

Effects of exotic solid-like matter in the post-inflationary universe

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This brief paper provides a new analysis of observational restrictions on inflationary and post-inflationary cosmological models based on results including the solid inflation model, as well as its combination with single scalar field. The first part of the paper shows that scalar spectral index and tensor-to-scalar ratio produced during inflation in the studied model are in a good agreement with the CMB observations. The second part of the paper deals with reheating scenario, so called solid remnant era with solid-like matter. A problematic feature is the possibility of superluminal propagation of perturbations, which considerably restricts the parameter space of studied models. Assuming constant pressure to energy ratio w , this superluminality is avoided for $w < (19 - 8\sqrt{7})/3 \doteq -0.722$, and the behavior of scalar, vector, and tensor perturbations considerably differs from the case with perfect fluid. There is increase in the vector-to-scalar ratio during the solid remnant era, $s \propto a^{-(1+3w)}$, while the tensor-to-scalar ratio remains unchanged. This illustrates possible challenges with comparing the observational data to models similar to solid inflation.

*XVIII International Conference on Topics in Astroparticle and Underground Physics (TAUP2023)
28.08_01.09.2023
University of Vienna*

*Speaker

1. Solid-like matter in cosmology

Solid inflation [1, 2] is one of numerous multifield inflationary models. Expansion of the universe is driven by a triplet of fields with the background configuration, $\Phi^i = \alpha x^i$, where α is a constant and x^i denotes standard comoving coordinates. Under the assumption of homogeneity and isotropy the matter Lagrangian depends on only three invariants, $\mathcal{X} = [B]$, $\mathcal{Y} = [B^2]/\mathcal{X}^2$, $\mathcal{Z} = [B^3]/\mathcal{X}^3$, where B denotes the body metric with its components defined as $B^{ij} = g^{\mu\nu} \Phi^i_{,\mu} \Phi^j_{,\nu}$, and square brackets stand for trace.

Features setting this model apart from other inflationary models include presence of vector perturbations, non-conservation of curvature perturbation modes after crossing the Hubble horizon, and novel shape of the primordial scalar bispectrum [2]. Further development of this model includes, for example, combining it with the single field inflation [3, 4].

2. Observational restrictions on special solid inflation with scalar field

Here we consider only a special case of the model combining solid inflation [1, 2] with scalar field [3, 4]. The system under consideration is given by the action

$$S = \int \sqrt{-g} d^4x \left(\frac{1}{2} M_{\text{pl}}^2 R - \frac{1}{2} g^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} - W(\varphi) \mathcal{X}^{\mathcal{A}} \right), \quad \mathcal{X} = g^{\mu\nu} \Phi^i_{,\mu} \Phi^i_{,\nu}. \quad (1)$$

The matter Lagrangian is given by the trace of the body metric, $\mathcal{X} = [B]$, and scalar field φ , with \mathcal{A} being a constant, and $W(\varphi)$ a function restricted by slow-roll conditions. The background configuration of fields is $\Phi^i \propto x^i$ and $\varphi = \varphi(t)$, and the metric of the background space-time is the flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric. Advantage of this special model is much smaller nonlinearity (primordial non-gaussianity) parameter f_{NL} than in the full model [2, 4].

We will call even more special case with only solid-like matter fields Φ^i without scalar field, $\varphi = 0$, $W(\varphi) = \text{const.}$, **special solid inflation**. It is given by the matter Lagrangian $\mathcal{L}_m \propto \mathcal{X}^{\mathcal{A}}$, and it is the simplest special case of solid inflation [1, 2].

The slow-roll approximation requires smallness of the Hubble-flow parameters, $\epsilon = -\dot{H}/H^2$, $\eta = \dot{\epsilon}/H\epsilon$, where $H = \dot{a}/a$ is the Hubble parameter. In the model given by the action (1) we have

$$\epsilon = \mathcal{A} + (1 - \mathcal{A}/3)p, \quad \eta = \frac{(1 - \mathcal{A}/3)p}{\mathcal{A} + (1 - \mathcal{A}/3)p} \eta_p, \quad p = \frac{\dot{\varphi}^2}{2M_{\text{pl}}^2 H^2}, \quad \eta_p = \frac{\dot{p}}{Hp}. \quad (2)$$

We call \mathcal{A} special solid parameter, and p scalar field parameter. They play the role of slow-roll parameters, $\mathcal{A}, p \ll 1$. The scalar field parameter and parameters derived from it describe derivatives of the function $W(\varphi)$. The first two can be expressed as $(M_{\text{pl}}^2/2) (W_{,\varphi}/W)^2 = p$, $(M_{\text{pl}}^2/2) (W_{,\varphi\varphi}/W) = p - \eta_p/4$, in the leading order of the slow-roll approximation. Structure of these relations is the same as slow-roll conditions for the single field inflation.

From the observational perspective, the most prominent physical quantities are two-point correlation functions of Fourier modes of scalar and tensor perturbations, $\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{q}} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{q}) \mathcal{P}_\zeta(k)$, $\langle \gamma_{\mathbf{k}ij} \gamma_{\mathbf{q}lm} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{q}) \Pi_{\mathbf{k}ijklm} \mathcal{P}_\gamma(k)$, where ζ and γ denote curvature and tensor perturbations respectively, $\Pi_{\mathbf{k}ijklm} = \sum_{\lambda=+,\times} e_{\mathbf{k}ij}^\lambda e_{\mathbf{k}lm}^{\lambda*}$ with polarization tensor e_{ij} , and $\mathcal{P}_\zeta(k)$ and $\mathcal{P}_\gamma(k)$ are corresponding power spectra. Information about them is encoded in scalar spectral tilt

n_s , defined through the relation $\mathcal{P}_\zeta(k) \propto k^{n_s-4}$, and tensor-to-scalar ratio r defined as $r = 4\mathcal{P}_\gamma/\mathcal{P}_\zeta$. Both n_s and r are defined in the long wavelength limit, $ak \ll H$.

The analysis of perturbations reveals, that the model given by the action (1) predicts

$$\mathcal{L}_m \propto \mathcal{X}^{\mathcal{A}} \Rightarrow n_s = 1 + \frac{2}{3} (1 + 2\mathcal{A}) \mathcal{A}, \quad (1) \Rightarrow r = \frac{16 (1 + 2\mathcal{A})^{5/2} (\mathcal{A} + p)^2}{\left| 9\sqrt{3}\mathcal{A} + (1 + 2\mathcal{A})^{5/2} p \right|}. \quad (3)$$

This result has been drawn from previous works dealing with a more general case [3, 4], and, for simplicity, the scalar spectral index is written only for the **special solid** model, which can be drawn also from [2]. We have taken the leading order in the slow-roll approximation, $\mathcal{A} \sim p \ll 1$, while keeping the accurate relation for the speed of propagation of scalar perturbations, $c_L^2 = (1 + 2\mathcal{A})/3$ (longitudinal sound speed). The comparison with the CMB observational data [5] is in Fig. 1. The left panel shows that even the **special solid inflation** without the scalar field is compatible with observations, and the right panel shows that smaller values of the tensor-to-scalar ratio are achieved with the scalar field included. This is the main benefit of including it into the studied model.

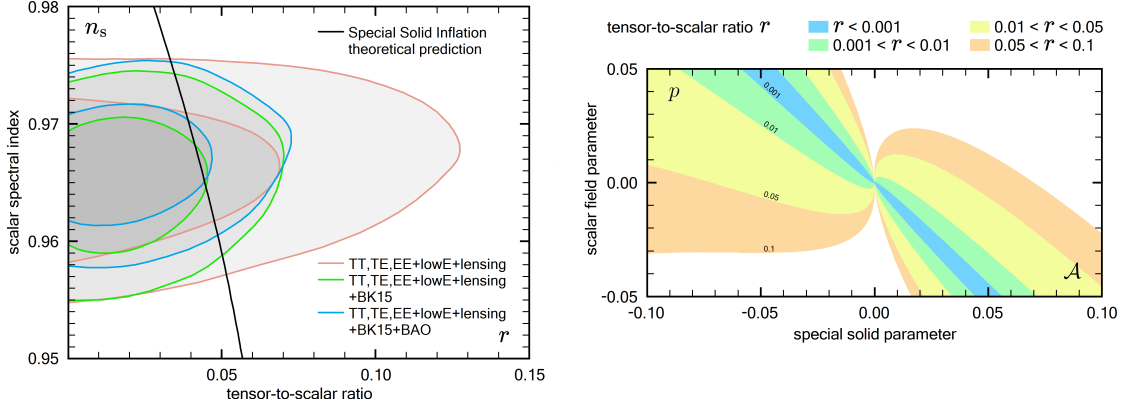


Figure 1: **Left panel:** Black line represents values of scalar spectral index n_s and tensor-to-scalar ratio r predicted by the **special solid inflation** (3) with $p = 0$. Coloured contours represent observational restrictions with two sets of lines corresponding to 68% and 95% confidence limits, and various colors for different datasets analyzed by *Planck Collaboration* [5] (Plik v3.01 likelihood software available at <http://pla.esac.esa.int>). **Right panel:** Values of the tensor-to-scalar ratio (3) in the parameter space of the model given by \mathcal{A} and p .

3. Reheating with solid remnant and growth of vector-to-scalar ratio

The matter Lagrangian of the inflationary model from the previous section has to break down towards the end of inflation. Energy of the inflaton fields transfers to the Standard Model particles or dark matter during the transitional reheating period. We assume the following structure

$$\mathcal{L}_m = \frac{1}{2} g^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} - W(\varphi) \mathcal{X}^{\mathcal{A}} \quad \longrightarrow \quad -C \mathcal{X}^{\mathcal{B}} \quad \longrightarrow \quad \Lambda\text{CDM Model}, \quad (4)$$

with two separate transitions. Scalar field φ disappears in the first transition, and the solid-like fields Φ^i in the second one. Evolution of perturbations in the universe filled with matter given

by the Lagrangian $\mathcal{L}_m = -C\mathcal{X}^{\mathcal{B}}$ is studied in [6]. We will call the period described by such model the **solid remnant era**. The pressure to energy density ratio is $w = -1 + 2\mathcal{B}/3$, and superluminality and instability of perturbations is avoided for $\mathcal{B} < \mathcal{B}_1 = 11 - 4\sqrt{7} \doteq 0.417$ or $w < w_1 = (19 - 8\sqrt{7})/3 \doteq -0.722$.

Since we study post-inflationary period, the long wavelength limit is the only regime with relevant observational consequences. During the **solid remnant era** sizes of the curvature perturbation ζ and tensor perturbation γ_{ij} (both defined in the standard way) depend on the scale factor as

$$\zeta \propto a^{P[\zeta]}, \quad \gamma_{ij} \propto a^{P[\gamma]}, \quad P[\zeta] = P[\gamma] = \frac{1}{2} \left(\mathcal{B} - 3 \pm \text{Re}\{\sqrt{\mathcal{B}^2 - 22\mathcal{B} + 9}\} \right), \quad (5)$$

The inflationary prediction for the tensor-to-scalar ratio then remains unchanged. However, for the vector perturbation S_i (defined as $g_{\tau i} = a^2 S_i$, $S_{i,i} = 0$, with conformal time τ) we have a different result, $S_i \propto a^{P[S]}$, $P[S] = P[\zeta] + 1 - \mathcal{B}$. The vector-to-scalar ratio s is proportional to $s \propto a^{2(P[S] - P[\zeta])} = a^{2(1 - \mathcal{B})}$. It is creasing, because $\mathcal{B} < \mathcal{B}_1 < 1$. The inflationary model studied in the previous section predicts $s = ((1 + 2\mathcal{A})/3)^{5/2}$. The overall theoretical prediction for the vector-to-scalar ratio at the beginning of the Λ CDM era is then

$$s = \frac{\overbrace{(1 + 2\mathcal{A})^{5/2}}^{\text{inflation}}}{9\sqrt{3}} \overbrace{\left(\frac{a_{\Lambda\text{CDM}}}{a_{\text{inf}}} \right)^{2(1 - \mathcal{B})}}^{\text{solid remnant era}} = \frac{(1 + 2\mathcal{A})^{5/2}}{9\sqrt{3}} \left(\frac{a_{\Lambda\text{CDM}}}{a_{\text{inf}}} \right)^{-(1+3w)}, \quad (6)$$

where a_{inf} and $a_{\Lambda\text{CDM}}$ is the scale factor at the end of the inflation and beginning of the Λ CDM era respectively. This is, of course, valid under the assumption that the transition from the **solid remnant era** to the Λ CDM era does not cause further nontrivial changes.

Acknowledgments

The work was supported by grants VEGA 1/0719/23, VEGA 1/0025/23, and Ministry of Education contract No. 0466/2022.

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