PROCEEDINGS OF SCIENCE

Condensation and early time dynamics in QCD plasmas

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High energy nuclear collisions produce far-from-equilibrium matter with a high density of gluons at early times. We identify for the first time two local order parameters for condensation, which can occur as a consequence of the large density of gluons. We demonstrate that an initial over-occupation of gluons can lead to the formation of a macroscopic zero mode towards low momenta that scales proportionally with the volume of the system—this defines a gauge invariant condensate. The formation of a condensate at early times has intriguing implications for early time dynamics in heavy ion collisions.

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1. Introduction

In high-energy nuclear collisions, far-from-equilibrium matter is produced that is expected to have a high density of gluons at early times. It has been demonstrated in [1] that an initial over-occupation of gluons in the deep infrared leads to the formation of a gauge-invariant condensate [2]. In this proceeding, we will present key results of our work [3]. We will introduce two local order parameters that are sensitive to infrared condensation, then we will extract the condensation observable and characterize the phenomenon.

2. Infrared order parameters

Infrared excitations of non-Abelian gauge theories can be computed from Wilson loops [4–7]. A previous study [1] showed that the spatial Wilson loop demonstrates the build-up of a macroscopic zero mode far from equilibrium that scales with the volume of the system. However, due to the fact that the Wilson loop is an extended object, it cannot be used in the formulation of an effective kinetic theory that can further unravel condensate dynamics.

In [3] we have considered a related object, the spatial Polyakov loop, which can be considered as a candidate degree of freedom with which to formulate a kinetic theory. As an observable for condensation, we use the two-point correlation function of these spatial Wilson loops that wind in a single spatial direction, akin to its temporal cousin that is an order parameter for confinement in thermal equilibrium. At sufficiently long distances, the two-point correlator of the spatial Polyakov loop is approximately equal to the expectation value of the Wilson loop.

Let us also introduce another candidate degree of freedom: a scalar field that is related to the spatial Polyakov loop. We can rewrite the untraced spatial Polyakov loop,

$$\tilde{P}_i(x) \equiv \mathcal{P} e^{-ig\int_0^L A_i(t,\mathbf{x})dx_i} = e^{i\phi_i(x)}, \tag{1}$$

such that the algebra element $\phi_i(x) = \phi_i^a(x)t^a$, where $t^a = \sigma^a/2$ are the generators of the gauge group and $\tilde{P}_i \in SU(N)$. We restrict ourselves to $SU(2)$ for the sake of simplicity, which allows us to introduce a gauge invariant scalar field φ_i via the relation,

$$\frac{1}{N_c}\mathrm{tr}\tilde{P}_i \equiv \cos\varphi_i. \tag{2}$$

This field is the holonomous eigenvalue of the spatial Polyakov loop. We elucidate its relation to the algebra element by noting that the untraced Polyakov loop transforms covariantly under the gauge transformation $\tilde{P}_i(x) \to U(x)\tilde{P}_i(x)U^\dagger(x)$ for $U(x) \in SU(2)$. It follows that the algebra element transforms similarly, $\phi_i(x) \to U(x)\phi_i(x)U^\dagger(x)$, for $U(x) \in SU(2)$. This transformation rotates the Polyakov loop into the Cartan subalgebra, and therefore

$$\phi_i(x) = \varphi_i(x)t^3. \tag{3}$$

For brevity, we refer to φ as the algebra field.

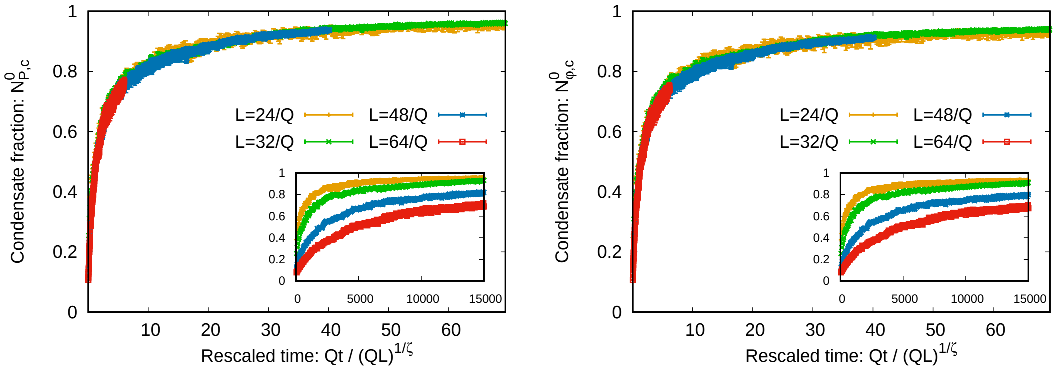

Figure 1: Condensate fractions (5) of the connected correlators $\langle PP^\dagger \rangle_c$ and $\langle \varphi\varphi^\dagger \rangle_c$ as functions of rescaled time for different lattice volumes. All curves fall on top of each other and therefore show the emergence of a volume-independent condensate fraction. The rescaled quantities have scaling exponents $\zeta = 0.31 \pm 0.09$ and $\zeta = 0.34 \pm 0.03$. *(Insets:)* Condensate fraction as a function of time, not rescaled, for the respective correlators.

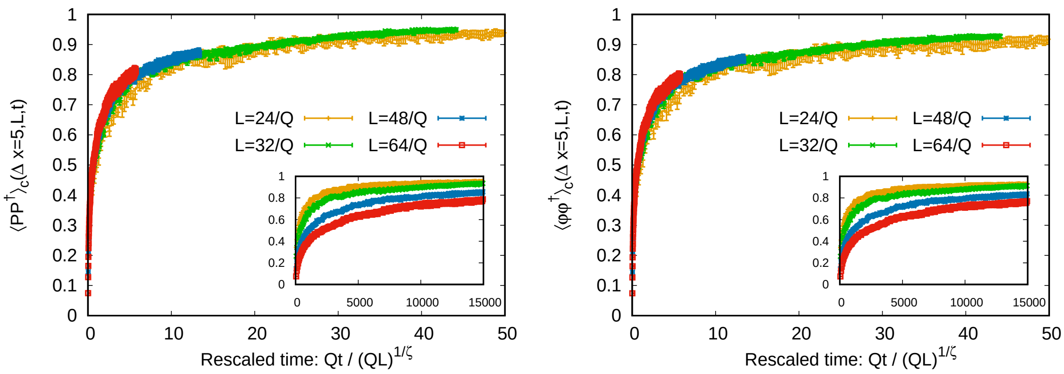

Figure 2: Rescaled connected correlators $\langle PP^\dagger \rangle_c$ and $\langle \varphi\varphi^\dagger \rangle_c$ normalized by their values at $\Delta x = 0$ for fixed $Q\Delta x = 5$, with a reduced time extent ($Qt \leq 15000$). The scaling exponents ζ are identical to those in Fig. 1. *(Insets:)* The same, not rescaled, for the respective correlators.

3. Condensation

Gluons produced in heavy ion collisions are expected to have typical momenta on the order of the saturation scale $Q \sim 1/\alpha_s$, and there is an over-occupation of gluons at time $t \sim 1/Q$. In this regime, the running gauge coupling is small, $\alpha_s \ll 1$. However, the system is strongly correlated due to the high occupancy of gluons. In this large-occupancy regime, the full dynamics of the theory is well-described by real-time classical-statistical lattice simulations. The gauge theory is discretized on a three-dimensional spatial lattice of length L and spacing a_s. Fields are initialized as a superposition of transversely polarized gluon fields. The characteristic initial over-occupation of gluons is translated into energy density and fluctuations to initialize the lattice gauge theory evolution. The real-time evolution is realized by solving the classical Heisenberg equations of motion in the temporal axial gauge, $A_0 = 0$.

We search for condensation in the system by comparing the fraction of the lattice volume that

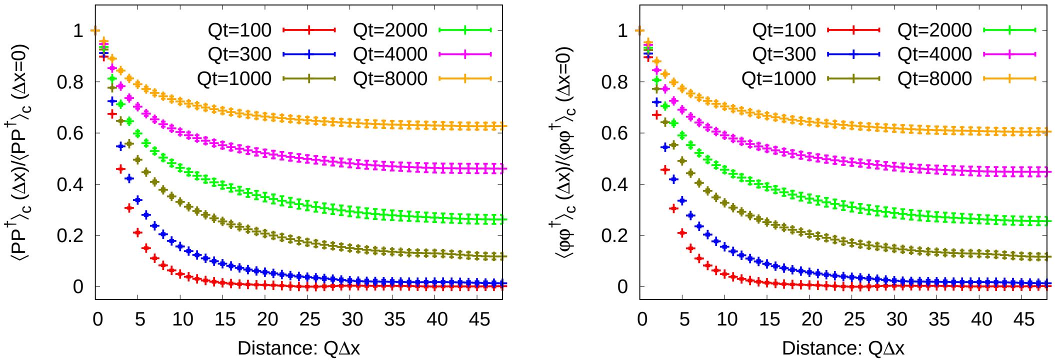

Figure 3: Time evolution of the normalized connected Polyakov loop correlator $\langle PP^\dagger\rangle_c(\Delta x)/\langle PP^\dagger\rangle_c(\Delta x{=}0)$ (left) and normalized connected algebra field correlator $\langle\varphi\varphi^\dagger\rangle_c(\Delta x)/\langle\varphi\varphi^\dagger\rangle_c(\Delta x{=}0)$ (right) shown for six times $Qt = 100, 300, 1000, 2000, 4000, 8000$ on a $N_s = 96$ lattice. The occurrence of condensation is demonstrated by the non-zero value of the correlator at later times.

is correlated for both the spatial Polyakov loop and the algebra field. For both cases we use the connected two-point correlation function,

$$\langle OO^\dagger\rangle_c(t,\Delta x,L) = \langle OO^\dagger\rangle(\Delta x) - \langle O\rangle\langle O^\dagger\rangle(\Delta x), \tag{4}$$

where $O = P,\ \varphi$. Then, the condensate fraction is defined by

$$N^0_{O,c} \equiv \frac{1}{L}\int_0^L d\Delta x \frac{\langle OO^\dagger\rangle_c(t,\Delta x,L)}{\langle OO^\dagger\rangle_c(t,\Delta x=0,L)}, \tag{5}$$

which contains the integral over the momentum modes divided by the zero mode. The integral is divided by an appropriate, system-dependent length element, which allows us to study how the condensate fraction grows over time independently with respect to the lattice size. We define a volume-independent condensate formation time following [1, 8],

$$t_{\rm cond} = (QL)^{1/\zeta}. \tag{6}$$

Rescaling time by $t/t_{\rm cond}$ gets rid of the remaining volume dependence in the evolution of the condensate fractions $N^0_{O,c}$. Volume independence indicates the formation of a condensate. This particular form of condensate formation, which is a genuinely far-from-equilibrium phenomenon, occurs in systems characterized by large occupation numbers and is often associated with an inverse particle cascade that occupies the zero mode. This is observed for relativistic and non-relativistic scalar theories [8, 9].

In Fig. 1, we show the emergence of a condensate with the connected Polyakov loop (left) and the connected algebra field correlators (right) computed using (5). In the insets of the figures, we show $N^0_{P,c}$ and $N^0_{\varphi,c}$ for four different lattice volumes as functions of time Qt, which yields a volume-dependent evolution. When we rescale these quantities by $t_{\rm cond}$, the curves become volume-independent, as illustrated by the collapse of the curves onto one another in Fig. 1. The exponent ζ in $t_{\rm cond}$ (6) characterizes the time scaling. We find for the condensate fraction of the connected Polyakov loop correlator, $\zeta = 0.31$, and for the connected algebra field correlator, $\zeta = 0.34$.

We then check the scaling of the connected correlators $\langle PP^\dagger \rangle_c$ and $\langle \varphi\varphi^\dagger \rangle_c$ in (4) at a fixed Δx, as shown in Fig. 2. Interestingly, both the Polyakov loop and the algebra field correlators show the same scaling as their integrated counterparts in Fig. 1. This is indicative that this scaling not only occurs at large distances, but at finite distances as well.

Finally, we show the time evolution for the connected Polyakov loop and algebra field correlators for a lattice of size $N_s = 96$ in Fig. 3. The connected correlators are plotted as functions of the distance Δx for six different times. For the earlier times $Qt = 100$ and 300, both correlators are zero at large distances. At the later times, we observe that the correlators are non-zero and grow in time. This indicates the formation and growth of a condensate.

4. Conclusion

We have identified gauge condensation in the early time dynamics of an $SU(2)$ gauge theory. This system qualitatively covers the dynamics of the initial stages of heavy ion collisions in which there is an over-occupation of gluons. The observables for identifying condensation are two two-point functions of spatial Polyakov loops and the scalar holonomous eigenvalue field, respectively. This will allow for the construction of an effective action in terms of the algebra field, which is naturally suited to describe infrared dynamics far from equilibrium. The algebra field is a local scalar field that could potentially be related to scalar Bose condensation in similar extreme systems. Therefore this is also an important step for the investigation of and understanding universality in systems far from equilibrium.

Acknowledgments

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