## Supplemental Material to article, 'Recent study and results for ionisation efficiency theory in pure materials<sup>\*</sup>'

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## Abstract

We present an analytic fit function to curve in Fig. (1) for the ionisation efficiency for Si and the corresponding curves that define the error band. For the main curve, the fit given here has a maximum deviation from numeric result up to 1%.

## Analytical Function

The function that describe the main curve of Fig. (1) in the text, in the recoil energy interval (0.047827385) keV  $\leq E_R \leq (10)$  keV, is given by

$$f_n(E_R) = \left(\frac{a_1(E_R + a_0) + a_2(E_R + a_0)^{a_3} + a_4(E_R + a_0)^{a_5}}{a_6(E_R + a_0) + a_7(E_R + a_0)^{a_8}}\right)^{a_9}, \quad (1)$$

where  $E_R$  is the recoil energy in keV,  $a_0 = -0.0473064$  (keV),  $a_1 = 1.4232691$ ,  $a_2 = -0.7285272$ ,  $a_3 = 0.9708002$ ,  $a_4 = -0.3957546$ ,  $a_5 = 0.9652145$ ,  $a_6 = -0.8896225$ ,  $a_7 = 1.6638736$ ,  $a_8 = 0.9832139$ ,  $a_9 = 2.1124542$ .

For the bands the fit function is easily obtained by the parametrization,

$$f_n^{\text{(bands)}}(E_R) = (b_0 + b_1(E_R + a_0)^{0.1})f_n(E_R; a_0), \qquad (2)$$

where for the upper band  $a_0 = -0.022$ ,  $b_0 = 1.4$  and  $b_1 = -0.15617419$ , and for the lower band  $a_0 = -0.078$ ,  $b_0 = 0.7$  and  $b_1 = 0.078512790$ , the other coefficients  $a_i$ , for i > 0, remain unchanged. For results see, fig 1.

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Figure 1: (red ) Analytical function Eq. (1) compare to numeric result (black) for  $f_n$  in Si, (blue) analytical functions for error bands Eq. (2) compared with the numeric result (dash).