is, even when using distinct cosmological structure formation codes, SMOc hydrodynamical simulations are unable to replicate the bar pattern speeds observed in real galaxies.

An interesting result concerning dynamical dissipation due to the expansive and massive dark matter halos of the SMOc are available for the nearby (≈ 3.3 Mpc distant) M81 group of galaxies. The distribution of HI gas throughout the system of more than three interacting galaxies constrains the past encounters between the galaxies, which, in the SMOc, are all together within the inner region of their dark matter halos such that dynamical dissipation is pronounced. Yun (1999 [56]) reports that the model system merges too rapidly to account for the observed tidal structures. This conclusion is shared by the independent numerical modelling by Thomson et al. (1999 [57]). Both of these groups thus come to the same conclusion, namely, that numerical modelling with dark matter halos does not lead to agreement with the observed system. Both of these are conference proceedings and neither group returned to the problem. The more recent modelling of the M81 system by Oehm et al. (2017 [58]) verifies the absence of dark-matter-based solutions. This work shows that the three major galaxies in the system must have approached each other from large (Mpc-scale) distances, in violation of the Hubble flow, to all meet at virtually the same time (the present time when we observe the system) just before they merge. This is arbitrarily unlikely

in the SMOc, leading to the strong exclusion of the existence of dark matter halos.

Independently of dynamical friction, a dark matter halo protects a dwarf galaxy from being tidally perturbed for most of its orbital history before it merges or disintegrates [59]. There is thus only a brief window of opportunity to observe significantly perturbed baryonic galaxies. The large number of tidally deformed dwarf galaxies in the Fornax galaxy cluster rules this out, as shown in the detailed study by Asencio et al. (2022 [60], Fig. 1 here). That the ultra-diffuse dwarf galaxies

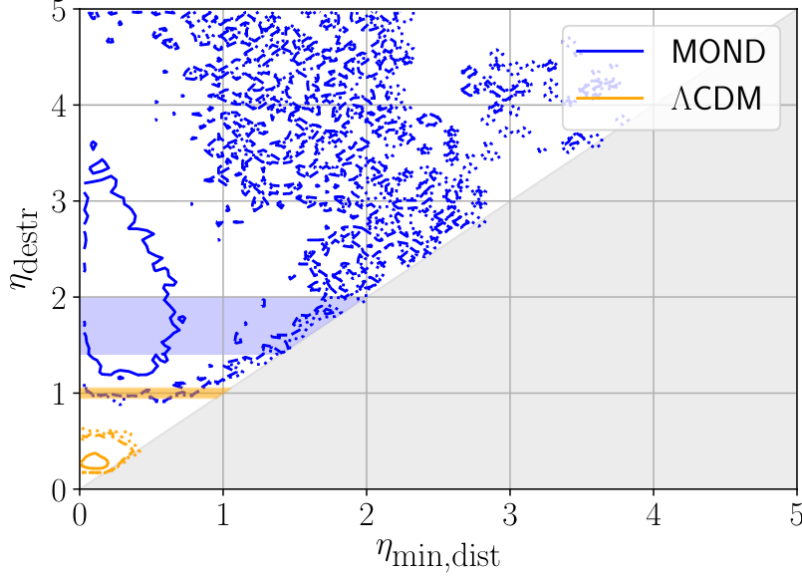


Figure 1: The tidal susceptibility, $\eta_{\text{tid}} = r_{\text{h}}/r_{\text{tid}}$, of dwarf galaxies in the Fornax galaxy cluster. Dwarf galaxies in dark matter halos have a large tidal radius, r_{tid} , compared to their observed half-mass radius, r_{h} , and thus a small η_{tid} value. A dwarf will be visibly tidally perturbed (i.e. close to destruction) if $\eta_{\text{destr}} \approx 1$ (the thick horizontal orange bar), while the observed galaxies have, when interpreted in terms of the SMOc being valid, $\eta_{\text{tid}} \approx 0.25$ (the orange contours). The observed degree of tidal perturbation together with the expected dark matter halo masses places the Λ CDM models well below the nominal $\eta_{\text{destr}} \approx 1$ value, therewith invalidating the dark-matter halo hypothesis at more than 5σ confidence. Without dark matter and in MOND (see Appendix A), on the other hand, the expected η_{tid} values are comfortably in the region where the dwarf galaxies are expected to be tidally perturbed (blue shaded region). We show the 1σ (inner solid line), 3σ (dashed line), and 5σ (outer dotted line) confidence region for MOND (blue) and Λ CDM (orange) models. The grey-shaded region is unphysical because for a dwarf within this region the threshold at which it starts to appear perturbed would be higher than the threshold at which the dwarf is destroyed by the tides. For more details see fig. 14 in [60]. With kind permission from Asencio et al. (2022 [60]).

(UDGs), some of which are reported to be lacking dark matter, are in strong disagreement with galaxies formed in the SMOc has been reported by Haslbauer et al. (2019a [61]). And, additional significant evidence against the existence of CDM and WDM particles comes from the very recently reported smooth lensing signal of multiply lensed images of background galaxies by Amruth et al. (2023 [62]). With the authors knowing dark matter exists, the lack of lensing sub-structure is explained with axions with a de Broglie wavelength of $\lambda_{\text{dB}} \approx 180$ pc. This however poses problems for the presumed dark matter content of ultra faint dwarf galaxies (UFDs) [63] with radii that are smaller than λ_{dB} . Dalal & Kravtsov (2022 [18]) conclude that the constraints provided by the UFDs make fuzzy dark matter have indistinguishable properties to cold dark matter on the scales probed.

Neither this nor the above constraints on the existence of dark matter particles using dynamical friction and Fornax dwarf galaxies are discussed by Amruth et al. (2023 [62]). Also, the reader is reminded of an early observation by Dubinski et al. (1995 [64]): the great lengths of the tidal tails produced during galaxy–galaxy encounters are not possible with the massive and extended dark matter halos that must surround major galaxies in the SMoC because the tidally expelled material cannot propagate out of the deep potential wells to the observed distances from the host galaxies.

Summarising, the sum of the above evidence using completely different and independent methods compellingly disfavors the existence of dark matter particles of any mass.

In addition to this failure of the dark-matter models to match the data, additional tensions have emerged:

(T2) Asymmetrical tidal tails: Open star clusters that orbit the Galaxy on near-circular trajectories lose stars through the energy equipartition process. As a result, the evaporation of stars into the leading or trailing tails in a cluster in the Solar neighbourhood is a stochastic process, as shown by Pflamm-Altenburg et al. (2023 [65]). That is, within Poissonian differences, there should be an equal number of stars in both tails. But the tidal tails of open star clusters in the Solar neighbourhood are asymmetrical with the nearest cluster, the Hyades, at a distance of about 45 pc showing more stars in its leading tail at the 6.5σ confidence level (Kroupa et al. 2022 [66]). These data rule out the validity of Newtonian gravitation on the pc scale unless another explanation for this asymmetry can be found. The only possibility would be that the Hyades suffered an encounter with a massive ($\approx 10^7 M_\odot$) perturbing object, as demonstrated by Jerabkova et al. (2021 [67]). But there is no independent evidence for such a massive perturber in the Solar neighbourhood. Also, the detailed structure of the observed tidal tails of the Hyades shows a radial symmetry between the leading and trailing tail despite the different number of stars in them, which is not plausible if the number asymmetry had been produced by an encounter [65].

(T3) Exponential galactic disks and rotation curves: The merger-driven dark-matter model has not yielded an explanation as to why observed disk galaxies have radial exponential surface mass density profiles (Wittenburg et al. 2020 [68]). Given the observed distribution of baryonic matter in a disk galaxy, it is not possible to calculate or predict from this the rotation curve of the galaxy because the model dark matter halos show a large dispersion of concentrations and shapes and spins (e.g. [52]). Given the dispersion of concentrations and masses, the model rotation curves do not fit the observed shapes [69, 70]. This mismatch suggests the observed rotation curves to not be related to dark matter halos. Small-scale ($\lesssim 1$ kpc) wiggles in the rotation curves are matched by density variations in the baryonic matter distribution and cannot be modelled with dark-matter halos (“Renzo’s Rule”, e.g. [71]).

(T4a) The Impossible Local Group: Disks of Satellites / Planes of Satellites: The Galaxy [72–75], Andromeda [76–78], Centaurus A [79, 80], M81 [81], NGC 253 [82] and the galaxy pair NGC 4490/85 (Karachentsev & Kroupa, submitted) and most other galaxies (Ibata et al. 2014 [83]) have a large fraction of their satellite galaxies arranged in vast rotating planar structures. Since the orbits of the satellites are confined to disks, these systems are reminiscent of planetary systems requiring a dissipational process for their formation rather than the spheroidal distribution of dark-matter bearing satellite galaxies expected from the merger tree in the SMoC (Pawlowski 2021a,b [84, 85]). Even on its own, the Milky Way disk of satellites presents a significant challenge for the SMoC as pointed out for the first time by Kroupa et al. (2005 [73], update [75]). The

combined tension of only three (the Galaxy, Andromeda and Centaurus A) existing simultaneously is above the 5 sigma confidence (Asencio et al. 2022, [60]). But in total there are now six such systems, and in fact, as pointed out by Chiboukas et al. ("in the few instances around nearby major galaxies where we have information, in every case there is evidence that gas-poor companions lie in flattened distributions"), the existence of flattened, rotating systems of satellite galaxies appears to be the norm rather than the exception. That is, the dark-matter-based structure formation models are inconsistent with the observations with more than 5 sigma confidence.⁶ The only known physically plausible process to create the observed phase-space correlated satellite galaxies is through them having formed as tidal dwarf galaxies in tidal arms pulled out in galaxy–galaxy encounters. Such tidal dwarf galaxies lack dark matter though and thus the SMOc predicts there to be primordial dwarf galaxies (being dominated by dark matter) and tidal dwarf galaxies (lacking dark matter) which therefore have different radii [7]. This *dual dwarf galaxy theorem* is evident in high-resolution SMOc simulations (Haslbauer et al. 2012b [89]), who show that the observed population of both types of dwarf galaxies (being indistinguishable [90]) poses a significant tension for the SMOc.

(T4b) The Impossible Local Group: it's highly symmetrical multiple-planar structure: Pawlowski, Kroupa & Jerjen (2013, [91]) discovered that all non-satellite galaxies in the Local Group are situated in two ≈ 100 kpc thick Mpc-scale planes (the *dominant plane*, LG1, containing more galaxies), as shown in Fig. 2. In Fig. 2, the line joining the Milky Way and Andromeda is perpendicular to the plane of the figure, and the Local Group moves w.r.t. the CMB along the upper right diagonal. The two planes, LG1 and LG2, are also perpendicular to the plane of the figure (both planes being seen edge-on) and are symmetrically and approximately equidistantly placed about the line joining Andromeda and the Galaxy. The origin of this Mpc-scale highly symmetrical multiple-planar structure is entirely unknown. Why do the galaxies not fall towards the barycentre of the Local Group? – the Local Group should be virialising, and in this process becoming pressure supported and spheroidal.

Just outside of the Local Group is another similar (i.e. thickness of ≈ 100 kpc) Mpc-scale plane of 5 galaxies, the NGC 3109 association, which is parallel to and offset by 0.3 – 0.5 Mpc from the dominant plane LG1 (Pawlowski & McGaugh 2014, [93]). A few Gyr ago the NGC 3109 association, itself a plane approximately parallel and similar to LG1 in extension and thickness,

⁶Recently, some authors have revisited the Disk of Satellite (DoS) problem arguing that the Milky Way satellite system is short-lived but normal in Λ CDM: Xu et al. (2023 [86]) find the probability of the observed configuration of 11 classical satellites to be about 1/231. Ignoring that the Milky Way, Andromeda and Cen A actually have more satellite galaxies in their disk configurations than only 11 in each, observing three such independent plane-of-satellite systems simultaneously in our immediate cosmological vicinity would have a probability of 8×10^{-8} (consistent with the more accurate calculation available in Asencio et al. 2022 [60]). Despite this, the Λ CDM authors argue that the DoS is consistent with the SMOc, therewith apparently introducing a renormalisation of the concept of probability. Sawala et al. (2022 [87]) likewise renormalise the likelihood of the Milky Way DoS elevating it to the status of being a normal occurrence, finding the DoS of 11 classical satellite galaxies to be a short-lived configuration but count this to be consistent with the SMOc. Since the configuration lasts a few hundred Myr only, it would constitute a remarkable coincidence of observing the DoS just now when higher-life appeared on Earth: it's emergence being heralded by the Great Constellation of Satellite Galaxies. However, they [87] base their analysis on erroneous calculations (e.g. the apogalactic distances of the Large and Small Magellanic Clouds are far too small and the errors on proper motion measurements of the satellite galaxies are taken inappropriately into account in the orbit integration causing the unnatural short-lived phase of the DoS) and misrepresent the previous work by [75] in terms of the combination of Gaia and HST proper motions. [88] confirm that a large fraction of the Cen A satellites are in a coherently rotating disk.

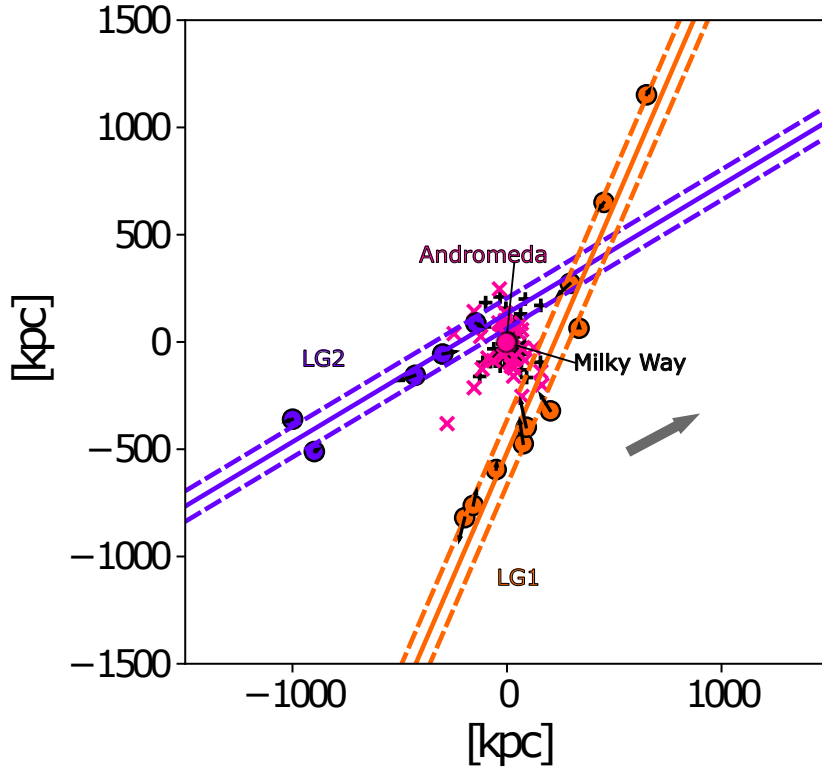


Figure 2: The highly symmetrical multiple-planar arrangement of matter in the Local Group (LG) of galaxies. The observer is at infinity and looking at the LG along the direction joining the Milky Way (MW, black) and Andromeda (M31, magenta). The MW satellite galaxies (with Galactocentric distance ≤ 300 kpc, most being of dwarf-spheroidal, dSph, type) are the black + signs, while the M31 satellites (most being of dSph type, with M31-centric distance ≤ 300 kpc, except for the Pegasus dwarf-irregular (dIrr) and Andromeda XVI, both of which are further but are considered to belong to the Great Plane of Andromeda, i.e., to M31's DoS, because of their positions and velocity vectors) are magenta crosses. All other galaxies (most being of dIrr type) in the LG are shown as orange and purple circles, being $d_{LG} \leq 1500$ kpc from the point midway between the MW and M31. These galaxies are in two major planes indicated by the purple (the "LG2" plane) and orange (the "LG1" plane) solid lines. These planes are about 100 kpc thick (root-mean-square heights ≈ 50 kpc, the dashed lines), and are approximately equidistant from the line joining the MW and M31 (the viewing direction here). No galaxy or system has been detected outside of these planes. The black solid arrows show the Galactocentric line-of-sight velocities of the galaxies (as seen from the Sun), and the large grey arrow shows the direction of the motion of the Local Group w.r.t. the CMB. Updated version of fig. 9 in [91] (using data from [92], updated until 2021).

was in the Local Group as it coherently moves away from it. The physical origin of this plane and the reason for its orientation and apparent association and movement relative to the dominant LG1 plane defy current understanding.

These data that uncover a rather extreme symmetrical organisation of matter over Mpc scales demonstrate that an infall-history in the sense of the SMOc in which groups form from mergers is clearly ruled out. The placement and motion away from the Local Group of the NGC 3109

association also rules out the action of Chandrasekhar dynamical friction on dark matter halos, because the coherent receding motion of the entire NGC 3109 association constituting galaxies with significantly different masses would not be possible under the action of dynamical friction, this conclusion being consistent with T1 above. Are other nearby galaxy groups also as organised? This is unclear because the 3D relative distances between galaxies in a group are too large for the detection of a Local-Group-like symmetric 3D structure.

Thus, both the highly symmetrical multiple-planar structure and the motion of the NGC 3109 association falsify the SMoC with essentially infinite confidence. At the moment there exists no theoretical framework able to explain these data.

(T5) *Only a few per cent of all galaxies are ellipticals. These formed on the short downsizing time-scale, and the formation of super-massive black holes (SMBHs) likewise occurred extremely early and rapidly:* It is known by observation since more than three decades that elliptical galaxies make up only a few per cent of all galaxies with the vast rest being rotationally supported disk galaxies [94, 95]. In 2014 Vogelsberger et al. [96] wrote “*Simulating the formation of realistic disk galaxies, like our own Milky Way, has remained an unsolved problem for more than two decades. The culprit was an angular momentum deficit leading to too high central concentrations, overly massive bulges and unrealistic rotation curves. The fact that our calculation naturally produces a morphological mix of realistic disk galaxies coexisting with a population of ellipticals resolves this long-standing issue. It also shows that previous futile attempts to achieve this were not due to an inherent flaw of the Λ CDM paradigm, but rather due to limitations of numerical algorithms and physical modelling.*” This is interesting in view of Stewart et al. having written in 2008 [50], “*Our results raise serious concerns about the survival of thin-disk-dominated galaxies within the current paradigm for galaxy formation in a Λ CDM universe. In order to achieve a 70 % disk-dominated fraction in Milky Way-sized Λ CDM halos, mergers involving $\approx 2 \times 10^{11} h^{-1} M_{\odot}$ objects must not destroy disks. Considering that most thick disks and bulges contain old stellar populations, the situation is even more restrictive: these mergers must not heat disks or drive gas into their centers to create young bulges.*” In spite of the positive depiction of the SMoC results by Vogelsberger, Haslbauer et al. confirm, in 2022 [97], the conclusion by Stewart et al. by demonstrating that the most modern and highest resolution hydrodynamical SMoC simulations are in more than 13σ conflict with the above observation that disk galaxies dominate by far the galaxy population (Fig. 3).

This severe conflict between the real galaxy population and the SMoC calculations is robust because it persists in the most recent highest resolution simulations that involve very different types of computer codes and sub-grid baryonic physics descriptions. Haslbauer et al. (2022 [97]) demonstrate that even a massive improvement in resolution will not be able to remedy this disaster. The reason for this model failure is that galaxies grow mostly through mergers and these destroy the existing thin disks such that the majority of galaxies formed in SMoC simulations are puffed-up, bulge-dominated or spheroidal. In contrast to the model, there is a pre-prominence of very thin disk galaxies in the real Universe, a significant fraction (roughly 30 per cent) lacking bulges [98].

While elliptical galaxies form readily in the SMoC, Eappen et al. (2022 [99]) demonstrate that the age distribution of the stellar particles in these is much broader than in real observed ellipticals which are known to follow downsizing such that the more massive galaxies formed faster and earlier ([100, 101] and references therein). The work by Eappen et al. shows that the distribution of ages of stellar particles in the model early-type galaxies is inconsistent at more than 5σ confidence with

the short formation time-scales of the real early-type galaxies. In the SMOc elliptical galaxies need time to build-up from many mergers.

A directly related problem for the SMOc is that there is no natural physical process that would form SMBHs because the conditions in the first dwarf dark matter halos cannot form these and mergers typically eject any forming massive black holes if present [102, 103]. While simulations of galaxy formation in the SMOc need to place pre-existing seed-SMBHs into the dark matter halos (e.g. [104]) without understanding where they come from, a physically more accurate calculation of the rapid formation of early-type galaxy components (ellipticals and classical bulges) shows SMBHs to arise very naturally and without exotic physics at their centres (Kroupa et al. 2020 [103]). However, in the SMOc early-type galaxy components cannot form rapidly enough, as discussed above.

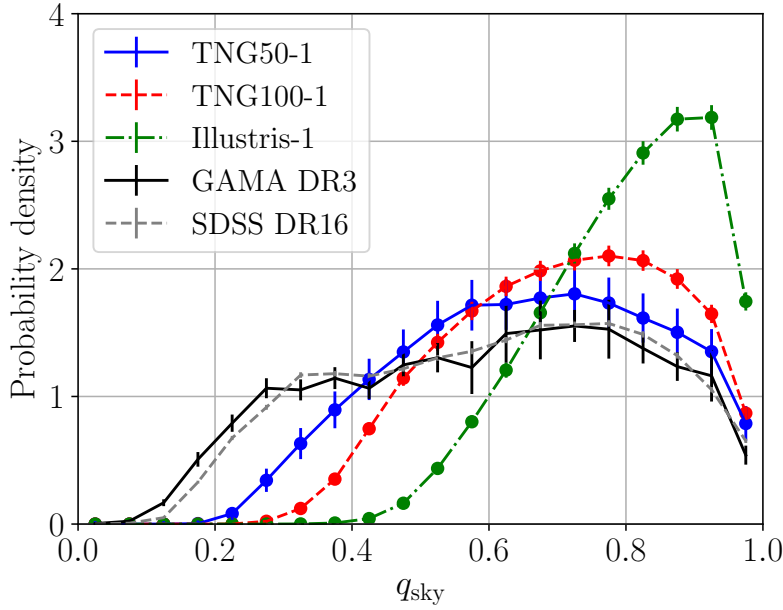


Figure 3: The probability density distribution of the on-sky ellipticity, q_{sky} , of galaxy images in the real Universe (solid black and dashed grey lines) in comparison to SMOc simulations (green, red and blue data, EAGLE simulations yielding comparable distributions, see fig. 4 in [97]). Perfectly thin disk galaxies without observational uncertainties give a distribution between $0.1 < q_{\text{sky}} < 0.5$ only (fig. 14 in [97]), while spherical galaxies have $q_{\text{sky}} = 1$. The SMOc models are in more than 13σ disagreement with the data (table 3 in [97]). Reproduced with permission from Haslbauer et al., being fig. 4 in [97].

(T6) The Hubble Tension and the KBC void: The measurement of the Hubble-Lemaître constant using supernovae 1a (SN1a) has become highly significantly discrepant with the same constant deduced from the observed CMB (if it is interpreted as being the standard CMB, i.e. the surface of last scattering, SoLS, or the photosphere of the Hot Big Bang, see Sec. 5). The SN1a measurement of the current expansion rate of the Universe out to a few hundred Mpc yields the local value of the Hubble-Lemaître constant, $H_0^{\text{local}} = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [105]. The global expansion rate is measured from the CMB to be $H_0^{\text{global}} = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [15]. Both values confirm that the present-day Universe is expanding more rapidly than one without dark

energy if these numbers are interpreted with the assumption that the SMOc be valid. However, the difference between H_0^{local} and H_0^{global} (the ‘‘Hubble Tension’’) is extremely significant. The usual interpretation of the Hubble Tension is that it points towards a more complicated physical theory for dark energy, such as early dark energy models, that temporarily change the expansion rate ([106] for an overview) and thus also affecting the age of the universe. Goldstein et al. (2023 [107]) rule out canonical dark energy models formulated to solve the Hubble Tension using the properties of absorption systems along the line of sight.

On the other hand, the difference between H_0^{local} and H_0^{global} is a natural consequence of the about 0.6 Gpc-scale under-density of matter by a factor of about two to 20 per cent (the Keenan-Barger-Cowie, KBC, void) within which the Local Group is situated because galaxies fall towards the sides of the void as a result of the gravitational field (Hoscheit & Barger 2018 [108], Haslbauer et al. 2020 [109]). The existence of such a Local Hole has been known since at least 2003 through the work of Tom Shanks and collaborators [110–114] and was also noticed by Igor Karachentsev (2012 [115]). While the Hubble Tension can naturally be explained by this under-density (i.e., there is no Hubble Tension since there is a void or hole), the scale and depth of this KBC void or Local Hole is in tension with the SMOc (see Fig. 4) at more than 5 sigma confidence (Fig. 5) since the SMOc universe becomes homogeneous on scales larger than about 100 Mpc [109].

Haslbauer et al. (2020 [109]) thus argue that the observed KBC void needs effectively stronger gravitation for this underdensity to be able to develop over a Hubble time. Note the consistency of the conclusions reached here with those above: that stronger effective gravitation is required for growth of the observed density difference (Haslbauer et al. 2020, [109]) is consistent with the exclusion of the existence of dark matter (T1) which also implies the necessity for stronger effective gravitation given the mass deficit observed in galaxies when assuming Einsteinian/Newtonian gravitation.

The existence of a 200 Mpc-scale under-density in the supernova data was initially suggested by Ó Colgáin (2019 [116]) and has been confirmed independently over 90 per cent of the sky [114]. Such a KBC void or Local Hole was declared to be incompatible with the supernova distance-redshift relation (Kenworthy et al. 2019 [117], but see Haslbauer et al. 2020 [109] who discuss their error), and Brout et al. (2022 [118]) argue that their upgraded Pantheon+ sample shows no evidence of any low-redshift variation in the matter density. But their fig. 16 is in actuality consistent with a significant increase of the matter density with increasing redshift up to a value of 0.3. Subsequent independent analyses have found that supernova data alone actually reveal a preference for a local void [119, 120]. Also, according to the results of [121], the preferred void parameters are remarkably close to those of the KBC void. This thus indicates a consistency between results obtained from galaxy number counts and from supernovae. Watkins et al. (2023 [122]) quantify the bulk flow of matter of about 450 km/s within a distance of about 270 Mpc finding it to be consistent with the SMOc with a probability of 2.1×10^{-6} . Noteworthy is that a similar bulk flow would be evident in the data by the Local Group being present in the KBC void (see fig. 8 in [109]; Mazurenko et al., submitted). There are indications that the void extends to $0.5 < z_e \lesssim 2$ through the Hubble-Lemaitre constant decreasing systematically with increasing redshift to the global Planck value (fig. A1 in [123], fig. 5 in [124], fig. 2 in [125], fig. 5 in [126, 127]; fig. 3 in [128]; fig. 6 in [129]).

Cosmology-model-independent measurements of the Hubble-Lemaitre constant are needed to shed light on the apparently somewhat nontransparent issue of cosmological expansion. Applying

reverberation-mapped quasars for cosmological constraints cannot determine the Hubble constant independently from the normalization of the observed broad-line region radius-luminosity relation. First, one can assume the value from other measurements (see e.g. Zajacek et al. 2021 [130]). Second, if the cosmological constraints of the quasar sample are consistent with the better established cosmological probe, one can merge the two. Cao et al. (2022, [131]) analyzed reverberation-mapped CIV+MgII quasars jointly with BAO+ $H(z_e)$ data and obtained the Hubble-Lemaitre constant value of $H_0^{\text{rev}} = 69.15 \pm 1.77 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for the flat Λ CDM model. The tight UV- and X-ray luminosity scaling relation of quasars (Lusso et al. 2019 [132], 2020 [133]) allow an independent method for measuring the expansion rate of the Universe, but the results suggest a significantly larger value of $H_0^{\text{UV-X}}$ than H_0^{local} for $z_e > 3$ than is compatible with any of the other measurements. Apart from the "Hubble Tension" above, these disparate measurements are for now not taken as being problematic for any cosmological model, since understanding the baryonic processes that lead to the reverberation process and the UV- and X-ray fluxes need to be improved significantly.

A model-independent constraint on the expansion rate and thus age of the Universe can however be obtained using cosmic chronometers and from the oldest stars (see Sec. 5).

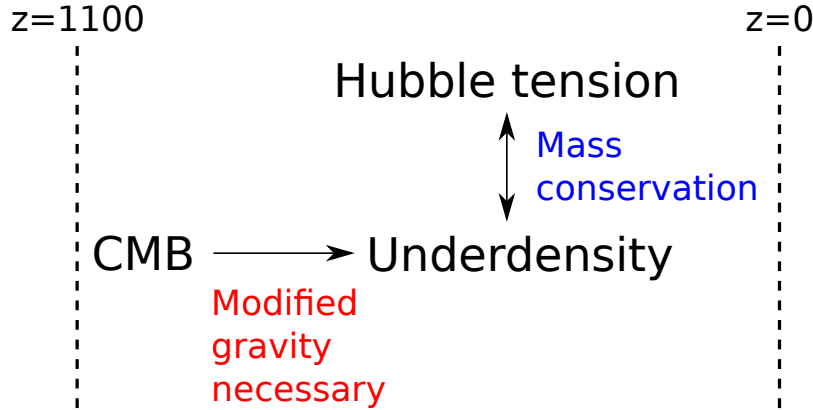


Figure 4: Taking the CMB as the initial boundary condition at redshift $z = z_e = 1100$ (left vertical dashed line) which defines the allowed initial density variations in the cosmological model, structure growth in the model is calculated (black arrow). Assuming all matter to be created during the inflationary era (for consistency with the assumed CMB boundary condition) implies mass conservation and the calculation yields the possible density contrasts and void sizes at $z = z_e = 0$. Given that the SMOc models cannot account for the magnitude and extension of the observed matter density contrasts, new models are required that need effectively stronger gravitational forces to grow structures from the assumed CMB boundary condition and assuming matter conservation to remain valid. An example of such a model is the AMoC, i.e. the ν HDM model [109]. Reproduced with permission from Haslbauer et al. (2020 [109], their fig. 11).

(T7) *Large-scale matter inhomogeneities:* More than 5 sigma evidence for large-scale ($> 0.5 \text{ Gpc}$) inhomogeneities in the observed matter distribution comes from line-of-sight motions of galaxy clusters (Migkas et al. 2021 [134]), the distribution and line-of-sight velocities of quasars and active galactic nuclei [135], as well as in the distribution of gamma-ray bursts [136]. Dupuy & Courtois (2023, [137]) use the CosmicFlows-4 catalogue to infer, from the observed galaxy kinematical field, the variations of the gravitational potential on Mpc scales within a distance of about 0.4 Gpc . They find that the inferred super-clusters are significantly larger in volume than allowed by the SMOc, indicating large-scale variations of the matter density that is not consistent

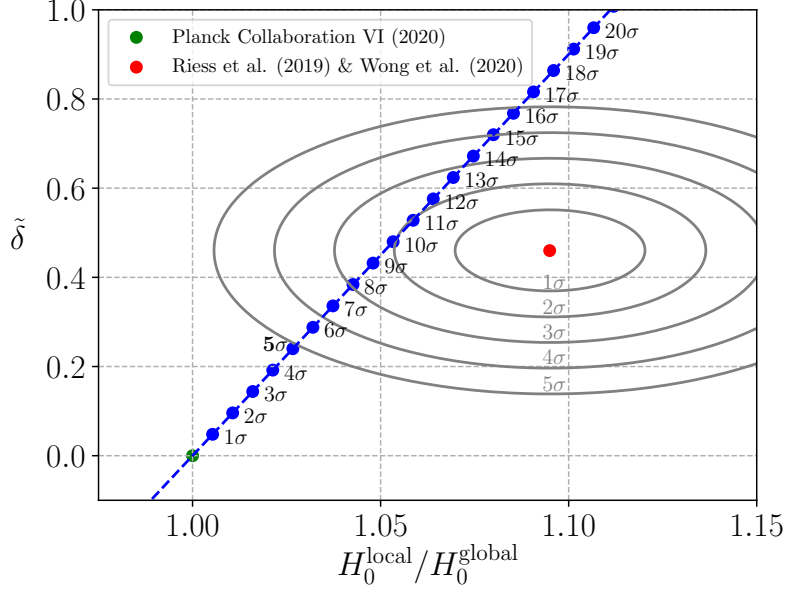


Figure 5: The observed local under-density not corrected for the redshift space distortion (RSD), $\tilde{\delta} = 1 - \rho/\rho_0$, versus the ratio $H_0^{\text{local}}/H_0^{\text{global}}$. ρ is the mass-density of matter between a radius of ≈ 40 Mpc and ≈ 300 Mpc from the Local Group and ρ_0 is the corresponding cosmic mean mass density. The green point is for no Hubble Tension and no under-density, i.e., it corresponds to the Λ CDM expectation on the probed spatial scale. The red point is the observed datum with error ellipses plotted in terms of the measurement confidence levels. The blue points show the apparent cosmic variance in Λ CDM at the indicated confidence level. Notice that a 5σ fluctuation is not enough to get within 5σ of the local observations. Thus, the observed KBC void (the red dot) cannot occur in a Λ CDM universe. This is fig. 2 in [109].

with the SMOc. The about 500 galaxies in the Local Cosmological Volume (within radius around us of ≈ 11 Mpc) have good estimates of their average and current star-formation rates, both being approximately equal [138]. Using this constraint, Haslbauer et al. (2023, [139] calculate the star-formation rate density backwards in time finding that the observed Lilly-Madau maximum of this quantity near a redshift of about 1.9 (at "cosmic noon") is not evident in the Local Cosmological Volume. Since the cosmological principle dictates the galaxies in the Local Cosmological Volume to be representative of galaxies elsewhere, Haslbauer et al. [139] deduce that the maximum of the star-formation rate density at "cosmic-noon" is due to a ≈ 5.1 Gpc-distant matter over-density. This inhomogeneity, observed by us at the present time, amounts to the Local Cosmological Volume being a under-dense region by a factor of a few relative to the regions observed about 5.1 Gpc in the past.

This evidence for large-scale inhomogeneity stands in contrast to the indirect evidence for a smoother Universe gleaned from weak lensing analysis (e.g. [140, 141]). This is related to the "sigma 8 tension" that comes from the analysis of CMB data yielding a more clumpy universe than galaxy-cluster observations and may be related to us being in the KBC void (see also "pT4" below). The recent review by Aluri et al. (2022 [142]) covers the above and further observational indications for a violation of the cosmological principle, these authors stating that "it is equally plausible that precision cosmology may have outgrown the FLRW paradigm, an extremely pragmatic but non-

fundamental symmetry assumption.” Related to the large-scale structure, (Mohayaee et al. 2021 [143]) document that the CMB reference frame is inconsistent with the AGN and quasar frame such that corrections for peculiar velocities of the SN1a data would invalidate, with high (but not quite 5σ) confidence the deduction that dark energy exists. Essentially, since the SN1a events occur in galaxies that share bulk flows that systematically deviate from the Hubble flow over large scales, inference on cosmological expansion based on SN1a data does not yield the correct value of the Hubble-Lemaître constant.

Meanwhile many groups have found the Hubble-Lemaître constant to decrease with redshift (see T6 above). This may be interpreted to be due to a changing equation of state of the Universe (Krishnan et al. 2021 [144]), but a plausible interpretation is also that we are in a 5-Gpc (in radial distance) under-density within which the KBC void or Local Hole is merely the small-scale (Gpc-scale across) local observation causing the Hubble Tension. Either way, the SMoC is invalidated by these observations and new physics is needed to explain them.

(T8) The El Gordo and Bullet galaxy clusters: The mass of the binary galaxy cluster El Gordo ($\approx 2 \times 10^{15} M_{\odot}$) and its existence so early in the Universe (at redshift $z_e = 0.87$) is in tension with the SMoC at 6.16σ confidence (Fig. 6), as shown by Asencio et al. (2021 [145], update: [146]). By $z_e = 0.87$ the SMoC model universe is not old enough to have grown two neighbouring galaxy cluster that each weigh near $10^{15} M_{\odot}$ and which have already fallen through each other. The binary Bullet galaxy cluster adds to the tension as it is also unlikely to occur in the SMoC [147], both binary clusters raising the tension with the SMoC to 6.43σ [145].

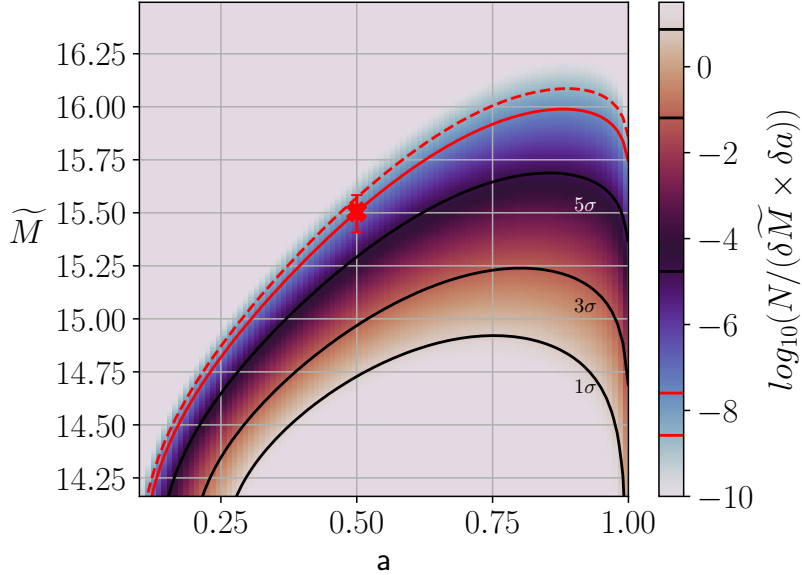


Figure 6: The tension of observing an El Gordo-type cluster collision in the Λ CDM model with mass $\approx 10^{15.5} M_{\odot} = 10^{\tilde{M}} M_{\odot}$, when the redshift $z_e = 1$ (expansion factor $a(t) = 0.5$), is 6.16σ (solid red contour line). The observed double cluster is at the red dot with 20 per cent uncertainty in mass. The levels of all the contours are shown on the colour bar. The 5σ contour is at $\tilde{M} = 15.293$ when $a(t) = 0.5$. The El Gordo double galaxy cluster cannot exist in the SMoC. For further details see fig. 7 in [145].

In the following possible tensions are listed:⁷

Possible tension 1 (**pT1**) arises in the *anomalies of the observed CMB* in view of the physics of the inflationary epoch: [148] report “*the lack of variance and correlation on the largest angular scales, alignment of the lowest multipole moments with one another and with the motion and geometry of the solar system, a hemispherical power asymmetry or dipolar power modulation, a preference for odd parity modes and an unexpectedly large cold spot in the Southern hemisphere*”.

(**pT2**) *Cosmological-scale hemispherical asymmetries*: The southern celestial hemisphere has a more than 5σ larger number of early-type galaxies with suppressed star formation rates than the northern hemisphere (Javanmardi & Kroupa 2017 [149]). While this was interpreted to be due to unknown biases in the galaxy catalogues, the southern celestial hemisphere also has more power and a higher temperature in the observed CMB than the northern hemisphere, the Planck data confirming the WMAP measurements [148, 150]. Furthermore, Shamir (2022 [151]) and Shamir & McAdams (2022 [152]) point out the existence of correlations of disk galaxy spins over large regions of spatial volume with significant differences between the two hemispheres. Are these asymmetries connected?

(**pT3**) A critical aspect constraining any model of structure formation is *how quickly the first galaxies emerge*. The observations performed with the James Webb Space Telescope (JWST) indicate that the real Universe appears to have already formed massive galaxies with stellar masses $\approx 10^{9-11} M_{\odot}$ at redshifts $10 \lesssim z_e \lesssim 20$, i.e. much earlier than possible in the SMOc (Haslbauer et al. 2022 [153]), as shown in Fig. 7 (see also [154] on spectroscopic confirmation). SMOc- Λ WDM models of structure formation come under more stress through such observations since in these the most-massive galaxies that can form at a given redshift are less massive than in the SMOc- Λ CMD models [155].

(**pT4**) Related to this, a possible tension concerning *growth of structure* from inflation through to the dark-matter-halo mass function constrained through weak-lensing observations has emerged (e.g. [159]). This problem is related to the “sigma 8 tension” and raises questions concerning the empirical normalisation (based on the abundance of galaxy clusters, e.g. [160]) of the power spectrum of perturbations generated during inflation and how this transfers during linear structure growth to the power spectrum of matter at $z_e \approx 130$ used to seed cosmological structure formation simulations (e.g. [161]).

(**pT5**) Only about 50 per cent of the baryons that ought to be in the universe according to the SMOc have been discovered (e.g. [162]). While this is seen as an important problem for the SMOc with the understanding that the missing baryons are in the filaments and between galaxies awaiting to be discovered, this “missing baryon” problem may simply be a consequence of the KBC void or an even larger-scale inhomogeneous matter distribution (T6, T7 above).

Reviews of the theoretical approaches to cosmology in view of the above tensions are available in [37], [25, 163] and [164]. An independent review of the tensions and failures of the SMOc, given astronomical observations, is provided by Peri & Skara (2022 [13]), while Banik & Zhao (2022 [165]) provide an assessment of these in comparison to a Milgromian universe.

⁷A “possible tension” is meant to be a tension that would falsify the SMOc with at least 5 sigma confidence provided further observations confirm the tension.

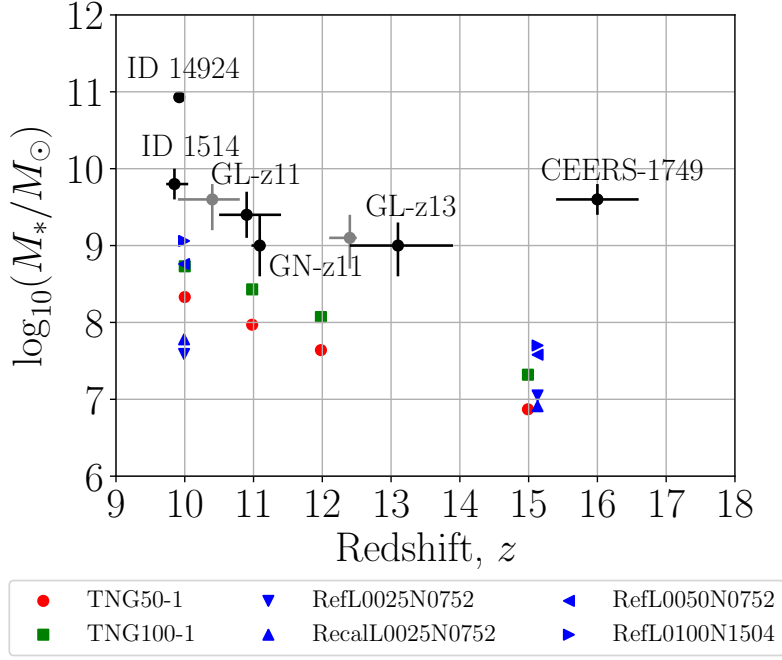


Figure 7: The stellar masses of the most massive galaxies already existing at redshift, $z = z_e$, in the Λ CDM simulations (coloured points). The observed galaxy candidates depicted as grey and black errorbars are more than 10 times more massive than the most massive galaxies formed in the Λ CDM simulations (coloured points). GN-z11 is a spectroscopically confirmed galaxy detected with the Hubble Space Telescope [156]. [157] updated the physical properties of the galaxy candidates GL-z11 (now labeled as GL-z10 in [157]) and GL-z13 (now GL-z12) in the accepted version of their paper. Their updated redshifts and stellar masses are visualised as grey errorbars in the figure. These authors updated these values after Haslbauer et al. (2022 [153]) got accepted for publication such that [153] were not able to include them in their analysis. However, the updated values do not change the conclusion of [153] in any way as can be seen by the grey error bars (GL-z10 and GL-z12). Haslbauer et al. (2022 [153]) also discuss if the tension between the observed very early existence of massive galaxies can be alleviated if the galaxy-wide stellar IMF (gwIMF) was top heavy (thereby leading to smaller real masses), but the calculations suggest that the reduction of the observed masses does not suffice to push them into the SMoC-allowed regime. More calculations with complex star-formation histories are needed to verify this conclusion. If the SMoC simulations had been performed with a systematically varying gwIMF then the stronger feedback energy from stellar populations with a top-heavy gwIMF would lead to smaller stellar masses of the model galaxies, thus not solving the problem, since both the observed and the model masses would be reduced. The [153] model stellar populations assume dust-free galaxies (as suggested by [158] to be the case). Credits: Modified version of fig. 2 in [153].

In Summary: taken at face value, the above compilation demonstrates there to be no dark matter on the scales of galaxies, and the real Universe to be significantly more inhomogeneous on scales up to a few Gpc than allowed by the SMoC. The SMoC thus fails on essentially all scales. This failure can be visualised using the theory-confidence graph [7]. Based on the SMoC-confidence graph originally shown in fig. 14 of Kroupa (2012, [7]), we present an updated version in Fig. 8. Fig. 14 from [7] is now the inset plot in our new figure. The numbers of the “tensions” known in 2012 are retained and the reader is directed for a description of these in [7]. None of these pre-existing tensions have been resolved as of the current year. Since black squares (1: inflation, 2:

dark matter, and 5: dark energy) denote phenomena that are treated as "new physics" in the SMoC, we exclude them from contributing to the loss of confidence. Some of the older tensions presented in [7] are here overwritten by more modern tests, so they are absorbed in the new tensions (T1–T8) documented in this contribution. Thus we have: tension 8 in [7] becomes now T4a (i.e. $8 \rightarrow T4a$), and likewise $17 \rightarrow T8$, $16 + 21 \rightarrow T6$, $22 \rightarrow T9$. For most of the tensions documented in Sec. 2.2 there are formal confidence results, and the downward drop corresponds to this amount of loss of confidence (e.g.: a 5σ tension is displayed here as a drop in confidence by 6 orders of magnitude). For the "possible tensions" (pT numbers) there are no formal confidence results and thus each is taken to correspond to a drop in confidence by 50 per cent.

The fundamental cosmological principle: Thus, as discussed above, observational evidence suggests that (i) the SMoPP is valid on all observed scales, but (ii) there is no sufficiently large scale where the Universe is homogeneous, i.e., the cosmological principle has not been verified on the scales reached by observation.

Therefore, all cosmological models based on the cosmological principle of homogeneity and isotropy (i.e., all models based on FLRW geometry) may be flawed. Future efforts to develop reliable cosmological models should consider in earnest the evidence for inhomogeneities on all scales. Given the detrimental performance of the SMoC, in the following Sec. 3 some alternative models are discussed.

3. Towards alternative models

Among the cosmological models which have been proposed over the years, we will be discussing some of those which challenge the three pillars of cosmology (inflation, dark matter, and dark energy) as well as the assumption of homogeneity and isotropy. Thus, in the following Sec. 3.1–4 a number of cosmological models are discussed that are relevant for the tensions documented in Sec. 2.2. Table 1 gives an overview of these.

3.1 Based on General Relativity / Newtonian gravitation

The tensions (T1–T8) between the observations and the SMoC listed in Sec. 2.1 thus comprise pc to Gpc scales and at all redshifts. These tensions imply that more exploration of the SMoC is called for. For example: [47] argue, given that detailed observations are consistent with the dark-matter plus dark-energy SMoC (in contradiction to the evidence reviewed above), the increasing data volumes need advances in machine learning to cast new light on the cosmological tensions. [166] propose extensions by including a dark big bang. Schiavone et al. (2023 [167]) suggest that a $f(R)$ dark energy model can lead to an apparent variation of the Hubble constant thus accounting for the Hubble tension (T6 above). [168] suggest that non-Gaussian exponential tails in inflationary perturbations generated by primordial quantum diffusion can account for the observed large-scale inhomogeneities and the existence of El Gordo-type galaxy clusters. All of these suggestions retain dark matter as an essential pillar of the models and do not address the falsifications of its presence reviewed above.

The deduction that dark energy drives an accelerated expansion is dependent on how to time- and spatially-average observational quantities in an evolving inhomogeneous Universe [169, 170]. The calculations suggest the following: if the Universe is thought to be homogeneous and isotropic,

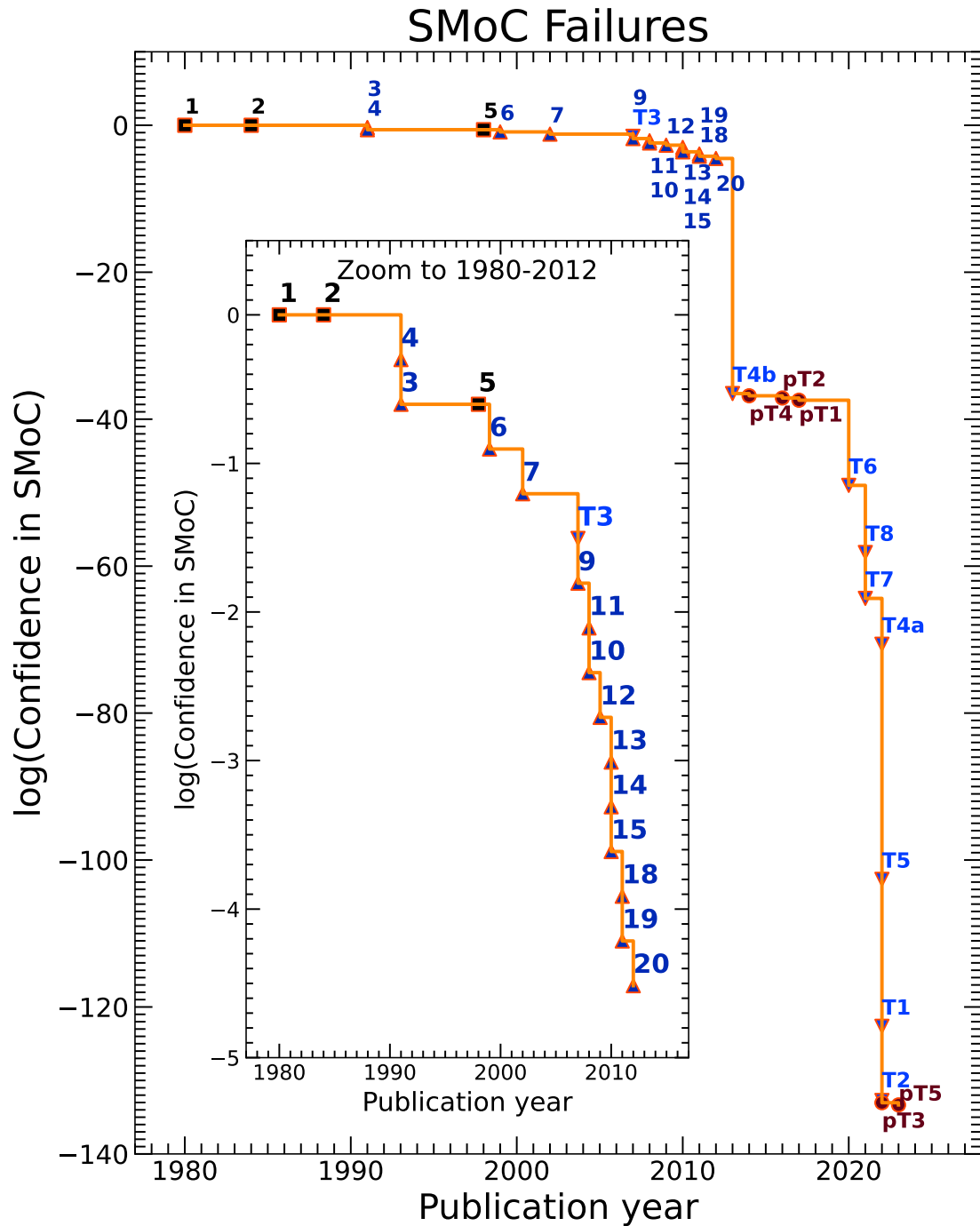


Figure 8: The SMoC-Confidence Graph: the cumulative loss in confidence that the Standard Model of Cosmology (SMoC) is a valid description of nature. The numbers 1-20 are based on a previous review (Kroupa, 2012, [7]), where an original form of the current plot appeared. Black squares (1, 2, and 5, representing inflation, dark matter, and dark energy, respectively) are treated in the SMoC as “new physics”, so they are not assigned a loss of confidence. Upward blue triangles indicate failures, still current, already recognized in [7], while downward blue triangles (T1–T8) represent newly identified tensions where the loss of confidence was computed formally, as presented in Section 2.2. From the same section come the possible tensions (pT1–pT5), shown with red circles. Wherever the loss of confidence was not computed formally, we assign a drop in confidence by 50%. The inset graph zooms into the falsifications up to 2012.

| Model | inflation? | DM? | dark energy? | role of CMB | simulations? |
|------------------------|-------------|-----|--------------|---|--------------|
| SMoC | yes | yes | yes | SoLS | many |
| timescape | yes | yes | no | SoLS | no |
| MMoC ($R_h = ct$) | no | yes | no | SoLS with dust reprocessing at $z_e \approx 16$ | no |
| SIV | in progress | no | no | in progress | no |
| AeST | yes | no | yes | SoLS | no |
| AMoC (ν HDM) | yes | no | yes | SoLS | a few |
| BMoC | no | no | no | starlight ($z_e < z_{*form} > 20$) with dust reprocessing at $z_e \lesssim z_{*form}$ | starting |

Table 1: The models discussed here. Each row represents the following cosmological models: discussed in Sec. 2.1: the Standard Model of Cosmology (SMoC), in Sec. 3.1: models based on inhomogeneity (timescape), the Melia $R_h = ct$ model (MMoC), the scale-invariant vacuum model (SIV), the Aether Scalar Tensor model (AeST), in Sec. 3.2 and 4: the Angus model (AMoC, alternatively known as ν HDM) and in Sec. 5: the Bohemian model (BMoC). "DM" stands for cold, warm or fuzzy dark matter which is introduced by the respective model to account for the mass deficit on galaxy and cosmological scales. "SoLS" is the surface of last scattering, i.e. the photosphere of the Hot Big Bang. z_{*form} is the redshift at which stars begin to form. The column "simulations?" lists if computer simulations of cosmological structure formation are available.

while in reality significant inhomogeneities develop over time in a model without dark energy, then the inferred observational quantities appear as if the Universe has dark energy and an accelerating expansion (Wiltshire 2007a,b, 2009, 2019, [171–174]). Wiltshire describes filaments (walls) and voids with separate homogeneous metrics, and their statistical average at large scales simplifies to a statistical homogeneity⁸. An interesting aspect of the timescape cosmological model is that the ages of different regions diverge such that in the model universe voids are older than galaxy clusters. The same oldest stars will thus appear older in voids than in dense regions, and measuring the global age of the universe is challenging and can only be achieved statistically.

This *timescape cosmological model* thus suggests dark energy to be an apparent effect and addresses (but we do not yet know if it also accounts) for tensions T6–T8. The timescape cosmological model makes a good case for dark energy to not be real, but it does not account how the large density contrasts can develop in a self-consistent model that has the observed CMB as an initial boundary condition. The reality of dark energy is also questioned by the analysis by Mohayaee et al. (2021 [143]) of the directionality of the bulk flow of galaxies and of the measured H_0 value and the lack of convergence to the CMB frame with increasing distance. A directionally-dependent value of the dark energy density parameter Ω_Λ based on SN1a data which aligns statistically significantly with the CMB dipole was previously reported independently by Javanmardi et al. (2015 [179]),

⁸Note that gravity in a homogeneous and isotropic metric, such as the FLRW metric, will produce inhomogeneities at sufficiently small scales. This is distinct from the inhomogeneity and anisotropy intrinsic to the geometry itself, proposed e.g. in the Lemaitre-Tolman-Bondi (LTB) metric [175–178].

supporting the conclusions reached by [143].

Since the light-horizon (R_h , the distance travelled by light over the age of the universe) equals the gravitational radius of the universe, Melia & Shevchuck (2012 [180]) emphasise that the coincidence is “*disturbing*” as it can only occur once in the SMOc (and, by implication, in any model such as the AMoC insisting on the same expansion history as the SMOc). They also point out that “*this equality may actually be upheld for all cosmic time t which, however, would not be entirely consistent with Λ CDM, or any other cosmological model we know of.*” They therefore argue that the observed Universe expands according to $R_h = ct$ such that the gravitational radius and the light-horizon radius are equal all the time (see also Melia 2012 [181]). This expansion fits the SN1a standard candles as well as the SMOc [182]. Neither dark energy nor an inflationary epoch are needed in this Melia model of cosmology (MMoC, [183]), and it therefore may allow the formation of galaxies at higher redshift than the SMOc [184], avoiding possible tension pT3 (Sec. 2.2). The MMoC thus appears to provide an interesting alternative to the standard expansion history of the SMOc (that has an initial very brief inflationary growth, then coasting expansion that slows due to eigengravity and then accelerates again due to dark energy) by avoiding fundamental tensions FT1 and FT2 (Sec. 2.2). Within the MMoC paradigm, interesting insights as to the origin of the observed CMB are emerging in terms of the primordial power-spectrum [185]. While in the MMoC the observed CMB is interpreted to form at $z_e \approx 1100$ as in the SMOc as the photosphere of a Hot Big Bang (but without inflation), Melia (2022 [186]) argues that the observed CMB anisotropies result from a dust screen formed from the first star formation near $z_e \approx 16$, therewith possibly accounting for pT2.

While the work on the timescape cosmological model and the MMoC needs to be kept in mind in the following, these models require dark matter to dominate the matter content by being based on Newtonian gravitation. That is, they retain FT3 and FT4 and thus suffer from tensions T1-T5 that are a result of needing dark matter to dominate the matter content of the model universe. Given the track-record of verified predictions of the MOND paradigm as emphasised in the award-winning book of Merritt (2020 [187]), see also Appendix A below), it may appear useful to study models of structure formation being based on this general framework.

In this context, Maeder (2017a,b,c [188–190], see also Gueorguiev & Maeder (2022 [191]) study cosmological models based on the hypothesis that macroscopic empty space is scale-invariant, a property related to MOND (Milgrom 2009 [192]). This Scale-Invariant Vacuum (SIV) theory is based on Weyl’s Integrable Geometry, furnished with a gauge scalar field. The main difference between Milgromian dynamics and the SIV theory is that Milgromian dynamics is equivalent to a global scale invariance of space and time, where the scale factor is a constant [192], while in the SIV theory the scale factor is a function of time [193]. Maeder (2023, [194]) points out that the SIV theory tends to MOND in the weak field limit. The SIV model of cosmology keeps the physical properties of General Relativity in a model universe void of matter, being based on scale invariance, homogeneity and isotropy of empty space. The scale invariance, given by the absence of matter such that a scale cannot be defined, is broken in the presence of matter and a characteristic acceleration scale emerges that is similar to Milgrom’s constant a_0 as a consequence of the equilibrium of the SIV and Newtonian dynamical accelerations. This approach leads to the deep-MOND limit being an approximation of the SIV theory at low densities and for systems with time-scales shorter than a few Myr. Cosmological models based on SIV have an accelerated expansion but no dark energy, while flat rotation curves of galaxies are given by the space-time scale invariance, similarly as in

MOND, thus avoiding the need for dark matter (and therewith solving T1). According to [195] SIV is challenged by Solar System data, and it appears difficult for the theory to account for the asymmetric tidal tails of star clusters (T2). Also, the accelerated expansion inherent to the model raises the question of energy conservation since space is associated with quantum vacuum energy (FT2). The requirement for homogeneity and isotropy of the SIV theory appears to be in tension with T6–T8. Inflation in SIV is being studied (Maeder & Gueorguiev 2021 [196]). The role of the CMB has not yet been explored in this theory (Maeder, private communication), but the growth of density fluctuations has been studied with favourable results [197], the analytical solution for the early Universe being documented by Maeder (2019 [198]). More detailed tests are needed, and these would be best approached with simulations of structure formation. This requires equations of motions to be available to integrate particles through time [188].

3.2 Based on non-Newtonian models

The level of the tensions on well observed galaxies suggest, when taken at face value, a need to explore models of structure formation that differ fundamentally from the SMOc, the timescape cosmological model and the MMOc, in order to open up or to reject possible new directions of investigation.

In order to avoid the tensions emanating from dynamical friction on dark matter particles in a Newtonian universe, a different theory of gravitation might be tried. Various alternatives to Newtonian dynamics have been suggested (e.g. emergent gravitation related to the entropy of space, [199]; gravitational dipoles, [200]; tensor-scalar-vector modified gravity, [201]; scale-invariant dynamics, [194]). But these formulations do not readily allow the equations of motion of particles to be written down for numerical integration thus inhibiting structure formation simulations down to the scale of galaxies, and/or they have been found to be in tension with observational data (respectively: [195, 202–204]).

A hint which models might be feasible to study in a first step towards this goal is the inferred absence of dark matter particles (T1) together with the documented success of MOND [205–208] on the scales of galaxies [71, 165, 209] as well as its promise on cosmological scales ([210] and [211]). While Milgromian dynamics is briefly introduced in Appendix A, here the available results are summarised. These results motivate us to explore cosmological models based on Milgromian dynamics.

Briefly, by postulating that Newtonian gravitation (represented by the $p = 2$ p-Laplace operator in the Poisson equation) shifts to the $p = 3$ p-Laplace operator when the gradient of the potential falls below Milgrom’s constant $a_0 = 1.21_{-0.27}^{+0.27} \times 10^{-13} \text{ km/s}^2 = 3.90_{-0.86}^{+0.86} \text{ pc/Myr}^2$ [212], one obtains a theory of gravitational dynamics which is, as far as all tests so far conducted, consistent with the data. By being able to calculate the local gradient of the gravitational potential from the matter density field, Milgromian dynamics thus allows the equations of motions of particles to be written down and integrated in a computer. This is an essential aspect of any new model of cosmology we might want to study. Assuming the equations of motion of particles to be based on Milgromian dynamics has immediate implications for the tensions catalogued in Sec. 2.2:

(T1) Dynamical dissipation is not active because dark matter particles are not needed to boost gravitation, and the large number of perturbed dwarf galaxies in galaxy clusters at any time are

natural in Milgromian dynamics (Fig. 1, as shown by Asencio et al. 2022 [60]). UDGs are natural in MOND [61], but this needs to be verified with cosmological simulations.

(T2) On the pc-scale, the evidence that stars evaporating from open star clusters preferentially populate their leading tidal tail in Milgromian dynamics as observed supports the necessity to switch away from the Newtonian equations of motion (Kroupa et al. 2022 [66]).

(T3) Recent hydrodynamical simulations with star formation of collapsing post-Big-Bang gas clouds have shown rotating disk galaxies with radial exponential density profiles with the observed physical sizes emanating naturally (Wittenburg et al. 2020 [68], Nagesh et al. 2023 [213]). The observed distribution of baryons allows an exact prediction of the rotation curves of galaxies in MOND (e.g. Lelli et al. 2017 [214]).

(T4, T5) Computations of interacting galaxies have shown galaxy mergers to be much rarer in Milgromian dynamics since dynamical dissipation on extended dark matter halos is non-existent [215]. Galaxies can thus orbit about each other multiple times without merging. This implies that galactic disks remain (even if warped). If a galaxy forms from a typically rotating gas cloud (see T3 just above) then the vast preponderance of disk galaxies would follow naturally. The disks/planes of satellite galaxies are naturally produced in galaxy–galaxy encounters as populations of co-orbiting tidal dwarf galaxies (Banik et al. 2018 [216], Bilek et al. 2018, 2021 [217, 218], Banik et al. 2022 [219], see also [220]). This also implies galactic bars to remain fast [55]. The dual dwarf galaxy theorem is automatically obeyed and is consistent with the observed dwarf galaxy population because all dwarf galaxies (tidal tail and primordial) follow the same dynamical laws and become indistinguishable apart from youth and metallicity differences [7, 89]. Polar ring galaxies have shown rotation velocities of their extended polar rings that are systematically larger than the rotation speed of their host which arises naturally in MOND (Lueghausen et al. 2013 [221]).

(T5) The downsizing time-scales of the rapid formation of elliptical galaxies arise naturally from initial non-rotating gas clouds (Eappen et al. 2022 [99]). Sub-grid physics describing star formation and gas heating and cooling are not critical for these outcomes (Wittenburg 2020 [68], Eappen et al. 2022 [99], Nagesh et al. 2023 [213]) such that cosmological structure formation simulations in Milgromian dynamics of a largely baryonic universe with galaxy-scale resolution are expected to lead to a population of galaxies that should have the correct properties. The rapid formation of spheroids (bulges, elliptical galaxies) produces the exact environment in which SMBHs form naturally and fast (Kroupa et al. 2020 [103]), while the rarity of mergers leaves these at the centres of their hosting galaxies as observed (Lena et al. 2014 [102]).

(T6, T7) The existence of, the size and depth of KBC-like voids arise naturally in a Milgromian universe (Fig. 4 here, Haslbauer et al. 2020, [109]). Concerning the Hubble Tension and the general problem of how quickly the overall Universe expands and thus how old it is, the oldest observed stars provide a cosmology-model-independent measure of the local age of the Universe. Their ages are reported to be consistent with the global value, H_0^{global} (see Sec. 5, with a tendency towards an older local Universe). The stellar ages thus indicate that the Planck cosmology may be closer to the background-level expansion. This means that the Hubble Tension is most likely due to the galaxies having developed peculiar velocities as they fall towards the matter overdensities surrounding the KBC void (Haslbauer et al. 2020 [109]). Solutions involving more complex dark energy models, such as early dark energy, that would affect the age of the universe and thus the ages of the oldest stars, are therefore not necessary. Systematic large-scale matter motions of nearly 1000 km/s over

500 Mpc as reported by Migkas et al. (2021 [134]) appear to be naturally obtained in MOND-based cosmological models of structure formation (Candlish 2016 [222]).

(T8) The existence of the El Gordo and Bullet clusters with their respective masses and redshifts appears to be well accounted for in a Milgromian universe (Katz et al. 2013 [223], Asencio et al. 2021 [145]). A minor tension has emerged in MOND with the gravitating masses of galaxy clusters being larger by about a factor of two than the observed baryonic masses (Sanders 1994 [224]). However, this tension may be due to the previous analyses lacking pressure corrections and/or having used inadequate profiles for the baryonic matter content (Lopez-Corredoira et al. 2022 [225]).

But some tension between MOND and the data have emerged as follows. MOND Tension 1 (MT1): Ultra-faint dwarf satellite galaxies (UFDs) have half-mass radii between $20 \lesssim r_h/\text{pc} \lesssim 500$ and masses in stars between about $10^{2.5}$ to $10^4 M_\odot$ (e.g. [63, 226]). Safarzadeh & Loeb (2021 [227]) show that the observed line-of-sight velocity dispersion in these objects is approximately constant between about 4 to 10 km/s while it should be decreasing with the mass in stars to values below 1 km/s. The analytical estimates of the theoretical velocity dispersion taking into account the external field of the Galaxy may not be correct though, and self-consistent simulations of the evolution of UFDs are needed to address this tension with rigour. This is needed since simulations of such objects done in Newtonian gravitation have uncovered hitherto unknown solutions of quasi-stable remnants of satellite galaxies that are not in virial equilibrium and appear to be dominated by dark matter but contain no dark matter (Kroupa 1997 [228], see also [229]). Such work needs to be redone in Milgromian dynamics.

(MT2): Based on hydrodynamical simulations of isolated disk galaxies that are based on initial conditions given by present-day observed galaxies and that allow for star formation to develop in the simulations, Nagesh et al. (2023 [213]) calculate the pattern speeds of the bars that form in these models. As for real galaxies, they find the bar length to be larger and the bar to be weaker for the more massive models, but bar pattern speeds, while being constant in time, to be slower than inferred for the real galaxies and in particle-simulations only. Formally the models disagree with the observed bar pattern speeds at more than 5σ confidence, but a rigorous comparison needs to be done with respect to galaxies formed in cosmological structure formation simulations, as was done above for the SMoC case (T1).

(MT3): Tension between MOND and data is also evident for the globular cluster NGC 2419 [230], but detailed stellar dynamical modelling [231] taking it's possible birth conditions into account may alleviate these [66].

(MT4): Also, some tension is evident in the Solar System with the external field from the Galaxy leading to the secular precession of the perihelion of Saturn in combination with that of Earth not favouring the usual interpolating function needed for modelling galaxies between the weak-field MOND and strong-field Newtonian regimes [232], but other interpolating functions are consistent with the Solar system and galactic data [233].

(MT5): Furthermore, the GAIA wide-binary-star data (binaries with separations larger than a few thousand astronomical units) have, as applied so far, been suggesting non-conformity with Newtonian dynamics, and appear to be in agreement with MOND ([234–237], Banik et al., submitted). This test sensitively dependents on the multiplicity properties of the low-mass stars being used (Clarke 2020 [238]) as well as on stellar astrophysics and evolution, such as the stellar mass-

luminosity relation and the possible pre-main sequence nature of the stars. A pre-main sequence star will be calculated to have a larger mass if it is wrongly thought to be a main sequence star, and so this bias hides the effectively stronger Milgromian gravitation. Further work is needed before final conclusions can be reached, but the most recent analysis of the Gaia DR3 wide-binary data by Chae (2023 [237]) strongly supports Milgromian gravitation.

(MT6): The Large Magellanic Cloud (LMC) is currently passing the MW at near-pericentre of its orbit at a distance of about 50 kpc from the MW. While the mass in stars and gas of the LMC ($\approx 4 \times 10^9 M_{\odot} \approx 5$ per cent of the MW baryonic mass) is consistent with the baryonic Tully-Fisher relation, given the observed rotation curve, its gravitating mass as deduced from the reflex motion of the MW suggests a larger gravitating mass (10–20 per cent of the MW, [239]). Does this suggest that some dark matter needs to be present to boost the gravitating mass of the LMC, therewith breaking the success of MOND? This deduction would be premature because the expected reflex motion of the MW needs to be calculated in MOND taking into account that the fully self-gravitating MW disk is breathing and oscillating in 3D which might feign some of the reflex motion. The same is true for the stellar streams that also constrain the gravitating mass of the LMC [239].

In Summary, from pc to Gpc scales, nature appears to comply to Milgromian dynamics rather than Newtonian dynamics plus dark matter, a deduction also reached by Banik & Zhao (2022 [165]) based on an independent assessment. In his award-winning book, Merritt (2020 [187]) applies the formalism of the philosophy of science to address the merit of the MONDian research programme in comparison to that of the SMOc programme, finding that only MOND has repeatedly predicted observational facts in advance of their discovery. Based on the criteria as applied in the philosophy of science, MOND is therewith the more viable and a progressive research programme.

Adopting Milgromian dynamics avoids most of the tensions of Sec. 2.2. A few tensions have emerged as noted here, but these need rigorous further analysis in order to ascertain if they might lead to a falsification with at least 5σ confidence of MOND in general, or of MOND in terms of its interpretation as a gravitational theory or as a theory of modified inertia. Also, at the moment it remains unclear whether the correct fraction of elliptical versus disk galaxies will truly emanate from structure formation in Milgromian dynamics. It is also unclear if a Milgromian-based cosmological model will account for the observed Gpc-scale density inhomogeneities and bulk matter flows. Self-consistent cosmological simulations are required to directly address this question as well as whether the distribution of galaxies in phase space and in galaxy clusters comes out comparable to that in the observed Universe.

The construction of a fully self-consistent cosmological model based on one set of relativistic field equations that quantify how matter curves space dynamically would be needed. This underlies the interpretation of gravitation as a strictly geometric effect.⁹ The recently developed new relativistic theory of modified Newtonian dynamics (the Aether Scalar Tensor – AeST) model by Skordis & Zlosnik (2021 [243]) proposes such a formulation and accounts for the standard CMB, the standard matter power-spectrum in the linear regime and that gravitational waves propagate with the speed of electromagnetic waves, c . It reproduces both the putative successes of the SMOc on cosmological scales and those of MOND on galactic scales. The AeST model requires an initial

⁹ Gravitation is not well understood and according to another interpretation it may be an effect emerging from spatial entropy differences [199, 240], or it might arise as a consequence of the wave-nature of matter [241, 242].

inflationary epoch, all matter being created during the Hot Big Bang, and dark energy to account for the observed expansion rate, as deduced from the observed CMB being interpreted as the surface of last scattering and from the SN1a data. It has a very similar expansion history as the SMOc because the AeST model satisfies the usual Friedmann equations. It is therewith subject to fundamental tensions FT1 and FT2. The AeST model fails to account for the observed inhomogeneities as it aims to reproduce the SMOc rather than observations (tensions T6–T8, [165]), and has also been found to encounter problems with weak lensing by galaxies (Mistele 2023 [244]). The AeST model requires more investigation, but here it is not discussed further as at present we do not have a computer programme that allows the equations of motion of baryons to be integrated self-consistently in view of the additional fields this model requires.

Given all the above, here a pragmatic and computationally-accessible approach is taken by combining theoretical knowledge that may be viewed to be reliable to provide first steps towards a holistic understanding of cosmological physics. We begin with the Angus model of cosmology.

4. The Angus model of cosmology (AMoC) – the ν HDM model

The AMoC constitutes a first conservative and thus careful step towards a Milgromian cosmological model by retaining postulates A1–A5 and auxiliary hypotheses AH1 and AH3. The AMoC removes AH2, i.e. cold or warm dark matter particles, from the computer experiments. In the AMoC the generalised Poisson equation based on the p-Laplacian which connects the baryonic overdensities with the Milgromian gravitational potential is applied rather than the standard Poisson equation (Appendix A). While postulate A3 (Einsteinian gravitation is valid globally) seems in conflict with this ansatz, the argument can be made that Milgromian dynamics emerges despite A3 due to the quantum vacuum affecting the motions of matter particles when space-curvature falls below the acceleration a_0 [245].

In order to match closely the observed CMB and to ensure reproduction of the same expansion history as the SMOc, the AMoC (Angus 2009 [246])¹⁰ is based on the assumption that the relic mass density of 11 eV sterile neutrinos¹¹ matches that of CDM in the standard Λ CDM model, and that dark energy contributes the remaining energy-density. In this model, the sterile neutrinos do not play a dynamical role in galaxies and Local-Group-type structures and thus do not lead to Chandrasekhar dynamical friction between galaxies, because they behave as hot dark matter and thus only contribute to the masses of galaxy clusters. Angus & Diaferio (2011 [247]) show that further refinements to the cosmological parameters are possible (see their fig. 1). Their result suggests that finding an optimal fit to the observed CMB in ν HDM would slightly increase the inferred H_0^{global} , therewith partially lessening the Hubble Tension. The AMoC is therefore defined by the same six parameters as the SMOc (Sec. 2.1) in order to describe the current state of the model universe in terms of its expansion rate, mass components, initial density fluctuations and curvature, with the only difference that Ω_c is replaced by the sterile neutrino mass density parameter $\Omega_\nu \approx 0.26$ (see also Katz et al. 2013 [223] and Wittenburg et al. 2023 [248]). The inflation-produced CMB-reproducing matter power-spectrum of the AMoC is very similar to that of the SMOc. Linear structure growth however

¹⁰The AMoC is often referred to as the “ ν HDM” model, e.g. Haslbauer et al. (2020 [109]).

¹¹Note that the possible existence of such sterile neutrinos as a hot dissipation-less matter component is motivated by neutrino physics rather than only by evidence from cosmological observations and galactic dynamics.

enhances the power on small scales in the SMOc due to the presence of cold or warm dark matter particles that allow gravitational clumping of these components on small scales already prior to $z_e = 1100$. However, by $z_e \approx 200$, the matter power spectrum of the AMoC falls off by orders of magnitude on scales smaller than about 30 Mpc in comparison to that of the SMOc (fig. 2 in Angus & Diaferio 2011 [247]) because the sterile neutrinos, being a hot dissipation-less matter-dominating component together with the baryonic plasma and radiation component hinder early gravitational clumping on small scales. In an AMoC simulation, neither galaxies nor other structures form in a simulation box smaller than 30 Mpc. The simulation boxes therefore need to be larger than 100 Mpc to capture structure formation since larger structures develop and collapse prior to smaller ones within them. The available computations do not yet reach a resolution sufficient for individual galaxies (Wittenburg et al. 2023 [248]).

Assuming Milgrom’s constant a_0 is time invariant and noting its value to be $a_0 \approx c H_0 / (2\pi)$ implies that the universe as a whole transitions into the Milgromian regime at $z_e \approx 3$ (McGaugh 1999 [211]), although density contrasts can develop much earlier as a consequence of the stronger effective gravitational field in MOND (Sanders 1998 [210]). It is safe to assume structure growth to remain in the Newtonian regime down to $z_e = 200$ when MOND-cosmological simulations are initialised [223, 249].

Pioneering studies of cosmological structure formation in Milgromian gravitation had already suggested larger and more massive structures to assemble more quickly than in the SMOc (Nusser 2002, [249]). Within these, Sanders (1998 [210], 2008 [250]) argues that galaxies form earlier and more rapidly than in the SMOc, with voids being much larger and emptier than in the SMOc. Some of this has been verified with actual cosmological simulations within the AMoC using dissipation-less particles to trace the baryonic matter by Angus et al. (2011 [251]) and Katz et al. (2013 [223]). These dissipation-less particle simulations indicate however that the present-day mass function of galaxy clusters might be too flat, extending to too large masses than observed (fig. 5 in Angus & Diaferio 2011 [247]; fig. 1 in Katz et al. 2013 [223]). As shown by Haslbauer et al. (2020, [109]), this may be alleviated though by noticing that the Local Group is situated in the about 0.6 Gpc KBC void such that the amplitude of the matter fluctuations within it are muted. This may also alleviate the σ_8 tension of the SMOc (pT4).

The AMoC thus avoids tension (Sec. 2.1) T1, accounts for T2 and appears to alleviate tensions T3–T8 (with its role on T4 being unclear), although it may overproduce massive galaxy clusters (T8). In an attempt to further address this problem and also the problem of galaxy formation in view of tension T5, modern (first-ever) hydrodynamical simulations of the AMoC have been performed in Bonn using the PoR code (Appendix A) and are being analysed in Bonn, Prague and St. Andrews (Wittenburg et al. 2023 [248]). These simulations do indicate that possible tension pT3 (JWST observations of very early galaxy formation) may worsen in the AMoC, since gravitationally bound structures are detected only at $z_e \lesssim 4$. Fig. 9 demonstrates an example of a $z_e = 0$ snapshot for the baryonic and sterile neutrino density distribution obtained with these simulations.

5. Conclusions

This contribution is an update of previous 2012–2015 assessments of the SMOc ([6–8]). It transpires that none of the problems previously identified have been solved, and many new problems

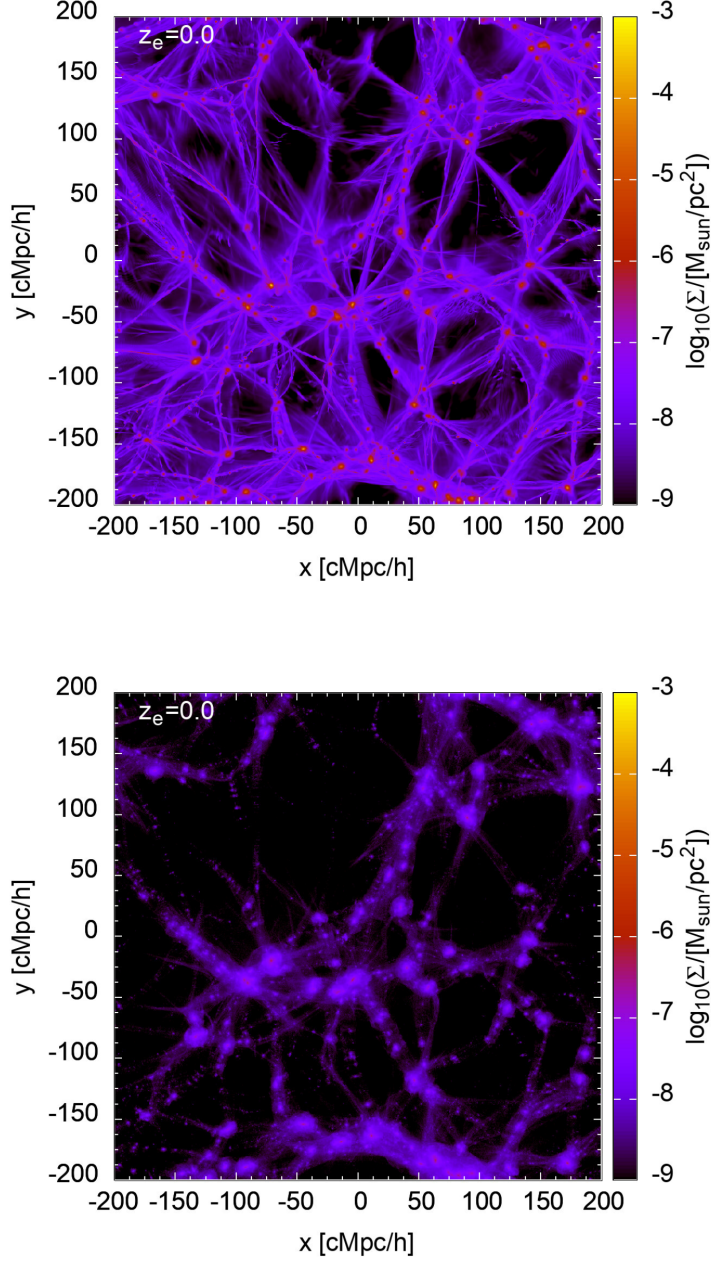


Figure 9: Hydrodynamical simulations of cosmological structure formation in the AMoC (i.e. ν HDM model, $z_e = 0$ snapshots of a 400 co-moving Mpc box). The simulations were performed by Nils Wittenburg in Bonn with the PoR code (Appendix A). *Upper panel:* the mass density of baryonic matter projected onto the xy plane. The density shown here is the mass-weighted average for the corresponding pixel. *Bottom panel:* Similar to the top panel, but for the sterile neutrinos. The dense clumps in both panels are galaxy clusters; galaxies are not resolved in these simulations. These simulations show that the mass-fraction of sterile neutrinos to baryons increases towards more massive galaxy clusters, being insignificant on the scale of galaxies (such that only MOND matters for their dynamics) while being consistent with the observationally-deduced fraction [252] of about 100 per cent for massive clusters. Movies of the simulations are available in Wittenburg et al. (2023 [248]). With permission from Wittenburg et al. (2023 [248]) their fig. 3).

arise on ever larger scales (T1–T8, pT1–pT5). This is visualised graphically by Fig. 8. The SMOc appears to be under significant stress on all scales and all redshifts. There appears little alternative but to face up to the possibility that the SMOc may need to be replaced by a different model. This conclusion, drawn from a multitude of independent tests of the SMOc against data on all scales, conforms with the SMOc having the unphysical property that it is energy non-conserving (FT2, Sec. 2.2). But can an alternative model be constructed?

An important constraint is that any viable model ought to obey the *fundamental cosmological principle* (Sec. 2.2) by allowing for significant structure growth on all observed scales, i.e. from kpc to Gpc scales, and probably also on the scale of the entire Universe (T7, pT2 in Sec. 2.2). This condition discourages most cosmological models that have been suggested to date, as most have been constructed on the premise that the model be homogeneous and isotropic on large (> 100 Mpc) scales to be in agreement with the wrongly-surmised success of the SMOc on these scales. Table 1 lists the models discussed here.

The perhaps most important clue as to how to construct a new model is the negative tests for the existence of dark matter particles with the significance of the independent tests amounting to well over the 5σ rejection threshold of the hypothesis that dark matter exists (T1, Sec. 2.2). Since there is also a lack of motivation from the standard model of particle physics for the existence of dark matter particles, it might appear useful to study models without dark matter.

Given the documented success of Milgromian dynamics (with some tensions existing though: MT1–MT6, Sec. 3.2), one way forward is to consider constructing cosmological models based on Milgromian equations of motion rather than Newtonian ones [209]. Milgromian dynamics is, to our knowledge, the only known other-than-Newtonian formulation of dynamics which is consistent with Solar System data and also explicitly formulates the equations of motion to be used to integrate a self-consistent system through time. Therefore a significant research effort (in Bonn, Prague and St. Andrews) is currently concentrating on exploring the ν HDM model (i.e. the AMoC, Sec. 4). This model is traditional in terms of minimalistic departures from the current SMOc by retaining the fundamental postulates A1–A5, and also the auxiliary hypothesis of inflation (AH1) and dark energy (AH3). The model improves the situation significantly as it avoids the problems associated with star clusters and galaxies and generates large voids with corresponding bulk matter flows (thus alleviating T1–T8). But further tests using simulations need to be made to affirm if this is truly the case. The possible tensions (pT1–pT5) remain though, and the results available from the simulations done so far suggest that the AMoC fails on pT3 (model galaxies form by a redshift of $z_e \approx 4$ and thus far too late), and in that the mass function of galaxy clusters may extend to too large masses and be too top-heavy compared to the observationally constrained one (Wittenburg et al. 2023 [248]). If each of the tensions associated with the AMoC correspond to a loss of confidence by 50 per cent (we do not yet have rigorous tests), then the MT1–MT6 tensions plus the probable failures with the mass function of galaxy clusters and the too-late formation of structures amount to a remaining confidence of 0.4 per cent that the AMoC is a correct description of the Universe. Furthermore, the AMoC retains the problems of the SMOc in terms of relying on inflation (FT1) and dark energy (FT2), none of these being physically understood with inflation and dark energy having no direct evidence by any other (non-cosmological) experiment. This model thus too does not conserve energy and runs into the “cosmological energy catastrophe” (Sec. 2.2). Also, it does not address the baryon asymmetry (FT4).

In Summary, all models that allow the hydrodynamical simulation of structure formation down to galaxy or galaxy-cluster scales and that rely on the initial conditions as set by the observed CMB appear to run into problems. In particular, assuming this initial CMB boundary condition, the SMOc produces too little structure on scales larger than 100 Mpc (apart from all the other problems listed in Sec. 2.2), while the AMoC produces too much of it and structures emerge at too small a redshift. It appears not possible to perform a SMOc or AMoC simulation that turns out a model universe which agrees with the Universe in terms of the large scales and the quick onset of star formation at $z_e > 10$ as uncovered by the JWST. Therefore it might seem useful to explore models that do not depend on inflation and on the CMB being the photosphere of a Hot Big Bang, and to consider suggestions on the origin of the CMB in terms of other processes.

Such ideas are not new, with Rees (1978 ([253], see also [254, 255] and references therein) having proposed that the CMB may have been produced by the first star formation with reprocessing of the stellar emission through dust. In the context of the MMOc (the $R_h = ct$ model), Melia (2022, [186]) suggests the CMB to be the (standard) surface of last scattering or photosphere produced at $z_e \approx 1100$, just as in the SMOc, but the temperature fluctuations in it to stem from $z_e \approx 16$ as a consequence of the CMB photons scattering on dust produced by the first generations of stars. [186] notes evidence for this through the frequency dependence in the CMB power spectrum observed by the Planck satellite. Vavrycuk (2018, [256]) suggests the CMB to be entirely dust-reprocessed star light over cosmological distances, given the observed traces of dust between galaxies in the present-day Universe (i.e., the present-day observed intergalactic dust density leads to an emission over cosmological distances that, according to the calculation presented in [256], is comparable to the observed CMB spectral energy distribution). With the possibility that the CMB might have a different origin, it is important to independently assess the validity of a cosmological model. One such approach would be to obtain a direct measurement of the expansion rate of the Universe (e.g. [257] and references therein). Another constraint is to obtain an independent measurement of the age of the Universe.

A cosmology-model-independent measurement of the expansion rate of the Universe may be available using the Cosmic Chronometers method proposed by Jimenez & Loeb (2002 [258]). The idea is to measure the ages of stellar populations in an ensemble of elliptical (i.e. passively evolving) galaxies at two different redshifts, to obtain a constraint on the time derivative of the redshift, dz_e/dt . The challenge of this method comes with needing to fully understand biases (e.g. Moresco et al. 2020 [259]) that enter through stellar-evolution models, chemical enrichment, the variation of the galaxy-wide IMF, possible on-going star formation at low levels as well as less-massive elliptical galaxies having typically formed later than more massive ones (Kroupa et al. 2013 [260], Jerabkova et al. 2018 [261], Yan et al. 2020 [100], Eapen et al. 2022 [99]). A more robust measurement of the actual age of the observed Universe that is independent of the applied cosmological model can be obtained from the ages of the oldest known stars. From table 1 (the entry "Very Metal Poor Stars") in Cimatti & Morsco (2023, [262]), the one-sigma upper limit on the age of the real Universe is $t_0 = 15.3$ Gyr (assuming the SMOc: global $H_0^{\text{global,stars}} = 60.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$) while the three-sigma upper limit is $t_0 = 16.5$ Gyr (assuming the SMOc: globally $H_0^{\text{global,stars}} = 55.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$). In this context it is interesting to note that Ying et al. (2023 [263]) measure the absolute age of the globular cluster M92 to be 13.80 ± 0.75 Gyr in tension with the nominal Planck age (t_0 , Sec. 2.1)

of the universe because the cluster stars are, with $[\text{Fe}/\text{H}] = -2.30 \pm 0.10$, not metal-free implying that a previous population of stars must have existed to enrich the interstellar medium from which the cluster formed.

The value of t_0 would be larger by a few hundred Myr since star formation can only begin after the birth of the Universe. Such a small value of the H_0^{global} might be qualitatively consistent with the larger local value measured using SN1a as standard candles, in view of the Local Group being in the $\gtrsim 0.6$ Gpc void (T6, Sec. 2.2). It will be necessary to test if such differences between apparent large local (as measured from the peculiar velocities of galaxies) and small global expansion rates arise as a consequence of the large matter inhomogeneities on many Gpc scales as is inherent to the timescape cosmological model, Sec. 3.1).

With the many tensions facing the SMoC and the other models discussed here, the Bohemian model of cosmology (BMoC) is being explored in Bonn, Prague and Nanjing. The results of this exploration will be reported in upcoming works, but here it is merely stated that the BMoC avoids inflation, dark matter and dark energy. It rests on Milgromian gravitational fields being sourced by matter density contrasts, just as in the AMoC (Sec. 4). Overall, there are encouraging aspects (expansion rate, star-formation histories of galaxies, an older Universe, large density variations over large scales) and stars can begin to form at a redshift $z_{\text{form}} > 20$. The first structure formation simulations using the PoR code (Appendix A) lead to positive results in that the BMoC appears to produce a universe full of disk galaxies.

Acknowledgments

We thank Eoin Ó Colgáin for useful comments and the staff at the Argelander-Institut for Astronomy in Bonn for their kind help with the computational system, and Indranil Banik for useful discussions. A very small financial support is acknowledged through the trans-disciplinary research area TRA-Matter at the University of Bonn. We acknowledge the DAAD-Bonn-Prague exchange program at the University of Bonn and at Charles University for supporting the Bonn-Prague exchange visits.

A. Milgromian dynamics (MOND) and the code PoR

Milgromian dynamics has been found to correctly predict (before the existence of the data) various relations that galaxies are observed to follow [71, 165, 187, 264–269]. Additionally, the kinematics of polar-ring galaxies are naturally understood in MOND, while they are not with Newtonian gravitation plus dark matter [221]. And, Newtonian gravitation does not account for the properties of the observed tidal tails of open-star-clusters, while Milgromian gravitation is very successful in doing so [66]. Finally, Milgromian gravitation appears to have been confirmed also on sub-pc scales using the wide-binary star test [237]. Together with MOND providing equations of motion, we thus have a strong motivation to investigate cosmological structure formation in this dynamics theory which is here taken to be a non-relativistic extension of Newtonian gravitation.¹²

¹²Interpretations of MOND as being a modified-inertia theory [270] are less well studied because they are technically very hard to investigate but are reported to be less preferred by the data [271]. Relativistic formulations of MOND have been developed and are consistent with gravitational waves travelling at the speed of light ([165] and references therein).

The reader needs to be aware that MOND is an effective framework that encompasses many different explicit formulations of gravitation (e.g. AQUAL vs QUMOND below, see Milgrom 2023 for the latest generalisations [272])). The different explicit formulations lead to subtle differences in the exact acceleration fields. For each such explicit formulation, the interpolating functions also offers degrees of freedom. Hereby it is not meant that different explicit formulations or interpolating functions can be chosen for different problems at will – only one such formulation and one interpolating function can be true. Research has not yet allowed the identification of which combination is true though, given the uncertainties in the data and the difficulties in performing calculations in this non-linear theory of dynamics.

Weak-lensing is often considered to provide evidence for dark matter halos around galaxies and galaxy clusters, but the statistically correlated distortions of background galaxy images by a foreground mass need to be re-evaluated in terms of another gravitational theory, in this case MOND. A bias would emerge due to possible correlations of disk galaxy spins over large regions of spatial volume (e.g. [151, 152, 273, 274]) that would affect the deduced mass distribution using weak lensing analysis if the (erroneous) assumption is made that galaxies are uncorrelated. The correct distribution of mass as deduced by weak-lensing surveys will then be different to that arrived at by using the (invalid) SMOc. The expected distribution of lensing mass as calculated in MOND of the galaxies in the nearby Universe has been documented by Oria et al. (2021 [275]), who point out that regions with effective negative Newtonian mass density are expected to exist in this gravitational framework.

A.1 MOND

With the baryonic mass density at the position vector, \vec{R} , being $\rho_b(\vec{R})$, the standard (Newtonian) Poisson equation reads

$$\vec{\nabla} \cdot \left[\vec{\nabla} \Phi_N \right] = 4\pi G \rho_b, \quad (1)$$

where $\Phi_N(\vec{R})$ is the Newtonian potential, the negative gradient of which provides the acceleration at \vec{R} . A generalised dynamics can be considered by rewriting the left hand side in terms of the well known p -Laplace operator,

$$\vec{\nabla} \cdot \left[\left(\frac{|\vec{\nabla} \Phi_p|}{a_0} \right)^{p-2} \vec{\nabla} \Phi_p \right] = 4\pi G \rho_b, \quad (2)$$

the constant, a_0 , with units of acceleration being necessary for dimensional reasons. This is a quasilinear elliptic partial differential equation of second order. In the case $p = di$, where di is the dimension of the domain, the corresponding p -Dirichlet integral is conformally invariant (e.g. [276]). Interestingly, this is related to the space-time scale invariance of the SIV model and of MOND [192, 193]. For $p = 2$ the standard Poisson equation (Eq. 1) is obtained that underlies Newtonian gravitation ($\Phi_N = \Phi_p$). The particular case $p = 3$ yields a non-linear gravitational theory which corresponds to the deep-MOND limit, when $|\vec{\nabla} \Phi_{p=3}| \ll a_0$ where $\Phi_{p=3} = \Phi_M(\vec{R})$ is the Milgromian potential, the negative gradient of which provides the acceleration at \vec{R} .

The transition from the $p = 3$ case to the $p = 2$ ($|\vec{\nabla} \Phi_p| \gg a_0$) case can be described by inserting an interpolating function, $\mu(x) \rightarrow 1$ for $x = |\vec{\nabla} \Phi|/a_0 \rightarrow \infty$ and $\mu(x) \rightarrow x$ for $x \rightarrow 0$. The

full Milgromian-Poisson equation thus becomes

$$\vec{\nabla} \cdot \left[\mu \left(\frac{|\vec{\nabla}\Phi_M|}{a_0} \right) \vec{\nabla}\Phi_M \right] = 4\pi G \rho_b. \quad (3)$$

Eq. 3 can be derived from a quadratic Lagrangian (AQUAL) as a generalisation of the non-relativistic Newtonian Lagrangian [208]. Thus we have energy- and momentum conserving time-integrable equations of motion of non-relativistic gravitating bodies in Milgromian dynamics.

By the properties of the divergence operator, the left hand side of Eq. 3 can be written

$$\vec{\nabla} \cdot \left[\mu \left(\frac{|\vec{\nabla}\Phi_M|}{a_0} \right) \vec{\nabla}\Phi_M \right] = \vec{\nabla}\mu(x) \cdot \vec{\nabla}\Phi_M + \mu(x) \left(\vec{\nabla} \cdot \vec{\nabla}\Phi_M \right), \quad (4)$$

such that

$$\vec{\nabla} \cdot \left[\vec{\nabla}\Phi_M \right] = 4\pi G \left[\frac{1}{\mu(x)} \left(\rho_b - \frac{\vec{\nabla}\mu(x) \cdot \vec{\nabla}\Phi_M}{4\pi G} \right) \right], \quad (5)$$

where the square brackets can be interpreted to be the total mass sourcing the Milgromian potential, Φ_M . This total mass is composed of the baryonic matter, ρ_b , modified by μ and the remaining term can be referred to as phantom dark matter (PDM) which however is not made up of real particles and does not lead to dynamical dissipation or friction. This formulation of Milgromian gravitation cannot be solved easily for Φ_M given ρ_b , because Φ_M is found in the dynamics term (the left-hand side) and also on the right-hand side (the source term) of Eq. 5.

This difficulty and the similarity of Eq. 5 with the standard Poisson equation (Eq. 1) leads to the motivation to develop a more direct approach. Essentially, this quasi-linear QUMOND (QUMOND) approach replaces the right-hand side of Eq. 5 by an approximation based on the Newtonian potential. A new Lagrangian leads to the QUMOND formulation [277] that allows the calculation of Φ_M more easily. According to QUMOND,

$$\vec{\nabla} \cdot \vec{\nabla}\Phi_{QM}(\vec{R}) = 4\pi G \left(\rho_b(\vec{R}) + \rho_{ph}(\vec{R}) \right), \quad (6)$$

where $\rho_{ph}(\vec{R})$ is the phantom dark matter density (PDM). Therefore the total Milgromian gravitational potential, $\Phi_{QM} = \Phi_N + \Phi_{ph}$, can be split into a Newtonian part, Φ_N , and an additional phantom part, Φ_{ph} . The matter density distribution, $\rho_{ph}(\vec{R})$, that would, in Newtonian gravitation, yield the additional potential, $\Phi_{ph}(\vec{R})$, and therefore obeys $\nabla^2\Phi_{ph}(\vec{R}) = 4\pi G\rho_{ph}(\vec{R})$, is known as the PDM density,

$$\rho_{ph}(\vec{R}) = \frac{1}{4\pi G} \vec{\nabla} \cdot \left[\tilde{\nu} \left(\frac{|\vec{\nabla}\Phi_N(\vec{R})|}{a_0} \right) \vec{\nabla}\Phi_N(\vec{R}) \right], \quad (7)$$

where $\tilde{\nu}(y) \rightarrow 0$ for $y \gg 1$ (Newtonian regime) and $\tilde{\nu}(y) \rightarrow y^{-1/2}$ for $y \ll 1$ (Milgromian regime). Note that μ and $\tilde{\nu}$ are algebraically related (see [71, 268]) with $y = |\vec{\nabla}\Phi_N|/a_0$. Remember that PDM is merely a mathematical aid for calculating the additional gravity in Milgrom's formulation, giving it an analogy in Newtonian dynamics. The PDM can become negative locally [222]. It is important to keep in mind that Φ_M and Φ_{QM} are not the same and subtle differences may emerge in the transition regime that will need scrutiny in the future.

The PDM density that would source the Q-Milgromian force field in Newtonian gravity can thus be calculated straightforwardly using standard grid-based methods (see eq. 35 in [71], and also [278]; [277]; [221]; [279], [280]). Starting with the known baryonic density distribution, $\rho_b(\vec{R})$, Φ_N is obtained by solving the standard Poisson equation (Eq. 1). Thereafter and at each time step, the new PDM distribution is calculated via Eq. 7, which, in sum with ρ_b , then yields the full Milgromian potential (Eq. 6).

An important consequence of the non-linear nature of Milgromian gravitation is that an external gravitational field (e.g. from a galaxy cluster) reduces ρ_{ph} of the system (e.g. a galaxy) immersed in this external field (e.g. [281–283]). The system loses most (but not all) of its non-Newtonian gravitational mass and becomes more Newtonian (the “external field effect” or EFE, see the reviews on MOND above and [66] for a discussion). The implication of the EFE for cosmological structure formation is important: overdensities inhibit structure growth in their vicinity therewith enhancing density differences [109].

A.2 The PoR code

The above QUMOND technique was implemented independently by Lueghausen et al. (2015 [280]) and Candlish et al. (2015 [284]) into the existing RAMSES code developed for Newtonian gravitation by Teyssier (2002 [285]). RAMSES employs an adaptively refined grid structure in Cartesian coordinates such that regions of higher density are automatically resolved with a higher resolution. In the PHANTOM OF RAMSES (PoR) code, [280] added a subroutine which, on the adaptive grid, computes the PDM density from the Newtonian potential (Eq. 7) and adds this (mathematical) DM-equivalent density to the baryonic one in terms of particles. These PDM particles are replaced by new PDM particles in the next time step, i.e., they do not carry an ID and are not integrated forward in time. The Poisson equation is then solved again to obtain the Milgromian potential, $\Phi_{QM} = \Phi_N + \Phi_{ph}$, i.e., the “true” (Milgromian) potential, which is used to integrate the stellar particles in time through space (each stellar particle experiencing the acceleration $\vec{a}_{star}(\vec{R}) = -\vec{\nabla}\Phi_M(\vec{R})$) and to solve the Euler equations for the dynamics of the gas.

It is to be noted that the AQUAL and QUMOND formulations are different theories complying to the MOND paradigm [268]. Thus, simulations with the AQUAL version (e.g. using the Candlish et al. 2015 [284] code) will show subtle differences to simulations relying on the QUMOND formulation by Lueghausen et al. (2015 [280]). Which formulation of MOND is closer to reality will need to be studied in the future (see Chae et al. (2022 [286])). Here we concentrate on the QUMOND version coded into PoR as the first step towards a very novel computable cosmological model.

The PoR code adopts as the default interpolating function

$$\tilde{v}(y) = -\frac{1}{2} + \left(\frac{1}{4} + \frac{1}{y}\right)^{\frac{1}{2}}, \quad (8)$$

which has already been used on a number of problems (for the manual and description see Nagesh et al. (2021 [287])). Other interpolating functions can be employed subject to observationally-given constraints (e.g. [233]).

The PoR code includes sub-grid baryonic physics algorithms that allow different degrees of realism in the description of star formation. Simulations have been done to model Antennae-like

interacting galaxies [215], the formation of exponential disk galaxies [68], the formation of elliptical galaxies [99] as well as of $10^7 M_\odot$ to $10^{11} M_\odot$ heavy galaxies on the main sequence [213]. These detailed tests of how the properties of a galaxy change with increased complexity of the in-built sub-grid baryonic processes have shown that galaxies evolved in MOND are not sensitive to these. In this work star-formation is turned off because the resolution achieved does not allow the formation of stellar particles to have a physical meaning. The PoR code has also been used without enabling star-formation to model the Milky-Way–Andromeda encounter which produced the disks of satellite galaxies around both [217, 219] and also to study the global stability of the M33 galaxy [288].

A.3 Cosmological simulations with PoR

Nusser (2002 [249]) details the implementation of the MOND field equation (Eq. 3 and 6) into computations of structure formation in an expanding universe. QUMOND has been applied in cosmological structure formation simulations using particles by Katz et al. (2013 [223]) and Angus et al. (2013 [289]) employing a code developed by Llinares et al. (2008 [290]) that accounts for co-moving coordinates in an expanding universe.

When including gas dynamical processes in order to compute how structures form and evolve in an expanding space, the equations of motion are here integrated in super-co-moving coordinates tailored specifically for an ideal gas with the ratio of specific heats being $5/3$ [291]. This is of advantage because, as [291] argue, in the absence of structure and in an expanding box, in co-moving coordinates the density is constant in time and the mass elements are at rest. In super-co-moving variables this is also true, and in addition, a gas with a ratio of specific heats as above also has its thermodynamic variables remain constant when structure is absent. RAMSES [285], and thus also PoR, adopts super-co-moving coordinates. The details of how QUMOND is implemented on calculating the Milgromian potential, Φ_{QM} , from the density difference relative to the mean density, are available in Candlish (2016 [222]) and Wittenburg et al. (2023 [248]).

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