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A look beyond ΛCDM: the quest for a more comprehensive model of cosmology

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The standard ΛCDM model of cosmology has been highly successful in explaining a wide range of data from various scales and epochs of the Universe. However, recent observations have revealed a number of discrepancies and anomalies that cannot be easily understood within this baseline scenario. This contribution aims to present a comprehensive and up-to-date overview of the situation emerging from the most recent analyses of the Cosmic Microwave Background temperature anisotropies and polarization angular power spectra. It delves into the key elements underlying current discrepancies, examining the challenges that must be addressed to restore cosmic concordance.

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1. Introduction

The ΛCDM model of cosmology depicts the Universe as homogeneous and isotropic on large scales. The majority of matter in the model is referred to as Cold Dark Matter (CDM) and is represented as a perfect fluid of collisionless particles that interact only through gravity. The model also includes the presence of Dark Energy (parametrized by a cosmological constant Λ) which is responsible for the observed accelerated expansion of the Universe at late times. On the other hand, to set the initial conditions, the model relies on cosmological inflation: an early-time phase of almost de-Sitter accelerated expansion able to drive the Universe towards flatness and homogeneity while also providing a well-supported explanation for the origin of primordial density fluctuations.

Despite involving such poorly understood physics, over the last decades, this model has been largely successful in providing a precise fit to a wide range of cosmological and astrophysical observations. Nonetheless, in recent years, as error-bars on cosmological parameters began to narrow, intriguing tensions and anomalies have emerged at different statistical levels [1]. The widely known tension between the value of the Hubble constant (H_0) measured by the SH0ES collaboration using luminosity distances of Type Ia supernovae [2] ($H_0 = 73 \pm 1$ Km/s/Mpc) and the value inferred by the Planck satellite from the Cosmic Microwave Background (CMB) observations [3] ($H_0 = 67.4 \pm 0.5$ Km/s/Mpc) has recently crossed the limit of 5 standard deviations, basically ruling out the possibility of a statistical fluke. It is also worth noting that various complementary observations of the late-time Universe are all in agreement with the SH0ES result and that none of these measurements gives a value lower than early Universe measurements. Additionally, Planck-independent observations of the CMB temperature and polarization anisotropies all predict a value of the expansion rate consistent with Planck and never higher than late-time probes. As a result, the H_0-tension seems to be caused by a mismatch between our understanding of the early and late Universe. This conclusion is further supported by the tension in the parameter $S_8 \equiv \sigma_8\sqrt{\Omega_m/0.3}$ between the value estimated by Planck and the weak lensing measurements of KiDS-1000 [4] and DES-y3 [5] (at the level of $2-3\sigma$, assuming a ΛCDM model).

Excitingly, discrepancies may suggest the presence of new physics, but they may also reflect unaccounted-for systematic errors in the data. To fully understand these differences, it becomes crucial to test both extensions to the theory in order to avoid theoretical biases and verify the consistency between independent experiments to clarify the role of observational systematic. In this contribution, we provide an overview of the situation emerging from the latest CMB observations. We examine the limits of the standard ΛCDM model of cosmology by scrutinizing its poorly understood pillars (*i.e.*, inflation, Dark Matter, and Dark Energy) and explore different parameterizations to identify any inconsistencies with observations and ΛCDM predictions.

2. Inflation

Cosmological inflation is a widely accepted theory that explains the evolution of the Universe in its very early stages. One of its key aspects is that this fast expansion period played a crucial role in shaping the Universe by stretching it to achieve spatial flatness and large-scale homogeneity. Additionally, inflation provides a well-supported framework for the physical origin of primordial perturbations, explaining the tiny anisotropies observed in the CMB and, ultimately, the presence

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of structure in the Universe. Despite its remarkable success, inflation still awaits experimental confirmation. A unique prediction of the theory is the existence of a stochastic background of primordial gravitational waves on super-horizon scales, whose detection would represent strong direct evidence. However, inflation does not predict the amplitude of these gravitational waves, and there has yet to be a detection of tensor modes. Therefore, testing the other indirect predictions, such as spatial flatness and the properties of primordial perturbations, through multiple independent observations of the cosmic microwave background becomes a crucial step for lending experimental weight to the inflationary paradigm and gaining a deeper understanding of the early Universe.

2.1 Spatial Flatness

Cosmological and astrophysical observations have largely supported the inflationary prediction of a spatially flat Universe. However, in recent years, the geometry of the Universe has become a matter of debate. The reason is that taking the Planck satellite data [3] at face value, they show a higher lensing amplitude at about 2.8 standard deviations [6, 7]. Since more lensing is expected with more Cold Dark Matter (CDM), this immediately recasts a preference for a closed Universe that also helps in explaining some large-scale anomalies in the data, such as the deficit of amplitude in the quadrupole and octupole modes. As a result, from Planck, we obtain an indication for a closed Universe at the statistical level of 3.4 standard deviations [8, 9]. Unexpectedly, the result conflicts with other complementary astrophysical observables such as Baryon Acoustic Oscillation (BAO) measurements [10–12], which show tension with a closed Universe when combined with Planck data [8]. Interestingly, the Planck data consistently indicates a preference for a closed Universe, even introducing additional parameters in the cosmological model, with significance levels exceeding 3 standard deviations [13]. This suggests that the anomalies driving this preference cannot be easily explained away by additional parameters, nor is it a result of a volume effect, as some have speculated. Additionally, forcing the Universe to be spatially flat can significantly skew the constraints on beyond-ΛCDM cosmologies, for instance biasing the constraints on the equation of state of the Dark Energy (DE) component and its dynamical nature [14].

It is therefore important to verify the inflationary predictions for spatial flatness through multiple, independent CMB experiments and gain a deeper understanding of this surprising result. The recent release of CMB temperature anisotropies and polarization data by the Atacama Cosmology Telescope (ACT) [15, 16] and the South Pole Telescope (SPT) [17, 18], when combined with the WMAP 9-year observations [19, 20], provides us with the means to do this. These experiments produce cosmological constraints that, while typically less stringent than those of the Planck collaboration, may still be sufficient for accurate independent tests of the Planck results. The combinations of ACT, SPT and WMAP data fully support the inflationary prediction of a spatially flat Universe and also agree with the ΛCDM expectations for lensing amplitude. Additionally, both ACT and SPT data remain consistent with spatial flatness when combined with WMAP data in extended parameter spaces and these results are robust under the combination of CMB and astrophysical observations [13]. This suggests that the tension between CMB data and the curvature of the Universe may be due to an undetected systematic error in the Planck results.

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2.2 Primordial Perturbations

Another remarkable success of inflation is its ability to explain the observed small fluctuations in the cosmic microwave background through the super-adiabatic amplification of quantum fluctuations. The quantum fluctuations in a de-Sitter spacetime have a flat spectrum, and since the inflationary dynamics is nearly de-Sitter, inflation naturally predicts an almost flat spectrum for primordial perturbations, with a small residual scale dependence quantified through the inflationary spectral index n_s defined as

$$\mathcal{P}(k) \propto k^{n_s-1} \tag{1}$$

where $\mathcal{P}(k)$ is the scalar spectrum at a given scale k. Determining how much the former index deviates from one dictates the theoretical, phenomenological, and experimental perspectives of the field. For instance, a cosmological model with $n_{\rm s} = 1$ – that will correspond to the phenomenological model proposed by Harrison, Zel'dovich, and Peebles [21–23] – will imply a major theoretical breakthrough, as it would imply that the origin of cosmic perturbations may lie in some unknown fundamental theory different from the standard inflationary picture or in extensions of the latter.

The Planck satellite data provide strong evidence ($\sim 8\sigma$) for $n_{\rm s} \neq 1$, supporting inflation as the leading theory of the early Universe. However, yet a new potential challenge for inflationary cosmology is arising from Planck-independent observations of the cosmic microwave background. Namely, a $\sim 2.7\sigma$ discrepancy in the value of the scalar spectral index $n_{\rm s}$ measured by Planck ($n_{\rm s} = 0.9649 \pm 0.0044$) [3] and by ACT ($n_{\rm s} = 1.008 \pm 0.015$) [16]. ACT measurements of small-scale CMB spectra suggest in fact a cosmology with a lower value of $\Omega_b h^2$ and a higher spectral index compared to Planck and WMAP observations. This recasts a preference of ACT for a lower amplitude of the first acoustic peak in the TT power spectrum: splitting the Planck data into low ($2 \leq \ell \leq 650$) and high ($\ell > 650$) multipoles confirms the discrepancy between the two experiments at the level of 3σ (2σ) for low (high) temperature data [24].

As with the curvature-tension mentioned above, the $n_{\rm s}$ tension could result from a yet unknown systematic effect in the ACT data, or a departure from the theoretical ΛCDM framework by assuming canonical inflation as the dominant mechanism for producing the perturbations in the early Universe.

Concerning the first possibility, this preference remains robust with the addition of large-scale structure information data both in the form of BAO measurements and shear-shear, galaxy-galaxy, and galaxy-shear correlation functions from the first year of the Dark Energy Survey. In addition, the inclusion of low multipole polarization data from the Planck measurements of E-modes at multipoles $2 \leq \ell \leq 30$, while breaking the degeneracy between the optical depth at reionization (τ) and the scalar spectral index, is not able to explain this anomaly [24]. Finally, in Ref. [16] it was argued that an overall calibration of the polarization spectra could eventually explain the mismatch in n_s. One can therefore neglect any information arising from ACT polarization measurements (TE EE) and combine ACT temperature anisotropies (TT) with SPT polarization data (TE EE). In this case, the disagreement with Planck is reduced below 2σ, but with the ACT and SPT data still preferring a value of n_s around unity [24]. It is also noteworthy that combining ACT and SPT produces a shift in the (n_s, Ω_b, h^2) plane, yielding a value of baryon energy density that is now in agreement with Planck, but still preferring $n_s \sim 1$. This result suggests that the degeneracy between the two parameters only partially contributes to the potential tension and restoring the agreement for $\Omega_b\, h^2$ doesn't seem enough to reconcile the n_s discrepancy [24].

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Concerning the second possibility, it's noteworthy that extending the inflationary sector of the theory by accounting for a running of the scalar spectral index, $\alpha_s = dn_s/d\log k$, can reconcile the differences between ACT and Planck. However, the tension in n_s maps into a controversy in the values of α_s with Planck preferring a negative running ($\alpha_s = -0.0119 \pm 0.0079$) and ACT a positive one ($\alpha_s = 0.058 \pm 0.028$) [24]. This preference for a positive tilted spectrum from small-scale CMB observations is shown to persist even combining ACT and SPT with WMAP 9-year observations [25], posing a challenge to the most typical inflationary models that predict a negative value of α_s. From the theoretical/model-building approach, more elaborate scenarios violating the standard inflationary predictions [26, 27] may provide a solution, while being testable by near future CMB B-mode and gravitational waves experiments.

3. Dark Matter and Neutrinos

Our current knowledge of structure formation in the Universe suggests that most matter interacts primarily through gravity. This challenges our comprehension of fundamental interactions as the dark matter has yet to be explained within the Standard Model (SM) of particle physics. Moreover, neutrino oscillation experiments have provided compelling evidence that neutrinos are massive particles, which contradicts their massless status in the Standard Model. These findings indicate that our knowledge of the fundamental interactions may be incomplete, as well. Cosmology remains an elegant and precise tool for exploring fundamental physics, including the nature of dark matter and the neutrino sector. For instance, theoretical frameworks that aim to address the shortcomings of the Standard Model of particle physics frequently involve elusive and lightweight degrees of freedom that may have played a significant role in the early Universe due to their feeble couplings and tiny masses.

We can parameterize the cosmological effects of additional light relativistic species at recombination by corrections to the effective number of relativistic degrees of freedom $N_{\rm eff}$, defined by the relation

$$\rho_{\rm rad} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right]\rho_\gamma \,, \tag{2}$$

with ρ_γ the present Cosmic Microwave Background (CMB) energy-density. The standard value for the effective number of neutrino species is $N_{\rm eff} = 3.044$ [28–30]. Any deviation from this value could affect cosmological observables, such as the CMB temperature angular power spectrum damping tail, the sound horizon, and the Silk damping scales. Furthermore, the predicted abundances of light elements from Big Bang Nucleosynthesis (BBN) could be impacted by extra light species, since the expansion rate during the BBN epoch heavily relies on the effective number of relativistic degrees of freedom, resulting in a higher freeze-out temperature for weak interactions and increasing the fraction of primordial helium. Therefore, accurately determining the number of relativistic species present during recombination, as well as the abundance of primordial nuclei forged during the Big Bang Nucleosynthesis, is a refined approach for unveiling new physics beyond the standard model, above all when combined with additional direct experiments and astrophysical measurements [31–33].

Interestingly, the Planck 2018 data align perfectly with the value $N_{\rm eff} = 3.044$ predicted by the standard model within a margin of uncertainty of $\Delta N_{\rm eff} \lesssim 0.3$ at a 95% CL. This finding is significant because it provides a robust foundation to set precise constraints on additional relativistic species during recombination and offers a means to limit various theoretical scenarios of the early Universe. Additionally, Planck's data did not reveal any evidence of neutrino mass and, when combined with BAO, the data seems to favor the normal ordering as the one governing the mass pattern of neutral fermions. These results remain basically unchanged by the inclusion of additional parameters in the model, setting robust cosmological limits on fundamental physics.

On the other hand, the Atacama Cosmology Telescope data systematically indicates a lower amount of radiation in the early Universe, with a statistical significance ranging from 1.8σ to 3σ, depending on the specific cosmological model [13]. In particular, this finding remains relatively stable both under the inclusion of additional parameters and under the inclusion of astrophysical observations (such as BAO and Pantheon). Furthermore, ACT, both alone and when combined with WMAP, consistently demonstrates a moderate-to-strong preference ($2.5\sigma - 4\sigma$) for massive neutrinos, indicating larger mass values of $\sum m_\nu \gtrsim 0.5$ eV. This is in tension with Planck and SPT results, as well. Additionally, it is intriguing to observe that such differences seem to correlate with the disagreement in the spectral index values observed between ACT and Planck. Notably, the consistency between Planck and ACT on n_s can be restored by adjusting the neutrino sector, such as by allowing their mass to be a free parameter in the cosmological model or permitting variations in the number of relativistic species [24].

In light of these results, we cannot disregard the possibility that the global tension between these two experiments could be rooted in the limitations of the standard cosmological model that may be not fully modelled to accurately represent the small-scale (high multipole) observations of the CMB as probed by ACT. In this regard, the analysis suggests a potentially prominent role of dark matter and neutrinos.

4. Dark Energy and late time expansion

Despite the variety of CMB experiments, the current level of constraint on the Dark Energy Equation of State is somehow limited, above all in extended parameter spaces. In extended cosmologies, the large error-bars of the results usually lead to consistency with a cosmological constant term in Einstein's equations within one standard deviation. Nonetheless, these bounds are insufficient to rule out different Dark Energy behaviours, thus leaving room for both phantom and quintessential models to remain consistent with observations [13]. When combined with BAO measurements, the constraints typically narrow down to around $w = -1$, with no significant deviations observed from this baseline value. On the other hand, when considering the Type Ia Supernovae distance moduli measurements from the Pantheon sample in combination with the CMB data, a preference for phantom Dark Energy ($w < -1$) is systematically observed at a statistical level ranging between 1.5σ and 2.5σ using Planck and ACT, whereas the combination of SPT and Pantheon data is typically consistent with the cosmological constant value at one standard deviation [13].

The rate at which the Universe is expanding, as inferred from various observations of the Cosmic Microwave Background, is highly sensitive to the cosmological model and its underlying assumptions. In more complex cosmologies, such as those that consider extensions beyond the standard ΛCDM model, the value of H_0 inferred from CMB data can be poorly constrained due

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to the significant error-bars [13]. As a result, the Hubble tension is often reduced in many of these models, which suggests that the current tension may be a consequence of the inadequacy of the standard ΛCDM model to accurately describe observations across a range of epochs in the Universe. This provides compelling evidence that using this model can introduce bias in the data constraints. However, the combination of CMB observations with astrophysical datasets can reduce errors, leading to an increase in tension. In addition, while the extensive analysis performed by the SH0ES collaboration [2] with ~ 70 different tests over all possible systematics in the literature suggests that the possibility of observational systematics in local measurements of H_0 is becoming increasingly unlikely, this hypothesis cannot be ruled out entirely, yet.

5. Conclusion

By conducting a meticulous examination of the latest cosmic microwave background measurements, it has become evident that multiple anomalies cannot be entirely accounted for by the current standard cosmological model. Based on current data, anomalies can be observed in several areas of the theory. For instance, Planck data seem to indicate a preference for a closed Universe by more than three standard deviations, contradicting the inflationary predictions for a flat Universe. This result is not supported by ACT and SPT data, which are perfectly consistent with a flat geometry. Nevertheless, ACT data suggest a Harrison-Zel'dovich spectrum with a unitary spectral index $n_s = 1$, which is unexpected and in strong tension with Planck. While this n_s anomaly may hint towards observational systematic errors, we cannot disregard the possibility that such a problem is rooted in the inability of the standard cosmological model to provide a good description of the small scales probed by ACT. For instance, expanding the neutrino and Dark Matter sector of the theory can be quite effective to restore the consistency with Planck on this parameter. Nonetheless, other unexpected anomalies can be observed, such as systematic evidence by ACT for a lower amount of radiation in the early Universe, in tension both with the standard model of elementary particles and with Planck.

In conclusion, a clear tension emerges between ACT and Planck. The global tension between independent CMB measurements can be quantified under a given cosmological model by using specific statistical techniques. For instance, assuming ΛCDM, the global tension between ACT and Planck can be estimated to be ~ 2.5 standard deviations [34, 35]. Expanding the parameter space and relaxing assumptions in different branches of the theory is quite ineffective in resolving this problem: the general disagreement between these two experiments is hard to accommodate to below 1σ by naively extending the cosmological model or by allowing additional parameters to vary and is reduced at the level of 1.8σ only when the effective number of relativistic particles ($N_{\rm eff}$) is significantly less than the standard value [35]. Therefore, this global "CMB tension" may indicate the standard model of the Universe provides an incorrect or incomplete description of Nature, and our analysis suggests that a satisfactory solution might require a more radical change in the theory. On the other hand, it is also plausible that significant unaccounted-for systematics in the data are producing biased results in one or both experiments and only independent high-precision CMB temperature and polarization measurements could provide a definitive answer.

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