

## Evolution of initial state fluctuations in the hotspot model of the proton structure

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The hotspot model has proven to be an efficient tool to study coherent and incoherent diffraction HERA data by modelling the initial state of the proton. The hotspot model in its original form is a non-perturbative model applicable for low momentum transfer and underestimates the incoherent cross section in orders of magnitude when extended for large momentum transfer studies for  $J/\psi$  photo-production at HERA. We present here a model of hotspot splittings based on the resolution for the evolution of the initial state of the proton. The incoherent diffraction at large momentum transfer probes the gluon wave function at smaller length scales as we increase the resolution which appears as hotspot splittings in our model. In addition to the geometrical fluctuations, we have additional sources of fluctuations in our model namely the hotspot width, number, and normalisation fluctuations which leads to a good agreement of our model's prediction with data.

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## 1. Introduction

Diffractive events in deeply inelastic scattering (DIS) events serve as an excellent probe for investigating the transverse structure of hadrons. The coherent and incoherent diffractive events observed in the H1 and ZEUS experiments at HERA have played a crucial role in understanding the spatial gluon distribution of the proton and its fluctuations within the Good Walker formalism [1]. In recent years, there has been significant interest in studying the transverse nucleon structure and its fluctuations, as they are highly relevant for establishing realistic initial conditions in the study of particle production in heavy-ion AA collisions, as well as in smaller systems such as pA and pp collisions at RHIC and LHC. Measuring the multi-dimensional structure of the protons and nuclei is one of the primary goals of the upcoming electron ion collider (EIC) at US [2, 3]. We still lack a complete understanding of the spatial distribution of gluons within hadrons at high resolutions and high energies. It remains unclear how their density varies and whether their distribution is isotropic. Understanding the spatial distribution and spectrum of gluon fluctuations at these scales is crucial for gaining a comprehensive understanding of the complex structure of hadrons and advancing our knowledge of quantum chromodynamics (QCD). In the dipole picture, the impact parameter serves as a Fourier conjugate to the momentum transfer at the target vertex. This allows for the investigation of the transverse structure of the target by examining cross sections that are differential with respect to the Mandelstam  $t$  variable. However, these differential cross sections are only experimentally accessible in exclusive diffractive events. The Hotspot model has emerged as a successful phenomenological model for describing the gluon distribution in proton geometry and its event-by-event fluctuations [4]. However, it has limitations and is applicable only at low momentum transfers. When extended to studies involving large momentum transfers, such as  $J/\psi$  photo-production at HERA, the Hotspot model significantly underestimates the incoherent cross section by several orders of magnitude.

In the dipole picture [5], the amplitude describing the exclusive vector meson production is given by the convolution of three sub-process, first the photon splits into quark-antiquark pair forming a dipole, then the dipole interacts with proton elastically via strong interaction and finally the dipole forms the final state.

$$\mathcal{A}_{T,L}(x_{\mathbb{P}}, Q^2, \Delta) = i \int d^2\mathbf{r} \int d^2\mathbf{b} \int \frac{dz}{4\pi} (\Psi^* \Psi_V)_{T,L}(Q^2, \mathbf{r}, z) e^{-i[\mathbf{b} - (\frac{1}{2}-z)\mathbf{r}] \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, x_{\mathbb{P}}) \quad (1)$$

The dipole cross-section in the saturated dipole model (bSat) is given by [6]:

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, x_{\mathbb{P}}) = 2 \left[ 1 - \exp\left(-\frac{\pi^2}{2N_C} \mathbf{r}^2 \alpha_s(\mu^2) x_{\mathbb{P}} g(x_{\mathbb{P}}, \mu^2) T_p(\mathbf{b})\right) \right] \quad (2)$$

and the linearised version of this dipole cross section is called the bNonSat model.

### 1.1 Fluctuations & The Hotspot Model

Diffractive events can be classified into two types: coherent events, where the target remains intact, and incoherent events, where the target dissociates. In the Good-Walker formalism [7], the coherent cross section is determined by the average interaction of the states that diagonalize the scattering matrix with the target. Thus, the coherent cross section provides information about the

average geometry of the target. On the other hand, the incoherent cross section is obtained by taking the difference between the second and first moments of the amplitude. It is sensitive to various kinds of fluctuations present in the target's wave function.

$$\frac{d\sigma_{\text{incoherent}}}{dt} = \frac{1}{16\pi} \left( \langle |\mathcal{A}(x_{\mathbb{P}}, Q^2, \Delta)|^2 \rangle_{\Omega} - |\langle \mathcal{A}(x_{\mathbb{P}}, Q^2, \Delta) \rangle_{\Omega}|^2 \right) \quad (3)$$

In a study by Mantysaari and Schenke [4], they introduced geometrical fluctuations in the proton wave function, primarily caused by event-by-event variations in the positions of the hotspots of spatial gluon density. In their simplistic model, they considered three hotspots, as motivated by the fact that the spatial gluon cloud forms around the valence partons. The transverse profile of the proton in the hotspot model is given by,

$$T_p(\mathbf{b}) = \frac{1}{2\pi N_q B_q} \sum_i^{N_q=3} e^{-\frac{(\mathbf{b}-\mathbf{b}_i)^2}{2B_q}} \quad (4)$$

The hotspot model provides a good description of incoherent  $J/\psi$  production in diffractive  $ep$  events within the momentum transfer range of  $0 \leq |t| \leq 2.2 \text{ GeV}^2$ . However, it underestimates the cross section at large momentum transfers, as demonstrated in Fig. 2. This discrepancy indicates that fluctuations at large momentum transfers are missing in the model. To address this, additional small-scale fluctuations were introduced in [8] by incorporating more substructure into the hotspots manually. This modified model successfully described all available data [9, 10] up to  $t \leq 30 \text{ GeV}^2$ . In this context, we propose a more dynamic model, called *the hotspot evolution model*, which combines the hotspot model with fundamental principles of quantum mechanics and optics.

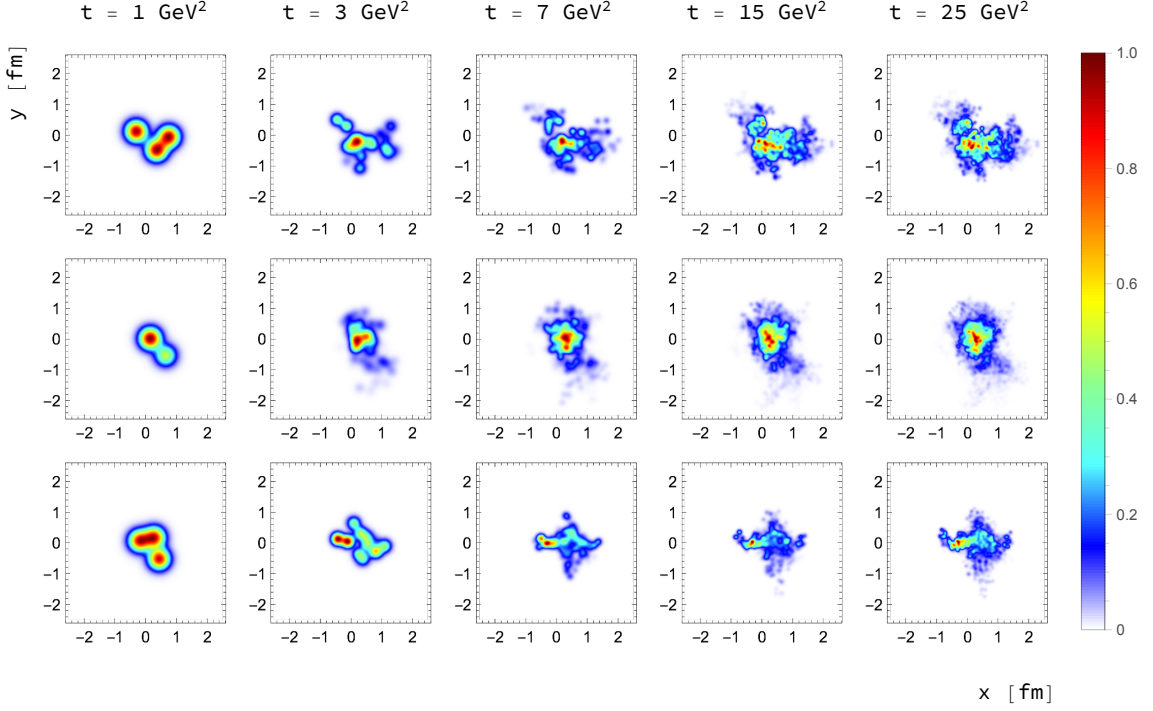
## 2. The Hotspot Evolution Model

The Mandelstam variable ' $t$ ', which represents the momentum transfer at the target vertex, serves as a measure of resolution in exclusive measurements. The key concept behind this model is that the transverse part of the gluon wavefunction is probed with an areal resolution  $\delta b^2 \sim \frac{1}{|t|}$ , and the increased resolution appears as hotspot spitting analogous to optics where at increased resolution one starts seeing additional features and can distinguish between two closely spaced objects.

Momentum transfer  $\leftrightarrow$  Resolution in optics

- *Hotspot model as the initial state for  $t = t_0$*
- *Evolution of the initial state as splitting of the hotspots based on the resolution for  $t > t_0$*

Now for the evolution of initial state and splitting of the hotspots in the evolution we need to define a probability splitting function for which we have some hints from the scaling behavior in our previous study of  $J/\psi$  production at large momentum transfer. We refer to [8] for more details on the scaling behavior for the number of hotspots. The actual probability of splitting ( $P_a$ ) is a product of probability of splitting ( $P_{\text{split}}$ ) and probability of no-splitting ( $P_{\text{no-split}}$ ).  $P_{\text{no-split}}$  needs to be taken into account as in evolution a particular hotspot will survive for some time before splitting into



**Figure 1:** Evolution of the initial state of proton in three different configurations of the proton in the hotspot evolution model. The proton thickness function  $T_p(b, t)$  is shown in the transverse plane for increasing momentum transfer ( $|t|$ ).

two offspring hotspots similar to the decay problem in nuclear physics. We consider two different models for evolution where in the Model 1 we have,

$$\frac{d\mathcal{P}_{split}}{dt} = \frac{\alpha}{|t|}, \quad \frac{d\mathcal{P}_{no-split}}{dt} = \exp\left(-\int_{t_0}^t dt' \frac{d\mathcal{P}_{split}}{dt'}\right) \quad (5)$$

$$\frac{d\mathcal{P}_a}{dt} = \frac{\alpha}{t} \left(\frac{t_0}{t}\right)^\alpha \quad (6)$$

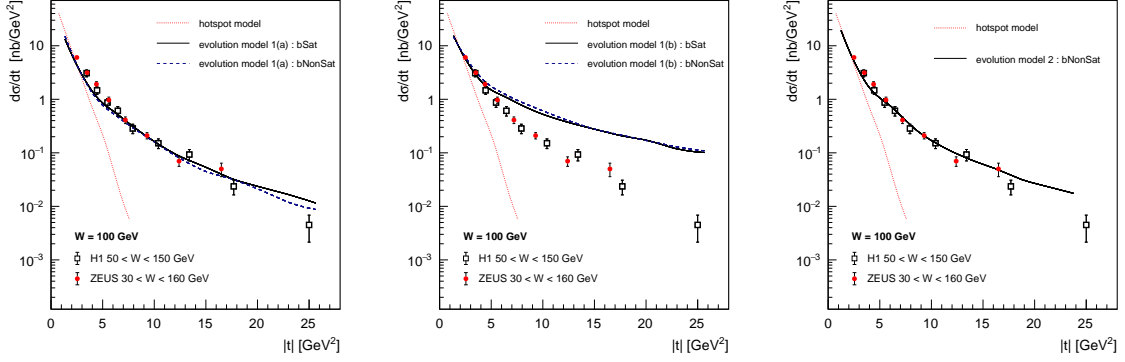
and in Model 2,:

$$\frac{d\mathcal{P}_{split}}{dt} = \frac{\alpha}{|t|} \frac{t-t_0}{t}, \quad \frac{d\mathcal{P}_{no-split}}{dt} = \exp\left(-\int_{t_0}^t dt' \frac{d\mathcal{P}_{split}}{dt'}\right) \quad (7)$$

$$\frac{d\mathcal{P}_a}{dt} = \frac{\alpha}{t} \frac{t-t_0}{t} \exp\left[-\alpha \left(\frac{t_0}{t} - \ln \frac{t_0}{t} - 1\right)\right] \quad (8)$$

where  $t_0$  is the initial scale and  $\alpha$  is the evolution parameter and will be determined through fit to the data. The underlying steps for the evolution of the initial state are as follow:

- offspring hotspots  $i, j$  created at distance  $d_{ij} = |b_i - b_j|$  sampled from parent hotspot with widths  $B_{i,j} = \frac{1}{|t|} \text{GeV}^{-2}$
- Probe & geometry resolution criterion :  $d_{ij} > 2\sqrt{B_{i,j}}$   
Reject if not resolved (effective hotspot repulsion)



**Figure 2:** The  $|t|$  dependence of incoherent  $J/\Psi$  photoproduction in different variants of the hotspot evolution models as compared to the HERA data from [9, 10]

The evolution of the proton profile for three distinct events within the hotspot evolution model is depicted in Fig. 1. At large momentum transfers, we observe the emergence of additional features corresponding to small-scale fluctuations. It is worth noting that our depiction of the proton at very high momentum transfers exhibits similarities to the IP-GLASMA model's description of the initial state where the proton structure is characterized by point-like color charges superimposed on top of the geometric hotspot structure [1].

### 3. Results & Conclusion

In Figure 2, we present a comparison between our model results and the experimental data from HERA on incoherent  $J/\psi$  production at large momentum transfers, considering both saturated and non-saturated versions of the dipole model. The first two sub-figures illustrate the implementation of the splitting function in Model 1 with the parameters as  $t_0 = 1.1\text{GeV}^2$  and  $\alpha = 2$ , which differs in terms of how the overall normalization is distributed. In Model 1 (a), we calculate the number of hotspots contributing at a specific instant of  $t$  and divide the number of gluons equally among all the hotspots. On the other hand, Model 1 (b) divides the gluons during the evolution, ensuring that the offspring hotspots always carry half the number of gluons compared to the parent hotspot. Although Model 1 (b) is more physically motivated, we observe that it introduces too many fluctuations due to long lived hotspots in the evolution. Consequently, we consider a more realistic model, referred to as Model 2, which suppresses these long lived hotspots and exhibits a peak centered at the instant value of  $t$  of the creation of offspring hotspots, resulting in more controlled offspring widths that fluctuate around  $1/|t|$ . The parameter values in model 2 are  $t_0 = 1.1\text{GeV}^2$  and  $\alpha = 18.5$  respectively. This model provides a good description of the experimental data and as compared to the original hotspot model there are additional sources of fluctuations in our models namely hotspot number, width and normalisation fluctuations. Furthermore, we note that both the saturated and non-saturated versions of the model yield good descriptions of the data, making it challenging to observe non-linear effects through this channel. However, it would be interesting to investigate the effects of this initial state evolution for nucleons in heavy-ion geometry, where enhanced saturation effects can be expected due to hotspots becoming even hotter as a result of a large number of gluons.

We have presented a model for the proton geometry which computes the incoherent cross section for vector meson production at large momentum transfer which is crucial to understand the very fundamental nature of fluctuations in the proton wavefunction and can be used for background estimations of coherent cross sections at large momentum transfer for future EIC. The EIC has a lot of potential for deciphering the transverse structure's complexity and revealing important details about the nature of strong force interactions.

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