Quarkonia dynamics in the Quark-Gluon Plasma with a quantum master equation

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We study the dynamics of a single quark-antiquark pair in the Quark-Gluon Plasma (QGP) within the open quantum system framework. The dynamics are governed by coupled partial differential equations describing the evolution of the pair density operator in the singlet and octet color channels. We study both the charmonium and bottomonium system in one dimension, using a one-dimensional potential specifically developed for this study and tailored to capture as best as possible characteristics of a full three-dimensional potential [1]. A first study of the suppression of bottomonia as function of the centrality, using EPOS4 temperature profiles for the QGP medium is presented.
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1. Introduction

One of the key observables in the study of the Quark-Gluon Plasma is the so-called quarkonia suppression, characterized by a lower production of quarkonium states in nucleus-nucleus collisions compared to proton-proton collisions. Several mechanisms are at play to explain the observed suppression. Inside the QGP medium, the high number of color charges leads to the screening of the interaction between the quark and antiquark forming a quarkonium, which can lead to the dissociation of the pair if the screening is too important (i.e. if the temperature is higher than the dissociation temperature of the considered quarkonium state). Alongside this "static" screening, the interactions between the pair and the medium light constituents can also lead to dissociation (for example via gluo-dissociation). Those two mechanisms are usually encoded inside a complex potential, with the real part describing the static screening and the imaginary part describing the pair-medium interactions. However, at high energy colliders such as the RHIC and the LHC, the increased production of \( c\bar{c} \) or \( b\bar{b} \) pairs can counterbalance the dissociation of pairs. Indeed, new pairs can form through recombination, leading to a smaller suppression than one would expect with increasing collision energy. This recombination can be of two natures: either a dissociated pair recombine ("diagonal" recombination) or a new pair is formed from two originally uncorrelated quarks ("off-diagonal" recombination \(^1\)). The treatment of the full dynamics is thus crucial for phenomenology. In recent years, a large effort was made to describe the dynamics of quarkonia in the Quark-Gluon Plasma within the open quantum systems framework. In the framework, quantum master equations can be derived, describing the evolution of a quarkonium inside the QGP through its density operator.

2. An open quantum system model

Our model relies on quantum master equations, which are extensions of quantum master equations originally derived by Blaizot&Escobedo [2], describing the dynamics of a single \( Q\bar{Q} \) pair in the QGP. The equations are derived in the quantum brownian regime (which is valid at high temperature) and assume a weak coupling between the pair and the medium constituents, which is described using Non-Relativistic QCD (NRQCD). Those master equations can be put in the following form:

\[
\frac{d}{dt} \begin{pmatrix} D_s(s,s') \\ D_o(s,s') \end{pmatrix} = L \begin{pmatrix} D_s(s,s') \\ D_o(s,s') \end{pmatrix},
\]

(1)

with \( D_s \) and \( D_o \) the density operators in the singlet and octet color channels and \( s \) the relative distance between the quark and antiquark of the pair (\( s' \) is the conjugated variable). \( L \) is a super-operator describing the different color transitions and can be decomposed in several sub-operators \( L = L_0 + L_1 + L_2 + L_3 + L_4 \). \( L_0 \) describes the pair free dynamics, while \( L_1 \) describes the static screening due to the medium. The \( L_2 \) to \( L_4 \) operators describe the interactions between the pair and the constituents of the medium, with the \( L_4 \) operator being an extension to the original equations from Blaizot&Escobedo, which ensures the conservation of the density operator positivity.

\(^1\)This effect should be weaker for bottomonia as the number of b quarks produced is much lower.
The singlet-octet transitions describe the dissociation or recombination effects in the medium. The equations are solved numerically in one dimension for both charmonia and bottomonia on a numerical grid of 501x501 points. The $s$ and $s'$ variables go from -10 to 10 fm with a spatial step $\Delta s = 0.04$ fm and a time step $\Delta t$ of 0.1 fm/c. The different $L_i$ sub-operators involve a complex potential, which in our model is a potential developed specifically for one-dimensional studies and aims at reproducing as best as possible the temperature dependant mass spectra and decay widths of quarkonium states obtained from [1].

3. Charmonium dynamics

We first study the charmonium system and its dynamics. We will consider two different mediums: a medium with fixed temperature $T = 400$ MeV or a medium with a time-dependent uniform temperature profile following a Björken profile. We take as initial state the ground state of our vacuum potential (1S-like state) in the singlet color channel. The initial singlet density operator is thus the projector on the ground state and the initial octet density operator is the empty matrix.

![Figure 1: Evolution of the singlet density operator $D_s$ (top panels) and octet density operator $D_o$ (bottom panels) over time for a fixed temperature medium ($T = 400$ MeV) and a 1S-like singlet initial state. From left panel to right panel: $t = 0.1$, 5 and 10 fm/c. The color scale changes from plot to plot for better readability.](image)

Figure 1 shows the evolution over time of the singlet and octet density operators. At early times, due to the dipolar nature of the color-changing transitions, the octet channel is populated by a P-like component. As time progresses, the initial state gets progressively delocalized alongside the $s = s'$ axis, and the density matrix becomes almost fully diagonal in coordinate space, indicating that decoherence occurs. Around $s = s' = 0$ fm, a correlation remains in the singlet channel, which is a remnant of the initial state still under the influence of the potential of the pair.

The left panel of Figure 2 shows the evolution over time of the populations of the vacuum eigenstates, defined as the instantaneous projections of the singlet density matrix on the different

\[^2\text{As only one pair is treated, only diagonal recombination is included.}\]
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Figure 2: Evolution over time of the populations of the first three eigenstates of the vacuum $c\bar{c}$ potential for a 1S-like singlet initial state. (Left panel) Fixed temperature medium ($T = 400$ MeV) (Right panel) Cooling medium ($T_0 = 400$ MeV)

eigenstates $P_n(t) = \langle \Psi_n | D_s(t) | \Psi_n \rangle$. The early evolution is characterized by a re-equilibration of the population. The 2S-like state is first populated followed by the 1P-like state. This delay is again explained by the singlet-octet transitions that are parity-changing, therefore it takes more time to obtain a 1P-like singlet state from the initial 1S-like singlet state. The late-time evolution is characterized by a decay phase, with a common decay rate for all states. We also consider a dynamical medium whose temperature (supposed spatially uniform) follows a Björken profile: $T(t) = T_0 \left( \frac{1}{1+t^{-1/3}} \right)$, with $T_0 = 400$ MeV. The right panel of Figure 2 also shows the evolution over time of the population of the first three eigenstates of the vacuum potential. The global evolution is similar to the fixed temperature medium case with several differences: The 1S-like state is less suppressed, due to the cooling of the medium and there is a clear inversion of the 1P-like and 2S-like populations, which was also present but less pronounced in the fixed temperature case. This gap between the populations of the 1P-like and 2S-like states can be also explained by the cooling of the medium, which slows down the evolution.

4. Bottomonium dynamics

We now study the bottomonium system. We will consider two different initial states, all in the singlet color channel: the $\Upsilon(1S)$-like state (ground state) or a mixture of both S and P states, defined as: $\Psi(x) \propto (1 + a_{odd} \frac{x}{\sigma}) e^{-\frac{x^2}{2\sigma^2}}$, with $a_{odd} = 3.5$ and $\sigma = 0.045$ fm. We will also consider two types of QGP medium: one with fixed temperature, and one following averaged temperature profiles obtained from EPOS4, for three different centrality classes: 0-10%, 20-30% and 40-50%. The EPOS4 profiles correspond to a rapidity interval $|y| < 2.4$, which is the rapidity interval of the CMS experiment.

Figure 3 shows the evolution of the populations of the first six vacuum eigenstates for each of the three different initial states. For the $\Upsilon(1S)$-like state, the global evolution is similar to the charmonium system one, with less suppression due to the higher binding energies of the
bottomonium states. In the case of a mixture of S and P states, the evolution is different, with excited states populations decreasing rapidly at early times. The \( \Upsilon(1S) \)-like evolution is also very similar to its evolution when it is the initial state of the system. To study the dependence of the dynamics on the centrality of the collision, we take an initial state constructed from the mixture of S and P state in the singlet color channel and let it evolve inside different mediums with temperature profiles corresponding to different centrality classes. Figure 4 shows the evolution of the populations of eigenstates for the different centrality classes. The global evolution is the same in all cases and is similar to what was seen in the fixed temperature case. We observe the decrease of the suppression when going to more peripheral collisions, which is what one would expect. However, the suppression of the \( \Upsilon(1S) \)-like state seems to be weaker than expected while the suppression of the \( \Upsilon(2S) \)-like state seems too strong. This could be explained by the effect the imaginary part of our potential being too strong, which can lead to an increased dissociation of excited states.

5. Conclusion

We studied the dynamics of a single quarkonium pair in the Quark-Gluon Plasma described by quantum master equations obtained in the open quantum systems formalism. Those equations describe both dissociation and diagonal recombination and are solved numerically in one-dimension. The evolution of the system is characterized by the delocalization of the initial density operator along the \( s = s' \) axis and its decoherence due to the presence of the QGP. An interesting feature is that for a given system (either charmonium or bottomonium), the evolution leads to similar final states, which indicate that the dependence on the initial state is washed by the medium influence. We observe a decrease of the suppression for bottomonia for more peripheral collisions, which is also seen in experiments, even though the suppression is too strong or weak depending on the state considered. A possible explanation is that the effect of the imaginary part of the potential is too
Figure 4: Evolution over time of the populations of the first three eigenstates of the vacuum $b\bar{b}$ potential for a cooling medium and a $\Upsilon_{1S}$-like singlet initial state. (Top left panel) 0-10% profile (Top right panel) 20-30% profile (Bottom panel) 40-50% profile

strong in our simulations. This hypothesis is currently under study and will be explored in a future publication.

References

