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BBN constraints in models that alleviate the H_0 tension

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We use Big Bang Nucleosynthesis (BBN) data in order to impose constraints on higher-order modified gravity, and in particular on: (i) $f(G)$ Gauss-Bonnet gravity, and $f(P)$ cubic gravity, arising respectively through the use of the quadratic-curvature Gauss-Bonnet G term, and the cubic-curvature combination. We perform a detailed investigation of the BBN epoch and we calculate the deviations of the freeze-out temperature T_f in comparison to ΛCDM paradigm. We then use the observational bound on $\left|\frac{\delta T_f}{T_f}\right|$ in order to extract constraints on the involved parameters. We find that all models can satisfy the BBN constraints and thus they constitute viable cosmological scenarios, since they can additionally account for the dark energy sector and the late-time acceleration, in a quantitative manner, without spoiling the formation of light elements during the BBN epoch. Nevertheless, the obtained constraints on the relevant model parameters are quite strong.

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1. Introduction

Modified gravity is one of the two main ways that are being followed in order to explain the early and late accelerated phases of universe expansion [1–3], with the other one being the introduction of inflaton or/and dark energy sectors [4, 5]. Amongst the various classes of gravitational modifications that can fulfill the above cosmological motivation, theories that incorporate higher-order corrections to the Einstein-Hilbert Lagrangian have an additional motivation, namely the potential for improving the renormalizability of General Relativity [6, 7]. Such theories may naturally arise as (ghost-free) low-energy effective field-theory limits of String Theory [8] and include Einstein gravity in the lowest-order in a derivative expansion. A particularly interesting sub-class of such ghost-free higher-derivative theories that are equivalent to General Relativity at the linearized level in the vacuum, with only a transverse and massless propagating graviton, are the (Lovelock) theories [9]. The most general, ghost-free covariant gravitational action in a Minkowski vacuum (that is, up to and including quadratic-order terms in fluctuations $h_{\mu\nu}$ of the graviton field $g_{\mu\nu}$ in the expansion $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, with $\eta_{\mu\nu}$ the Minkowski metric), which also involves higher-curvature as well as non-local terms with improved Ultraviolet behaviour (UV), but recovers Einstein's General Relativity in the Infrared (IR), has been given in [10].

The construction of higher-order gravities is based on the addition of extra terms in the Einstein-Hilbert Lagrangian, such as in $f(R)$ gravity [11–13], in Lovelock gravity [9, 14], in Weyl gravity [15, 16], in Galileon theory [17, 18], etc. Restricting to quadratic-in-curvature corrections a well-studied class is obtained by using functions of the Gauss-Bonnet combination, resulting to the $f(G)$ gravity [19], which proves to have interesting cosmological phenomenology [20–34]. Similarly, using cubic terms one may construct the particular cubic curvature invariance P, which is a combination that is neither topological nor trivial in four dimensions and when used as a Lagrangian leads to a spectrum identical to that of General Relativity [35]. Cubic and $f(P)$ gravity have been also showed to lead to interesting cosmological [36–42] and black-hole applications [43–50].

An important and necessary test of every modified gravity is the confrontation with cosmological observations, since such a confrontation provides information on the involved unknown functions, as well as the allowed regions of the model parameters. Although, investigations related to late-time cosmological data [51] have been performed in some detail in the case of higher-order gravities [20, 23, 32], the use of early-time, and in particular, of Big Bang Nucleosynthesis (BBN) considerations has not been done as yet. Hence, in the present work we address this crucial issue, namely we impose constrains on a power law model of $f(G)$ gravity, and on a power law model of $f(P)$ gravity.

The plan of the article is the following: In Section 2 we briefly present $f(G)$ and $f(P)$ gravity, and we apply them in a cosmological framework. In Section 3, after a brief introduction to the basics of BBN, we examine in detail the BBN constraints on those two models, extracting the bounds on the involved model parameters. Finally, Section 4 is devoted to the Conclusions.

2. Higher-order gravity and cosmology

In this section we present higher-order gravity and we apply it in a cosmological framework. As we mentioned in the Introduction, such theories are obtained through the addition of higher-order terms, that are constructed by contractions of Riemann tensors, in the Einstein-Hilbert Lagrangian [9]. Throughout the work we consider the flat homogeneous and isotropic Friedmann-Robertson-Walker (FRW) geometry with metric

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \,, \tag{1}$$

where $a(t)$ is the scale factor. In the following subsections we examine the quadratic and cubic cases separately.

2.1 $f(G)$ gravity and cosmology

Let us first consider quadratic terms in the Riemann tensor. The corresponding combination is the Gauss-Bonnet one, given as[1]

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}. \tag{2}$$

Although this term is topological in four dimensions and thus it cannot lead to any corrections in the field equations, the extended action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + f(G)\right], \tag{3}$$

with $M_P \equiv 1/\sqrt{8\pi \mathrm{G_N}} = 2.4\times 10^{18}$ GeV the (reduced) Planck mass, and $\mathrm{G_N}$ the gravitational constant, corresponds to a new gravitational modification, namely $f(G)$ gravity. Variation of the action with respect to the metric leads to

$$\begin{aligned} M_P^2 G^{\mu\nu} = &\frac{1}{2}g^{\mu\nu}f(G) - 2f'(G)\,RR^{\mu\nu} + 4f'(G)\,R^{\mu}_{\rho}R^{\nu\rho} \\ &-2f'(G)\,R^{\mu\rho\sigma\tau}R^{\rho\sigma\tau}_{\nu} - 4f'(G)\,R^{\mu\rho\sigma\nu}R_{\rho\sigma} \\ &+2\nabla^{\mu}\nabla^{\nu}f'(G)\,R - 2g^{\mu\nu}\nabla^2 f'(G)\,R + 4\nabla^2 f'(G)\,R^{\mu\nu} \\ &-4\nabla_{\rho}\nabla^{\mu}f'(G)\,R^{\nu\rho} - 4\nabla_{\rho}\nabla^{\nu}f'(G)\,R^{\mu\rho} \\ &+4g^{\mu\nu}\nabla_{\rho}\nabla_{\sigma}f'(G)\,R^{\rho\sigma} - 4\nabla_{\rho}\nabla_{\sigma}f'(G)\,R^{\mu\rho\nu\sigma}, \end{aligned} \tag{4}$$

with $f_G \equiv \partial f(G)/\partial G$. Applying it to a cosmological framework, namely to the metric (1), and considering additionally the matter and radiation perfect fluids, we find the Friedmann equations

$$3M_P^2H^2 = \rho_m + \rho_r + \rho_{DE} \tag{5}$$

$$-2M_P^2\dot{H} = \rho_m + p_m + \rho_r + p_r + \rho_{DE} + p_{DE}, \tag{6}$$

[1] Our notation and conventions throughout this work are: signature of metric $(-,+,+,+)$, Riemann Curvature tensor $R^{\lambda}{}_{\mu\nu\sigma} = \partial_{\nu}\Gamma^{\lambda}_{\mu\sigma} + \Gamma^{\rho}_{\mu\sigma}\Gamma^{\lambda}_{\rho\nu} - (\nu \leftrightarrow \sigma)$, Ricci tensor $R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}$, and Ricci scalar $R = R_{\mu\nu}g^{\mu\nu}$. We also work in units $\hbar = c = 1$.

with ρ_m and p_m respectively the energy density and pressure of the matter fluid, ρ_r and p_r the corresponding quantities for radiation sector, and where we have introduced the corresponding quantities of the effective dark energy sector as

$$\rho_{DE} \equiv \frac{1}{2}\Big[-f(G) + 24H^2\left(H^2+\dot{H}\right)f'(G) \\ -24^2H^4\left(2\dot{H}^2+H\ddot{H}+4H^2\dot{H}\right)f''(G)\Big], \tag{7}$$

$$p_{DE} \equiv f(G) - 24H^2\left(H^2+\dot{H}\right)f'(G) \\ +8(24)^2\left(2\dot{H}^2+H\ddot{H}+4H^2\dot{H}\right)^2 f'''(G) \\ +192H^2\Big(6\dot{H}^3+8H\dot{H}\ddot{H}+24\dot{H}^2H^2 \\ +6H^3\ddot{H}+8H^4\dot{H}+H^2\dddot{H}\Big)f''(G), \tag{8}$$

where primes denote differentiation with respect to the argument. Note that in FRW metric the Gauss-Bonnet combination becomes

$$G = 24H^2(H^2+\dot{H}), \tag{9}$$

which has indeed squared powers comparing to the Ricci scalar $R = 6(2H^2+\dot{H})$.

2.2 $f(P)$ gravity and cosmology

We now proceed to the investigation of cubic terms. A general such combination is written as [9]

$$P = \beta_1 R_\mu{}^\rho{}_\nu{}^\sigma R_\rho{}^\gamma{}_\sigma{}^\delta R_\gamma{}^\mu{}_\delta{}^\nu + \beta_2 R_{\mu\nu}^{\rho\sigma} R_{\rho\sigma}^{\gamma\delta} R_{\gamma\delta}^{\mu\nu} \\ +\beta_3 R^{\sigma\gamma} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho}{}_\gamma + \beta_4 R R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\ +\beta_5 R_{\mu\nu\rho\sigma} R^{\mu\rho} R^{\nu\sigma} + \beta_6 R_\mu^\nu R_\nu^\rho R_\rho^\mu \\ +\beta_7 R_{\mu\nu} R^{\mu\nu} R + \beta_8 R^3. \tag{10}$$

Hence, using it as an argument of an arbitrary function we can construct the action of $f(P)$ gravity as [36]

$$\int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + f(P)\right]. \tag{11}$$

The above cubic combination possesses many coupling parameters. Nevertheless, we can significantly reduce their number by requiring that in the case of simple cubic theory (i.e. with $f(P) = P$) the resulting theory possesses a spectrum identical to that of general relativity, that this combination is neither topological nor trivial in four dimensions, and that its definition is independent of the dimensions [35]. Focusing additionally on FRW geometry we finally find that [36]

$$P = 6\tilde{\beta}H^4\left(2H^2+3\dot{H}\right), \tag{12}$$

which has only one free parameter. As expected is cubic in terms comparing to the Ricci scalar.

The two Friedmann equations of $f(P)$ gravity in the case of FRW geometry take the standard form (5),(6), however now the energy density and pressure of the effective dark energy fluid are written as

$$\rho_{DE} \equiv -f(P) - 18\tilde{\beta}H^4(H\partial_t - H^2 - \dot{H})f'(P), \tag{13}$$

$$p_{DE} \equiv f(P) + 6\tilde{\beta}H^3\left[H\partial_t^2 + 2(H^2 + 2\dot{H})\partial_t - 3H^3 - 5H\dot{H}\right]f'(P). \tag{14}$$

3. Big Bang Nucleosynthesis constraints

In this section we will investigate the Big Bang Nucleosynthesis (BBN) constraints on scenarios that are governed by higher-order modified gravity. BBN is realized during the radiation epoch [52–55]. In the case of standard cosmology, i.e. in the case of Standard Model radiation in the framework of general relativity, during the BBN the first Friedmann equation is approximated as

$$H^2 \approx \frac{M_P^{-2}}{3}\rho_r \equiv H_{GR}^2. \tag{15}$$

Additionally, we know that the energy density of relativistic particles is

$$\rho_r = \frac{\pi^2}{30}g_* T^4, \tag{16}$$

with $g_* \sim 10$ the effective number of degrees of freedom and T the temperature. Hence, we obtain

$$H(T) \approx \left(\frac{4\pi^3 g_*}{45}\right)^{1/2}\frac{T^2}{M_{Pl}}, \tag{17}$$

where $M_{Pl} = (8\pi)^{\frac{1}{2}}M_P = 1.22 \times 10^{19}$ GeV is the Planck mass.

Since the radiation conservation equation finally leads to a scale factor evolution of the form $a \sim t^{1/2}$, we can finally extract the expression between temperature and time, namely $\frac{1}{t} \simeq \left(\frac{32\pi^3 g_*}{90}\right)^{1/2}\frac{T^2}{M_{Pl}}$ (or $T(t) \simeq (t/\text{sec})^{-1/2}$ MeV).

During the BBN, the calculation of the neutron abundance arises from the protons-neutron conversion rate [54, 55]

$$\lambda_{pn}(T) = \lambda_{(n+\nu_e\to p+e^-)} + \lambda_{(n+e^+\to p+\bar{\nu}_e)} + \lambda_{(n\to p+e^-+\bar{\nu}_e)} \tag{18}$$

and its inverse $\lambda_{np}(T)$, and therefore for the total rate we have $\lambda_{tot}(T) = \lambda_{np}(T)+\lambda_{pn}(T)$. Assuming that the varius particles (neutrinos, electrons, photons) temperatures are the same, and low enough in order to use the Boltzmann distribution instead of the Fermi-Dirac one), and neglecting the electron mass compared to the electron and neutrino energies, straightforward calculations lead to the expression [56–60]

$$\lambda_{tot}(T) = 4A\,T^3(4!T^2 + 2\times 3!QT + 2!Q^2)\,, \tag{19}$$

where $Q = m_n - m_p = 1.29 \times 10^{-3}$GeV is the mass difference between neutron and proton and $A = 1.02 \times 10^{-11}$ GeV^{-4}.

Let us now calculate the corresponding freeze-out temperature. This will arise from the comparison of the universe expansion rate $\frac{1}{H}$ with $\lambda_{tot}(T)$. In particular, if $\frac{1}{H} \ll \lambda_{tot}(T)$, namely if the expansion time is much smaller than the interaction time we can consider thermal equilibrium [52, 53]. On the contrary, if $\frac{1}{H} \gg \lambda_{tot}(T)$ then particles do not have enough time to interact and therefore they decouple. Thus, the freeze-out temperature T_f, in which the decoupling takes place corresponds to $H(T_f) = \lambda_{tot}\left(T_f\right) \simeq c_q T_f^5$, with $c_q \equiv 4A\,4! \simeq 9.8 \times 10^{-10}\,\mathrm{GeV}^{-4}$ [56–60]. Using (17) and (19), the above requirement gives

$$T_f = \left(\frac{4\pi^3 g_*}{45 M_{Pl}^2 c_q^2}\right)^{1/6} \sim 0.0006\ \mathrm{GeV}. \tag{20}$$

Now, in any modified cosmological scenario one obtains extra terms in the Friedmann equations. During the BBN era these extra contributions need to be small, compared to the radiation sector of standard cosmology, in order not to spoil the observational facts. In particular, from a general modified Friedmann equation of the form (5) we obtain

$$H = H_{GR}\sqrt{1 + \frac{\rho_{DE}}{\rho_r}} = H_{GR} + \delta H, \tag{21}$$

where H_{GR} is the Hubble parameter of standard cosmology. Thus, we have

$$\delta H = \left(\sqrt{1 + \frac{\rho_{DE}}{\rho_r}} - 1\right) H_{GR}. \tag{22}$$

This deviation from standard cosmology, i.e form H_{GR}, will lead to a deviation in the freeze-out temperature δT_f. Since $H_{GR} = \lambda_{tot} \approx c_q T_f^5$, we easily find

$$\left(\sqrt{1 + \frac{\rho_{DE}}{\rho_r}} - 1\right) H_{GR} = 5 c_q T_f^4 \delta T_f, \tag{23}$$

and finally

$$\frac{\delta T_f}{T_f} \simeq \frac{\rho_{DE}}{\rho_r} \frac{H_{GR}}{10 c_q T_f^5}, \tag{24}$$

where we used that $\rho_{DE} << \rho_r$ during BBN. This theoretically calculated $\frac{\delta T_f}{T_f}$ should be compared with the observational bound

$$\left|\frac{\delta T_f}{T_f}\right| < 4.7 \times 10^{-4}, \tag{25}$$

which is obtained from the observational estimations of the baryon mass fraction converted to ^4He [61–67]. In the following subsections we use the above formalism, and in particular expression (24), in order to impose constraints on ρ_{DE} and thus on the underlying modified gravity, in specific models.

3.0.1 $f(G)$ Gravity

We consider the power-law model [19] where

$$f(G) = \alpha G^n, \tag{26}$$

with $n \neq 1$. In this expression n is the only free model parameter, since as long as $n \neq 1$ then α can be expressed in terms of the present value of the Hubble parameter H_0 and the present value of the dark energy density parameter $\Omega_{DE0} \equiv \rho_{DE0}/(3M_P^2H_0^2)$ (in the case $n = 1$ the above model cannot account for dark energy, in view of the aforementioned total derivative nature of the four-dimensional Gauss-Bonnet invariant). In particular, by applying (7) at present we find

$$\alpha = \frac{3H_0^2\Omega_{DE0}}{M_P^{-2}\left[(n-1)\,G_0^n - n\,(n-1)\,\gamma_0 G_0^{n-2}\right]}, \tag{27}$$

where

$$G_0 = 24H_0^2\left(H_0^2 + \dot{H}_0\right), \tag{28}$$

and

$$\gamma_0 = 24^2 H_0^4\left(2\dot{H}_0^2 + H_0\ddot{H}_0 + 4H_0^2\dot{H}_0\right). \tag{29}$$

Inserting (26) into (7) and then into (24) we finally obtain

$$\begin{aligned}\frac{\delta T_f}{T_f} &= -\Omega_{DE0}\,(\zeta)^{4n-1}\,(T_f)^{8n-7}\\ &\cdot\left[(-1)^n + n(-1)^{n-1} + 8n\,(n-1)\,(-1)^{n-2}\right]\\ &\cdot(H_0)^{2-2n}\left(H_0^2+\dot{H}_0\right)^{-n}\left[10c_q\,(n-1)\right]^{-1}\\ &\cdot\left[(1-2n)\left(\dot{H}_0^2+2\dot{H}_0H_0^2\right) - n\ddot{H}_0H_0 + H_0^4\right]^{-1},\end{aligned} \tag{30}$$

with

$$\zeta \equiv \left(\frac{4\pi^3 g_*}{45}\right)^{\frac{1}{2}} M_{Pl.}^{-1}\,. \tag{31}$$

In this expression we set [51]

$$\Omega_{DE0} \approx 0.7, \quad H_0 = 1.4\times 10^{-42}\ \mathrm{GeV}, \tag{32}$$

and the derivatives of the Hubble function at present are calculated through $\dot{H}_0 = -H_0^2\,(1+q_0)$ and $\ddot{H}_0 = H_0^3\,(j_0 + 3q_0 + 2)$ with $q_0 = -0.503$ the current decceleration parameter of the Universe [51], and $j_0 = 1.011$ the current jerk parameter [68, 69]. In Fig. 1 we plot $\delta T_f/T_f$ appearing in (30) vs the model parameter n, as well as the upper bound inferred from (25). As becomes evident from the figure, the expression (30) satisfies the bound (25) for $n \lesssim 0.45$.

We stress here that in this work we desire to impose BBN constraints on higher-order modified gravities models that can describe dark energy. Hence, concerning the present $f(G)$ model we require the fulfillment of condition (27), which imposes a dependence of the model parameters α and n. That is why BBN analysis leads to a so strong constraint on n. If we relax condition (27) then the BBN constraints can always be fulfilled for every n by suitably constraining α, and equivalently the BBN constraints can always be fulfilled for every α by suitably constraining n. However, under the condition (27), namely under the requirement that the Gauss-Bonnet terms describe dark energy at the late Universe, then n is constrained close to zero, in which case $f(G)$ correction becomes a constant and the scenario becomes ΛCDM.

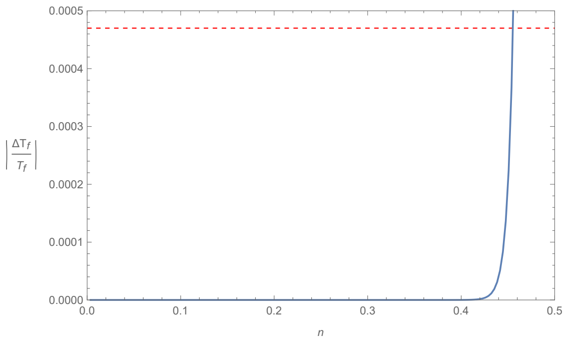

Figure 1: *$\delta T_f/T_f$ from (30) vs the model parameter n (blue solid curve), in the case of the power law model of $f(G)$ and the upper bound for $\delta T_f/T_f$ from (25) (red dashed line). As we observe, as long as $n \neq 1$ constraints from BBN require $n \lesssim 0.45$ (in the case $n = 1$ the above model cannot account for dark energy due to the topological nature of G).*

3.1 $f(P)$ Gravity

We consider the power-law model

$$f(P) = \alpha P^n, \tag{33}$$

where n is the only free model parameter, since α can be expressed in terms of H_0 and Ω_{DE0} given in (32) by applying (13) at the present epoch. Inserting (33) into (13) and then into (24) we acquire

$$\begin{aligned}
\frac{\delta T_f}{T_f} &= 2.1\,(\zeta)^{6n-1}\,(T_f)^{12n-7} \\
&\cdot \left[(-24)^n - 216n\,(n-1)\,(-24)^{n-1} + 18n\,(-24)^{n-1}\right] \\
&\cdot (30c_q)^{-1}\,(6)^{1-n}\,(H_0)^{2-4n}\left(2H_0^2 + 3\dot{H}_0\right)^{2-n} \\
&\cdot \Big\{[216n\,(n-1) - 18]\,\dot{H}_0 H_0^2 \\
&\quad +54n\,(n-1)\left(4\dot{H}_0^2 + \ddot{H}_0 H_0\right) - 12H_0^4\Big\}^{-1}.
\end{aligned} \tag{34}$$

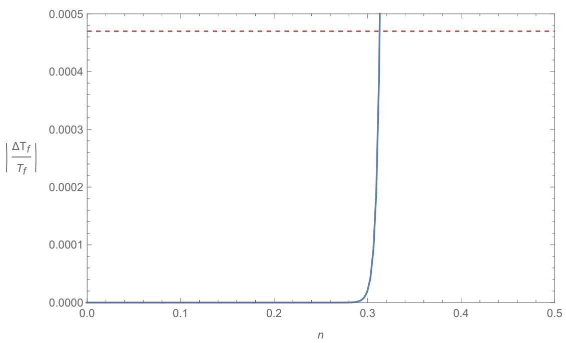

Figure 2: *$\delta T_f/T_f$ from (34) vs the model parameter n (blue solid curve) in the case of $f(P)$ Model of (33), and the upper bound for $\delta T_f/T_f$ from (25) (red dashed line). As we observe, constraints from BBN require $n \lesssim 0.31$.*

In Fig. 2 we draw $\delta T_f/T_f$ from (34) vs the model parameter n, as well as the upper bound from (25). It follows from the figure that the expression (34) satisfies the bound (25) of $n \lesssim 0.31$. As we observe, n is constrained to small values if we want the model to describe dark energy, in which case α and n are not independent but related through (13) at present. If we relax this relation then the BBN constraints can always be fulfilled for every n by suitably constraining α, and for every α by suitably constraining n. However, under their relation, i.e. under the requirement that the $f(P)$ terms describe dark energy at the late Universe, then n is constrained close to zero, in which case $f(P)$ becomes a constant and the scenario becomes ΛCDM.

4. Conclusions

In this work we investigated the implications of higher-order modified gravity to the formation of light elements in the early Universe, namely on the Big Bang Nucleosynthesis (BBN). Such gravitational modifications are proved to be both theoretically motivated as well as phenomenologically very efficient in describing the later times evolution of the universe. Nevertheless, in order for such scenarios to be able to be considered as viable, one should examine that they do not spoil the early universe behaviour, and in particular the BBN epoch.

The present analysis shows that models of higher-order modified gravity, apart from being closer to a renormalizable gravitational theory, they can be viable candidates of the description of Nature too, since they can quantitatively account for the dark energy sector and the late-time acceleration of the Universe, without altering the successes of the BBN epoch and the formation of light elements. Nonetheless, we mention that in most of the cases the corresponding model parameters are constrained in narrow windows, which is expected since it is well known that BBN analysis imposes strong constraints on possible deviations from standard cosmology. However, even in this case the results of the present work reveal the capabilities of such constructions and offers a motivation for further investigation, at a more detailed level, of the evolution of cosmic perturbations and their role in the large-scale structure of the Universe.

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