

Heavy quarks probe the equation of state of QCD matter in heavy-ion collisions

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We perform a state-of-the-art Bayesian statistical inference to extract the equation of state (EoS) and heavy quark transport coefficient of quark-gluon plasma (QGP) created in relativistic heavy-ion collisions. Based on our quasi-particle linear Boltzmann transport (QLBT) model combined with (3+1)-dimensional viscous hydrodynamic simulation of the QGP and a hybrid fragmentation-coalescence approach for heavy flavor hadronization, the QGP EoS and the heavy quark transport coefficient can be simultaneously constrained from the D meson R_{AA} and v_2 data at RHIC and the LHC, both consistent with the lattice QCD results. Our study provides a novel perspective on constraining the QGP EoS from hard probe observables.

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1. Introduction

Heavy quarks, as an ideal and clean hard probe of Quark Gluon Plasma (QGP) in relativistic heavy-ion collisions, has been extensively studied. Since the masses of heavy quarks are much larger than the energy scale of QCD and the typical temperature of the QGP, it is generally believed that heavy quarks are mainly produced from the primordial hard scatterings at the early stage of nuclear collisions, and can be systematically calculated via the analytical perturbative QCD method. Another advantage of heavy quarks is that due to their flavors conserved, they will experience the entire evolution process of fireball by interacting with the QGP medium.

One of the central tasks of heavy-ion physics is to probe the equation of state (EoS) and transport properties of QCD medium produced in relativistic nucleus-nucleus collisions. One common strategy for determining transport coefficients is to compare the numerical calculations from the mult-stage models with the experimental data on various observables. For example, the JET Collaboration performed the classical χ^2 -fit statistical method to extract jet transport coefficient \hat{q} from the nuclear modification factor R_{AA} of single inclusive hadrons at high p_T at RHIC and the LHC [1]. Recently, tremendous effort has been devoted to extracting the average values and distributions of transport coefficients by utilizing the Bayesian inference technology, such as the temperature-dependent shear and bulk viscosities by JETSCAPE Collaboration [2]. The EoS of the QGP medium can also be successfully determined from the Bayesian method via the spectra of low p_T identified particles [3]. In the above studies [1–3], the parameterized EoS and transport coefficients are extracted from soft bulk observables, while jet transport coefficients are extracted from hard observables. Here we present our recent work [4], in which by combining the quasi-particle linear Boltzmann transport (QLBT) model [5] with the (3+1)-dimensional CLVisc hydrodynamics simulation [6, 7] and hybrid fragmentation-coalescence hadronization approach [8], we provide an alternative strategy to extract the QGP EoS and transport coefficients via Bayesian inference using heavy flavor observables.

2. Framework

In our study, we utilize the QLBT model to simulate the dynamic evolution of heavy quarks through the QGP medium. In the QLBT model, the strongly-interacting massless partons in the QGP medium are replaced with the non-interacting massive quasi-particles with temperature-dependent masses [9],

$$m_q^2(T) = \frac{N_c^2 - 1}{8N_c} g^2(T) T^2, m_g^2(T) = \frac{1}{6} \left(N_c + \frac{1}{2} N_f \right) g^2(T) T^2. \quad (1)$$

Here q , g denote the thermal quarks and gluons; N_c and N_f are the number of color and flavor factors. The temperature-dependent coupling $g(T)$ is the key quantity in the QLBT model as it directly relates to the QCD EoS. Inspired by the perturbative QCD calculation, our $g(T)$ is parameterized as:

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f) \ln \left[(aT/T_c + b)^2 / (1 + ce^{-d(T/T_c)^2}) \right]}. \quad (2)$$

Here T_c is the transition temperature from the QGP phase to the hadronic phase; a, b, c, d are free parameters that can be used to constrain the QCD EoS from experiment observables. For the dynamical evolution of heavy quarks within the QGP medium, QLBT model numerically solves the following relativistic Boltzmann equation based on the Monte Carlo method [10–15]:

$$p_1 \cdot \partial f_1(x_1, p_1) = E_1 (C_{\text{el}} [f_1] + C_{\text{inel}} [f_1]). \quad (3)$$

Here C_{el} and C_{inel} denote the collision terms from the contributions of elastic and inelastic scatterings between heavy quarks and the QGP constituents. In the elastic process, the scattering rate $\Gamma_{12 \rightarrow 34}(\vec{p}_1)$ is calculated as:

$$\begin{aligned} \Gamma_{12 \rightarrow 34}(\vec{p}_1) &= \frac{\gamma_2}{2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \int \frac{d^3 p_3}{(2\pi)^3 2E_3} \int \frac{d^3 p_4}{(2\pi)^3 2E_4} \\ &\times f_2(\vec{p}_2) [1 \pm f_3(\vec{p}_3)] [1 \pm f_4(\vec{p}_4)] S_2(s, t, u) \\ &\times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) |M_{12 \rightarrow 34}|^2. \end{aligned} \quad (4)$$

Here 1,3 label the heavy quarks and 2,4 label the thermal partons. γ_2 is the spin-color degeneracy factor. $|M_{12 \rightarrow 34}|$ is the matrix element taken from the linear order pQCD 2->2 calculations. $S_2(s, t, u)$ is the kinematic constraint to avoid the divergence for small-angle scattering cases with a regulated Debye mass $\mu_D^2(T) = 2m_g^2(T)$. In the current work, the thermal parton masses are included in the distribution function f_i and kinematic constraint $S_2(s, t, u)$. For the inelastic scattering process, the medium-induced gluon radiation is calculated using the higher-twist energy loss formalism [16, 17]. To simulate the interaction of heavy quarks with the QGP medium, the coupling constant $\alpha_s(E) = g^2(E)/(4\pi)$ connected to heavy quarks is parameterized as follows:

$$g^2(E) = \frac{48\pi^2}{(11N_c - 2N_f) \ln[(AE/T_c + B)^2]}, \quad (5)$$

where A and B are two parameters, while the coupling constant $\alpha_s(T) = g^2(T)/(4\pi)$ connected to the thermal partons is set as a function of the local temperature of QGP medium, which can be obtained from (3+1)-D CLVisc hydrodynamics simulations with smooth AMPT initial conditions [6, 7]. The initial momentum distributions of heavy quarks are simulated with the LO perturbative QCD calculations, and the initial production positions are estimated from the locations of binary collisions in the Monte-Carlo Glauber model.

Next, we perform the Bayesian analysis to extract the six parameters (a, b, c, d, A and B) in the parameterized coupling constants to constrain the QCD EoS using the heavy flavor R_{AA} and v_2 data at RHIC and the LHC. To speed up the QLBT simulation, 815 parameter sets for Hot-QCD (HQ) case with $T_c = 154$ MeV and 821 sets for Wuppertal-Budapest (WB) case with $T_c = 150$ MeV are used to train the Gaussian emulator. Therefore, the error estimation in the likelihood in Bayesian analysis should consider the combined contributions from experimental measurements and the Gaussian emulator.

3. Results

In Fig. (1), we show the model calibration of the QLBT calculation with $T_c = 154$ MeV in Hot-QCD case based on the D meson R_{AA} and v_2 experimental data at RHIC [21, 22] and LHC [23, 24].

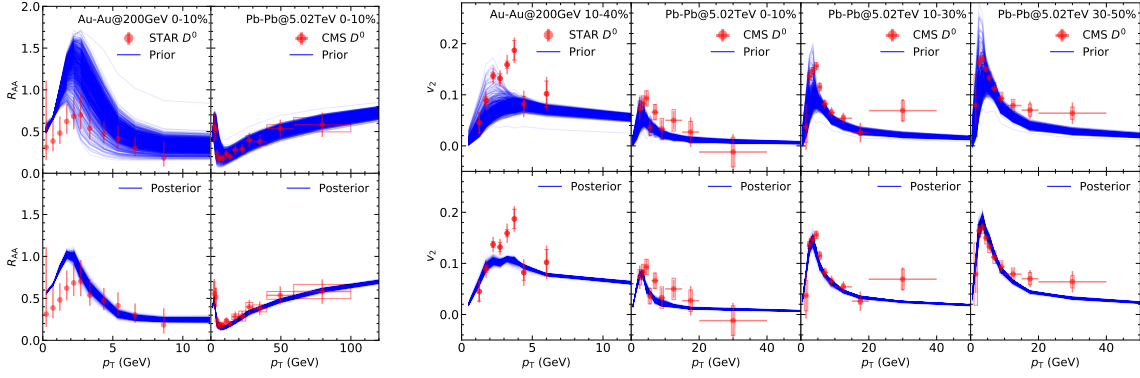


Figure 1: QLBT model calibration for Hot-QCD case with $T_c = 150$ MeV from the D meson R_{AA} and v_2 data at RHIC [21, 22] and the LHC [23, 24]. Upper panel: QLBT calculation from prior parameter sets. Lower panel: QLBT model calculation of R_{AA} and v_2 using the posterior parameter sets after the calibration.

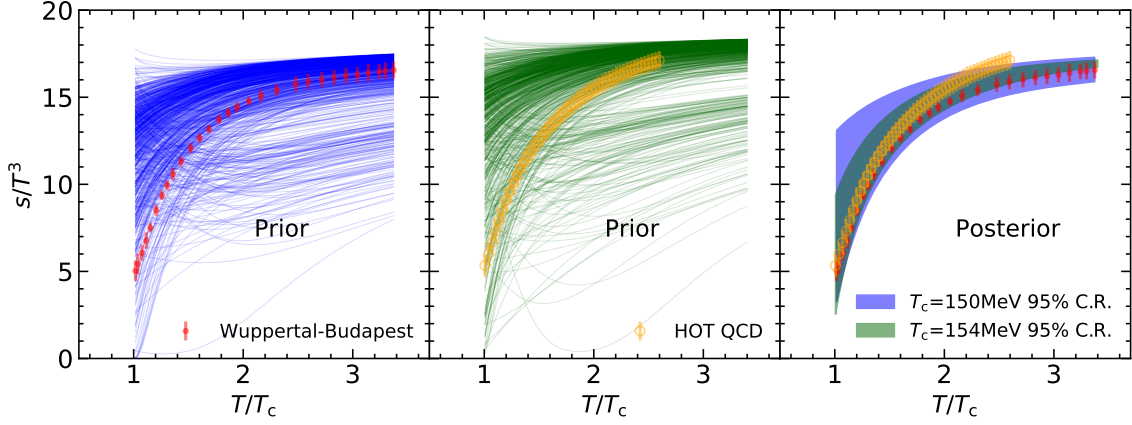


Figure 2: The left two panels show the prior distributions. The right panel shows T^3 -scaled entropy density from posterior range constrained by the heavy flavor observables. The QCD EoS from Hot-QCD and Wuppertal-Budapest lattice data are also shown for comparison [18, 19].

The WB case is very similar and not shown here. The upper panel shows the calculation using the prior distribution of parameter sets of the QLBT model. The lower panel shows the QLBT calculation using the posterior distributions after Bayesian analysis. One can see that our model calculation can agree well with D meson R_{AA} and v_2 data at both RHIC and LHC energies.

Once the parameter sets are determined after model calibration through Bayesian analysis, the entropy density of the non-interacting quasi-particle system can be obtained, as shown in Fig. (2) shown. The left and middle panels show the results using the prior parameters compared to the WB and Hot-QCD lattice data [18, 19]. The right panel shows the posterior scaled entropy density under 95% Credible Region. It is found that the prior distributions of the EoS converge into two bands. Another interesting observation is that the posterior QCD EoS (both the width and magnitude of posterior bands) is sensitive to the value of transition temperature T_c . Note that there is clear difference between HQ and WB lattice QCD results. In the future, we will remove the restriction on T_c and treat it as a free parameter to be constrained from the experimental data.

Furthermore, the transport coefficients of heavy quarks and the shear viscosity of the quasi-

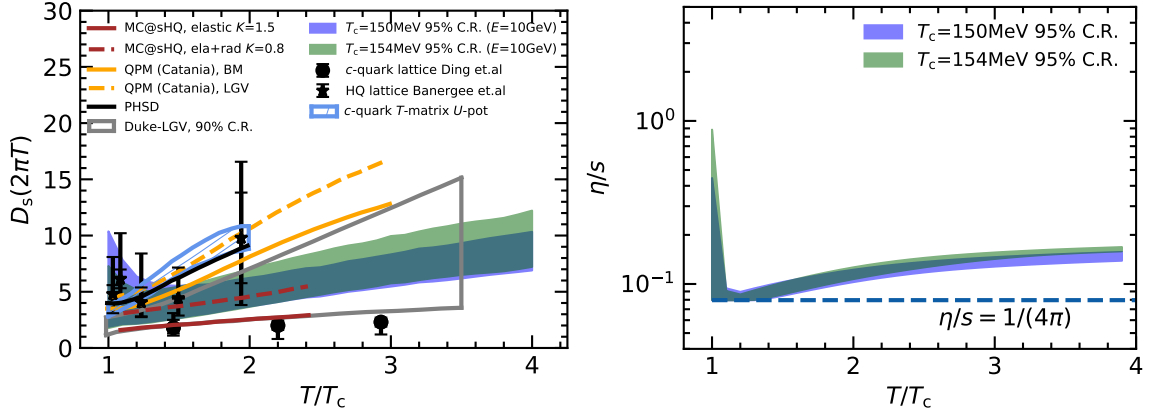


Figure 3: The spatial diffusion coefficient D_s of charm quarks (left) and the specific shear viscosity η/s (right) as a function of the medium temperature for two different T_c scenarios after QLBT model calibration. Other model calculations of charm quark diffusion coefficient D_s are taken from Ref. [20].

particle system can be also simultaneously constrained after our QLBT model calibration, as shown in Fig. (3). The left plot shows the temperature-dependent spatial diffusion coefficients $D_s(T)$ for heavy quarks with energy $E = 10$ GeV, compared with various results in the literature. The right plot shows the temperature-dependent specific shear viscosity η/s . We have checked that using the posterior shear viscosity in the hydrodynamics simulation, we obtain similar final soft hadron spectra and elliptic flow as compared to the hydrodynamics calculation using a constant specific shear viscosity $\eta/s = 0.08$.

4. Summary

Based on our QLBT model combined with the (3+1)-dimensional CLVisc hydrodynamics simulation and a hybrid fragmentation-coalescence hadronization approach, we have performed a Bayesian analysis on the heavy favor R_{AA} and v_2 data at RHIC and the LHC and simultaneously extracted the QGP EoS, specific shear viscosity and heavy quark transport coefficient in the same model. It is found that the posterior EoS constructed from a quasi-particle system is in qualitative agreement with previous lattice QCD data. The extracted spatial diffusion coefficient of heavy quarks also shows similar temperature dependence as compared to the other model calculations and the lattice QCD data. In the future, we would like to release the limitations on the transition temperature T_c and perform more comprehensive analysis, aiming to obtain more accurate extraction of the EoS and transport coefficients of QGP produced in relativistic heavy-ion collisions.

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