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Optimal control theory for the angular control of the full payload for AdV+ Phase II

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In the present work it is presented a formulation of a new control strategy for the angular degrees of freedom of a Fabry-Perot cavity in the presence of radiation pressure effect for Advanced Virgo+ (AdV+) Phase II experiment. The main difference with Phase I configuration is the introduction of large terminal masses. The different physical dimensions of the two masses and the consequent different momenta of inertia introduce a not negligible asymmetry of the mechanical system which is translated in an increase of difficulties of decoupling all the degrees of freedom. Given this difficulty, the possibility of designing SISO controllers (Single Input-Single Output) is left out. A new approach of designing MIMO controllers (Multi Input-Multi Output) in time-domain is investigated. Optimal Control Theory is used in order to design controllers which allow, by the minimization of a specific cost function, to obtain direct closed-loop stability with the optimal phase margin available. The present work will explain different topics: starting from the analytical description of the Fabry-Perot cavity, first (i) a State Space formulation of the mechanical system is obtained; then (ii) the design of a LQI control (Linear Quadratic Integral regulator) is described. Results and conclusions will be devoted to evaluate the robustness of the strategy and limitation of the mechanical system regarding the controllability requirements.

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1. Introduction

The Advanced Virgo detector is a long scale enhanced Michelson interferometer placed in Italy, close to Pisa, with the aim of detecting gravitational waves from astronomical sources. The Advanced Virgo interferometer has detected, together with the LIGO interferometers located in the USA [1], an impressive collection of gravitational wave emissions in the last observation runs O2 and O3. These detectors rely on the Michelson interferometer design, offering unprecedented sensitivity with a high duty cycle. These modern laser interferometers utilize resonant optical cavities, with the primary mirrors suspended as pendulums to isolate them from seismic disturbances. These instruments generate a useful signal only when the optical components are precisely positioned at predetermined locations relative to each other, referred to as the operating point. For instance, acceptable deviations from the operating point along the optical axis are typically on the order of $1 \cdot 10^{-12}$ meters in longitudinal and $1 \cdot 10^{-9}$ radians in angular, while the free motion of the suspended mirrors is many orders of magnitude larger. Sophisticated electro-optical control systems are necessary to continuously measure and restore the mirror positions.

2. Analytical description of the mechanical payload of AdV+

Consider the mechanical system reported in the left side of Figure 1. The suspended Fabry-Perot cavity is described considering the bottom stage of the payload. Each element of the system is connected between each other through an equivalent elastic-damping element represented by a torsional stiffness k_i and a damping coefficient d_i. The two mirrors are connected by radiation pressure effect, which can be described by an equivalent optical spring with stiffness k_{OS} [2].

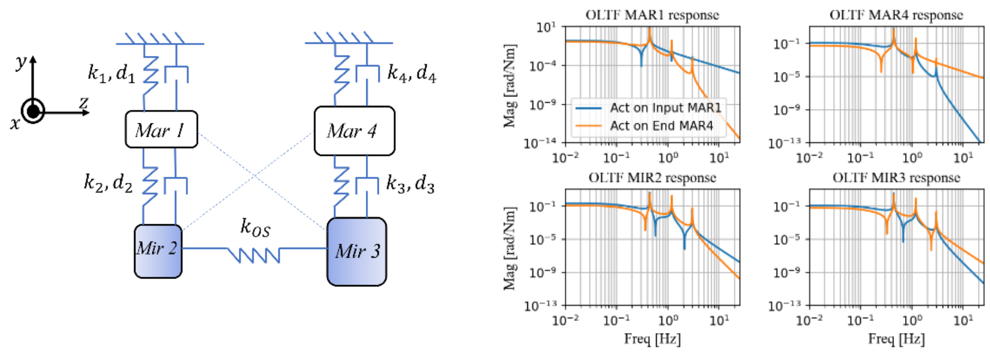

Figure 1 – Schematic of the suspended Fabry-Perot cavity, bottom stage (left) and Open-loop transfer function of the opto-mechanical system, for the TY DoF (right).

The present system can be described mathematically with a mechanical model of the coupled payload, which for simplicity is not reported in the present paper but can be found in [3][1]. We want to obtain the State-Space notation for the full system in matrix notation by defining the state vector $x^T = [\theta_1\ \dot{\theta}_1\ \theta_2\ \dot{\theta}_2\ \theta_3\ \dot{\theta}_3\ \theta_4\ \dot{\theta}_4]$. We can rewrite the complete law of motion with respect to the second derivative of the angular displacement in matrix form:

[1] Full details of the present study are reported in [3]: Virgo Technical NoteVIR-0219A-23: https://tds.virgo-gw.eu/ql/?c=19089.

$$\dot{x} = Ax + Bu;$$

Where the State Matrix *A*, the Input matrix *B* and the control vector *u* are explicitly written:

$$\begin{Bmatrix} \dot{\theta} \\ \ddot{\theta} \end{Bmatrix} = \begin{bmatrix} 0 & II \\ -J^{-1}K_{tot} & -J^{-1}\Gamma \end{bmatrix} \begin{Bmatrix} \theta \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} 0 \\ J^{-1} \end{bmatrix} \{T(t)\}$$

Where J is the Inertia matrix, Γ the Damping matrix and K_{tot} is the total opto-mechanical stiffness matrix respectively. At this point, it is possible to compute the Open-Loop transfer functions of the full coupled system. The Output VS Input relationship between an arbitrary torque and the correspondent rotation of the masses is given by:

$$\frac{\theta}{T} = C(s \cdot II - A)^{-1}B + D$$

Where II is the identity matrix of dimension $2N_{DoF} \times 2N_{DoF}$. According to this last equation, the opto-mechanical Open-loop transfer functions are reported in the right side of Fig.1: the four subplots represent all the input-output pairings due to the actuations on the two marionettas. In details, blue plots represent the masses response for an applied torque to the Input marionetta, while the orange plots are the ones for the actuation on the End marionetta.

3. Diagonalization of the opto-mechanical system

In order to decouple the equations of motion describing the opto-mechanical system, modal analysis methodology can be used. Thus, we can apply the eigenvalues problem [4], by solving the following equation:

$$\det|K_{tot} - \lambda \cdot J| = 0$$

Given the structural asymmetry, it is not possible to find a couple of eigenvectors, i.e. a couple of torques applied as input to the two marionettas, which allows to excite one resonance mode with respect to another. This implies that the whole system can not be reduced to a problem in which we can control separately the different modes of vibration with a SISO-like control strategy. Instead, it is possible to work with the fully MIMO coupled system, by designing two controllers applied separately to the two marionettas. In the following section it will be described the *Linear Quadratic Regulator* controller, i.e. LQR, with integral action added.

4. Control regulators design and controllability requirements

In order to design the control loop for the present configuration, we can consider a second order LTI mechanical system described by $\ddot{x} = f(x, u)$. We can describe the system with the following equations:

$$J\big(x_0, u(t)\big) = \int_0^T x'(\tau)Qx(\tau) + u'Ru(\tau)\, d\tau$$

Where $J\big(x_0, u(t)\big)$ is defined the *performance index*, dependent by the state and the control action which represents the cost function to be minimized in order to reach a specific target for the state and the control by conveniently tuning the penalty matrices *Q* and *R* [5,6]. To design the LQR control regulator with the integral action (i.e. LQI) we reformulate the system with the augmented version $\tilde{A}$ of the State matrix and the state vector $\tilde{x}$ [7] as follows:

$$\begin{Bmatrix} \dot{x} \\ \dot{v} \end{Bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{Bmatrix} x \\ v \end{Bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u$$

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u$$

Where v is the integral of the error signal e: $\dot{v}_n(t) = e_n(t)$. By solving the Riccati equation it is possible to find the control matrix $K_{LQI} = R^{-1}\tilde{B}^T\tilde{P}$, such that the feedback control equation assumes the form: $u = -K_{LQI}(\tilde{x} - \tilde{x}_T)$. Once Riccati equation has been solved, and the feedback control equation has been computed, to close the loop we define a new State matrix A_{CL}, which considers of the feedback matrix:

$$A_{CL} = \tilde{A} - \tilde{B}K_{LQI}$$

Given the A_{CL} matrix, it is possible to compute the Closed-loop gain transfer functions, which gives us information about the stability of the system and the attenuation performance of the control loop.

5. Results and discussions

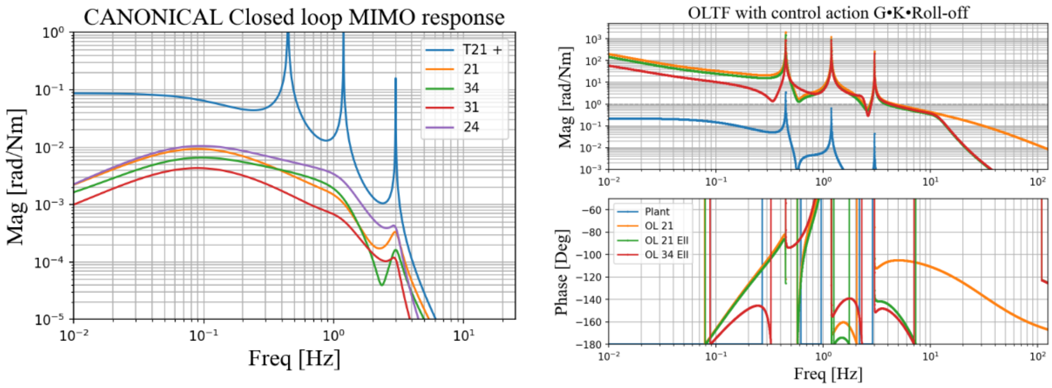

Figure 2 - Closed loop response of the LQI controller implemented (left) and Open-loop transfer functions of the mirror response.

Results of the implemented strategy are here reported. In Fig. 2 (left side) are plotted the Closed-loop gain transfer functions obtained through the implementation of the LQI control strategy. Such formulation allows to tune the controller by acting directly on the modes that we want to control. Performance of the controller can be also evaluated by analyzing the Open-Loop transfer functions of the system. In the right side of Fig. 2, Bode plot of the mirrors transfer functions due to the control actions implemented are shown (Input mirror in green, End mirror in red, Plant MAR2MIR input for reference in blue). In order to optimize the low-frequency and the high-frequency regions of the loop, the necessity of using dedicated structures in the control filters such as Lag Filters (aka Boost Filters to reduce the residual motion of the system) and Roll-off structures that reduce the reintroduction of control noise in the loop is needed. Since we want to reduce the noise re-introduction generally above 10 Hz, this will inevitably cause a huge constraint on the controllability of the higher resonance mode at 3 Hz. We need indeed to find a trade-off between the possibility to have the loop bandwidth ahead of such mode, and to roll-off efficiently the higher frequencies. Cutting the noise ahead 10 Hz with the use of an elliptic filter, will determine a consistent reduction of the Phase-margin of the loop from 68 deg (orange plot of right side of Fig.2) to around 25 deg. Consequently, the Gain-margin will be very limited. One solution to increase the overall range of stability margins, with the same rate of roll-off, would be to put the bandwidth of the loop before the last resonance mode. This choice will imply however to increase the difficulties to control the 3 Hz mode.

6. Conclusions

In the present document, a new strategy for the angular control of an asymmetric opto-mechanical system, i.e. AdV+ Phase II configuration, has been proposed. The whole control architecture starts from the analytical description of the full payload (bottom stage) which allows to obtain a State Space modeling of the system. However, given the asymmetric mechanical configuration of the investigated system (large terminal masses), some problems arise during the diagonalization process: indeed, considering the full system, it is not possible to fully decouple the different degrees of freedom (modes of vibration). This difficulty is translated in an impossibility to use SISO-like control strategies. For this reason, the possibility to study the full-coupled system, so to design MIMO-like controllers, is investigated instead. The design of an optimal LQR (Linear Quadratic Regulator) controller with an additional integral term (i.e. LQI) has been performed. Such control is based on the minimization of a specific cost function (built taking into account the state convergence and the control effort requirements) which allows to obtain direct closed-loop stability. However, one limitation arises since in real working conditions we want ideally to reduce the noise re-introduction as much as possible above 10 Hz. To fulfill such requirements, dedicated structures such as Roll-off filters (e.g. Low-pass, Elliptic filters...) need to be used. The use of such structures will cause a consistent loss of the Phase-margin available that, in case we want to make sure to control the high resonance mode (i.e. 3 Hz), will lead to a very restricted range of stability margin. One solution will be to put the UGF of the loop before the 3 Hz mode, that is to reduce the bandwidth of the loop and so to increase the difficulties to be able to control such mode. Results of the implemented strategy, although preliminary, showed the potential of the proposed control technique.

References

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