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Recent theoretical developments on heavy quarkonia in QGP

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In these proceedings, we review recent developments in the theoretical description of heavy quarkonium. First, we will briefly described the different approaches that have been used to study quarkonium suppression and some relations between them. Later, we will discuss some recent developments in the field, many of then related to the inclusion of E/T corrections in the master equation of quarkonium.

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1. Introduction

The focus of this conference is the study of hard probes, probes that are generated at the early stages of heavy-ion collisions, primarily due to their high-energy requirements. These probes undergo significant modifications within the medium and possess characteristics that make them relatively straightforward to detect. Specifically, our focus centers on the probes associated with heavy quarkonium.

Quarkonium, consisting of a heavy quark and its antiquark bound together, serves as an excellent probe for studying the properties of the quark-gluon plasma (QGP) formed in heavy-ion collisions. Several key factors contribute to its suitability as a probe:

Firstly, the mass of a heavy quark, such as charm or bottom quark, denoted by m, is significantly larger than the characteristic scale of Quantum Chromodynamics (QCD) known as Λ_{QCD}. Secondly, the temperature T of the QGP is considerably smaller than the mass of heavy quarks ($T \ll m$). Thus, the medium has a negligible influence on the production of heavy quarks. However, it significantly affects their properties such as diffusion and the likelihood of forming a bound state. Thirdly, in the case of quarkonium, additional energy scales come into play, namely the inverse of the typical radius $\frac{1}{r}$ and the binding energy E. By utilizing heavy quarks, we can effectively explore and analyze the properties of the medium at different energy scales, shedding light on the behavior and characteristics of the quark-gluon plasma formed in heavy-ion collisions.

Let us now review the mechanisms that affect the formation of bound states in a QGP. Color screening was the first mechanism discussed in the literature. It was proposed by Matsui and Satz [1] that the suppression of heavy quarkonium is a signal of the formation of a QGP and that this suppression is due to the screening of chromoelectric fields at large distances. From the perturbative point of view, the potential at short distances changes from a Coulomb potential at zero temperature to a Yukawa potential in the QGP. This implies that there is a distance, called the Debye radius, such that if the heavy quarks are separated by a distance larger than the Debye radius they can not form a bound state. Another mechanism leading to dissociation is inelastic scattering with medium partons. Collisions with medium constituents induce a transition from a color singlet to a color octet state that produce a finite thermal decay width. This mechanism that is responsible for the imaginary part of the potential, which was first discussed in [2], was already known prior to the studies focusing on it. However, it was only after these studies that its significance became apparent, with the realization that it could be as crucial as screening or even the dominant factor.

Finally, we must discuss recombination. Recombination is the process in which two unbound heavy quarks recombine to form a new bound state inside of the medium. We can distinguish two types of recombination, uncorrelated and correlated. By correlated we mean that the two heavy unbound quarks belonged initially to the same bound state. Uncorrelated recombination dominates when the density of heavy quarks is large. At LHC energies, this means that uncorrelated recombination is relevant for charmonium but not for bottomonium.

2. Theoretical description

There are many different approaches that can be used to study quarkonium in heavy-ion collisions. Here we will briefly review the more common ones. We can classify the different theoretical approaches according to two different aspects. The first one is the way in which the medium-quarkonium interaction is modelled. For example, we can describe the interaction using perturbation theory. In this case, we must take into account the Hard Thermal Loop resummation [3–5] for degrees of freedom with energies of order gT, with g the coupling constant. In some situations, lattice QCD data can also be used [6]. Another complementary approach is to use Effective Field Theories (EFTs) [7, 8]. In this way it is possible to disentangle long and short distance physics. This is very convenient in the case of quarkonium because we know that physics at the scale m is perturbative and not affected by the medium while for the other energy scales it might not be the case. In particular, the combination of EFTs with lattice computations is very useful. Finally, we will discuss the potential model approach. In this approach, a Schrödinger equation and a potential are postulated for quarkonium. Let us note that these approaches are not completely unrelated and that they can be combined in several ways. For example, potential models can be deduced from EFT at leading order (LO) and the needed potential can be computed using perturbation theory or using lattice QCD data on the static potential.

A second way in which we can classify the existing theoretical approaches is according to how the evolution of quarkonium is described. In general terms, we can regard quarkonium as a quantum system interacting with another quantum system, the medium. It is convenient to describe the state of quarkonium by using the so-called reduced density matrix which is obtained from the full density matrix by performing a trace over the medium degrees of freedom. The evolution of quarkonium is then given by a quantum master equation. However, in some situations, the reduced density matrix contains more information that what is actually needed and the evolution of quarkonium can be accurately described using a semi-classical approach like a Boltzmann or a Langevin equation.

Let us now review with some more detail different approaches. Within the semi-classical approaches we can consider the Boltzmann equation

$$\frac{\partial}{\partial t}f_{nl}(\mathbf{x},\mathbf{p},t)+\mathbf{v}\cdot\boldsymbol{\nabla}_{\mathbf{x}}f_{nl}(\mathbf{x},\mathbf{p},t)=C^{(+)}_{nl}(\mathbf{x},\mathbf{p},t)-C^{(-)}_{nl}(\mathbf{x},\mathbf{p},t)\,, \tag{1}$$

where f_{nl} is the probability density to find a quarkonium state with quantum numbers nl in position $\mathbf{x}$ with center of mass momentum p and $C^{(+)}$ ($C^{(-)}$) is a collision term corresponding to the increase (decrease) of the number of that particular bound state. An example of the use of this approach can be found in [9]. An additional simplification can be made by assuming that the particles sourcing the production term are in equilibrium. Then, we obtain a rate equation of the type

$$\frac{\partial}{\partial t}p_n=-\Gamma(p_n-p_n^{eq})\,, \tag{2}$$

where p_n^{eq} is the probability of the considered bound state in thermal equilibrium. Examples of the use of rate equations for the study of quarkonium suppression can be found in [10, 11]. Both Boltzmann and rate equation assume implicitly that the decay width is a perturbation. We are going to see that only when this approximation is done we can assign a well-defined decay width. When the effects that can not be encoded in a Hermitian Hamiltonian, like que decay width, are not a perturbation we need to consider an approach that takes into account all the information of the reduced density matrix.

We can regard quarkonium as an open quantum system interacting with an environment (the medium). As we said, the state of quarkonium at a given time is encoded it is reduced density matrix. The equation that describes the evolution of the density matrix is called master equation. The master equation for QCD has been derived using perturbation theory and HTL [12–15] and EFTs in the regime $Tr \ll 1$ [16, 17]. If the temperature is much larger than the binding energy $T \gg E$, the interaction of quarkonium with the medium is very fast compared to the time scales of the evolution of quarkonium. Then, from the point of view of the quarkonium, the interaction is instantaneous and, therefore, the evolution is Markovian. It is well-known that all Markovian master equations that maintain some basic properties of density matrices (trace equal to unity, being hermitian and complete positivity) are so-called Lindblad equations [18, 19]

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_n \left(C_n \rho C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho\} \right) , \tag{3}$$

where H is an Hermitian Hamiltonian and C_n's are so-called collapse operators. As we are going to discuss later, some of the recent developments are related with the study of the properties of the Lindblad equation of quarkonium or with its phenomenological consequences. The master equation allows to threat screening and the dissociation induced by inelastic scattering on equal footing.

It is also interesting to relate the Lindblad/master equation approach to other ways to understand the evolution of quarkonium.

- We can recover the Boltzmann equation as an specific limit of the master equation. If the evolution induced by H is much faster than the rest of the evolution, then off-diagonal elements of the density matrix (in the basis that diagonalizes H) oscillate very fast and we can ignore them. This is often called the Rotating-Wave Approximation (RWA) [15, 20]
- Starting with the Lindblad equation, we can obtain a Schrödinger equation with a non-Hermitian Hamiltonian if we ignore the term $C_n \rho C_n^\dagger$.
- The Lindblad equation obtained in the regime $T \gg E$ naturally leads at large times to a density matrix that is almost diagonal in coordinate space. In this situation, the evolution can be described by a Langevin-like equation [14].

Finally, let us discuss the statistical hadronization model (see [21] and references therein). In this model, the focus is not so much in the evolution of quarkonium but rather in its final state at freeze-out. It is assumed that the distribution of quarkonium is thermal at freeze-out. This is compatible with observations for the case of charmonium. However, the model has to be corrected for bottomonium by including a fraction of non-thermalized bottom quarks.

3. Recent developments

In this section, we will review some recent developments in the study of quarkonium suppression. Many interesting investigations were also reported in *Hard Probes 2023* and can be found in these proceedings [28–32]. We would like to be able to describe the evolution of quarkonium in a medium in a realistic way. That would imply being able to simulate a three dimensional quantum evolution that takes into account the non-Abelian nature of QCD. At the same time, we would need to simulate a large number of heavy quarks in order to take into account uncorrelated

Work	System	Quantum	Dimensions	Dilute limit?	Equilibration
[22]	B	✓	3D	Yes	?
[23]	B,C	✓	1D	Yes	?
[24]	B,C	✓(Remler)	3D	No	✓
[25]	B_c (B,C,exotics)	X	3D	No	✓
[26]	B,C	✓	1D	Yes	✓
[27]	C	X	3D	No	✓

Table 1: Table showing the fulfillment of some relevant conditions that a realistic master equation should have by recent works studying the medium evolution of mesons made of heavy quarks.

recombination. Finally, an important cross-check is that we can prove that the master equation leads to thermalization. Unfortunately, such a study is not yet available in the literature. However, there are many works that fulfill some conditions but not all of them. A brief review of the state-of-the-art regarding the conditions we have just discussed can be found in table 1. Furthermore, it is essential to consider the evolution of various quarkonium species (such as charmonium, bottomonium, Bc, and others), as they probe the medium at distinct energy scales. In the following, we will discuss some recent developments and how they have contributed to improve the state-of-the-art.

3.1 New species

Traditional quarkonium species studied in heavy-ion collisions are charmonium and bottomonium, both the fundamental and the exited states. On recent years, there is a growing interest in studying other states as, for example, B_c systems and exotic quarkonia. B_c is a meson formed by a bottom quark and a charm antiquark (or vice versa). Although it is not a quarkonium state, it is also a meson made of heavy quarks ant it shares many of its characteristics. The B_c is interesting because while we can considered the bottom quark as dilute at LHC heavy-ion collisions this is not the case for the charm quark. Therefore, B_c is interesting as a state that interpolates between charmonium and botttomonium. Recently, B_c has been observed in heavy-ion collisions [33]. In [25], the B_c system has been studied with the use of a rate equation whose coefficients have been determined using the T-matrix approach. They have obtained results compatible with observations.

Exotic quarkonia states are states that can not be accommodated in the traditional quark model. In many cases, their nature is still not clear. However, there are many theoretical ideas about their structure. Examples of them are; tetraquarks, hybrid quarkonia, molecular states, diquark bound states... The prospects of measuring these states in heavy-ion collisions could provide a new powerful tool to determine the nature of quarkonia exotic since each of the proposed structures would interact in a different way with the medium. This has been done in [34], where $X(3872)$ is studied in high multiplicity pp collisions with the aim of determining whether this state is a molecule or a tetraquark.

3.2 Remmler's approach

In recent years, Remmler's approach has been applied to the study of quarkonium suppression [24, 28]. The starting point of this approach is the quantum master equation, but in this case some specific assumptions are taken regarding the form of the Wigner function of quarkonium, and

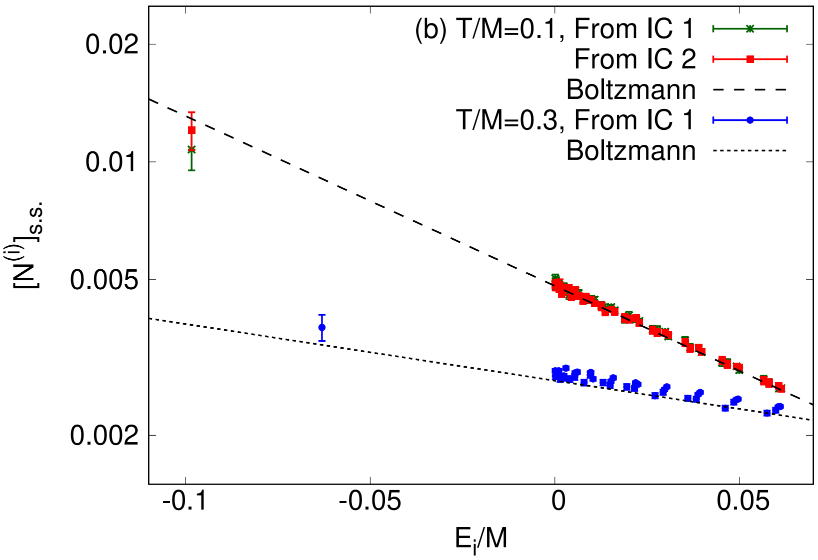

Figure 1: Picture taken from [26] where the approach to equilibrium of the occupancies of the lowest-lying singlet and octet states is shown.

therefore its density matrix. The advantage of doing this is that it allows to have a simpler evolution equation in which a large number of heavy quarks can be incorporated.

3.3 E/T corrections in the master equation

At the moment, there are tools that allow to efficiently simulate any quantum master equation for quarkonium as long as it is of the Lindblad type [35]. However, in the $T \sim E$ case we obtain a non-Markovian evolution. Approximations that make it Markovian either lead to a Boltzmann equation, eliminating all quantum coherence information, or do not conserve complete positivity of the density matrix, as for example the Caldeira-Leggett equation. An alternative that has been explored by several groups is to consider the case $T \gg E$, then including higher order corrections in the E/T expansion we hope to capture some of the physics present in the $T \sim E$ case. The advantage of proceeding in this way is that we can obtain an equation of the Lindblad type that we know how to simulate very efficiently. Moreover, it is known that in the QED case a Lindblad equation including only the first corrections in the E/T expansion leads to thermalization [15].

In the approach reported in [31], a one-dimensional case with adapted equations is studied. By performing a numerical analysis of the evolution, they have confirmed that the Lindblad equation leads at large times to density matrices that are almost diagonal in coordinate spaces, as was argued in [14]. However, they have also observed that near the origin (of the relative coordinate) there is a region in which quantum correlations survive. This is compatible with the observation reported in [14] that Langevin-like equations did not give consistent results close to the origin.

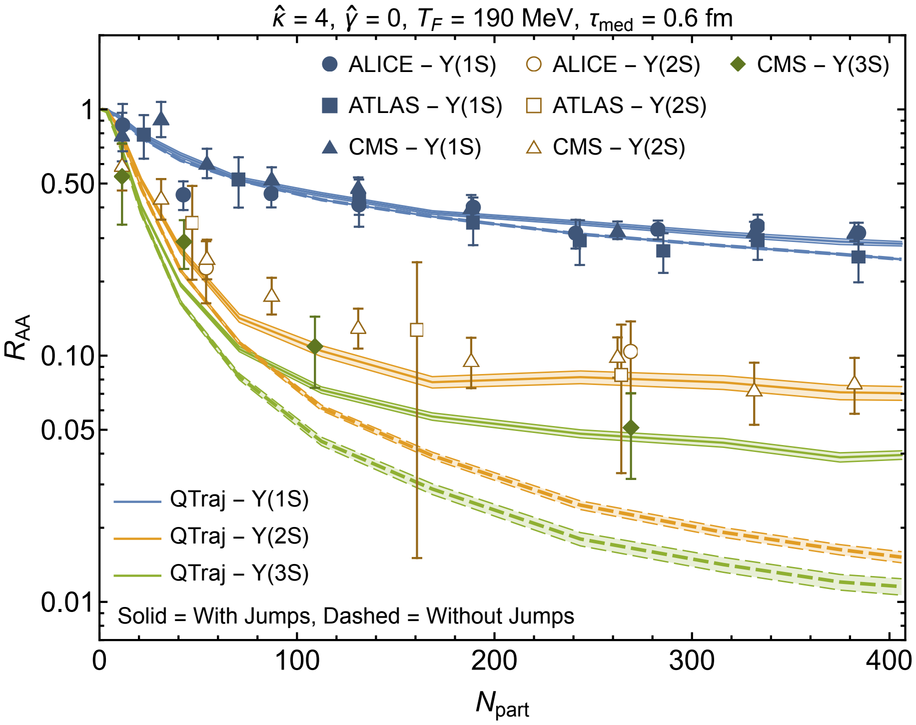

Figure 2: Figure taken from [22]. Results for R_{AA} obtained within the pNRQCD approach are compared to experimental results from [36–39]. We can clearly see the need of recombination (jumps) to reproduce experimental results.

In [26], the authors studied a one-dimensional master equation valid in the regime $m_D r \sim 1$ with a focus on studying thermalization. They found several interesting results. At very long times quarkonium thermalizes to an almost Boltzmann distribution using the master equation. This is illustrated in fig. 1. However, for the case of bottomonium, the time scale of this thermalization is around 15 times longer than the lifetime of the fireball of heavy-ion collisions. They also observed, that within their setting, the dipole approximation and the full result coincide at small times.

Let us now discuss the results in [22]. The master equation used is valid in the regime in which the medium sees quarkonium as a small color dipole $rT \ll 1$. The impact of E/T has been studied with a focus on obtaining phenomenological results for bottomonium's R_{AA} and a discussion of thermalization has been postponed to future work. The study uses a three-dimensional master equation that takes into account the non-Abelian nature of QCD. It has been found that up to temperatures lower than 190 MeV the corrections induced by the first E/T correction is smaller than 50%. Another important result is that correlated recombination is needed to reproduce experimental results for excited states even if it is a very small correction for $\Upsilon(1S)$. A summary of the results for R_{AA} can be seen in fig. 2, where it can be observed the need of recombination to reproduce excited state data.

4. Conclusions

Heavy quarkonium is an excellent probe of the medium created in heavy-ion collisions. It allows to test the medium in a wide region of energy and length scales. On one hand, quarkonium is a non-relativistic system where a wide range of energy scales appear. On the other hand, there is a large variety of quarkonium species (bottomonium, charmonium, Bc, excited state...) each state having different size and binding energy. Some of the new developments in the fields are related with the study of new species of quarkonium like, for example, Bc states and exotics.

There are also many recent developments related with the application of the open quantum system formalism. There is now a detailed understanding of the connection of this approach with more traditional ones and we understand that quantum effects are important when the decay width is not a perturbation. There has been an interest by several groups on studying E/T corrections and their impact on thermalization and phenomenological results.

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