Unlocking the Light(er) Sterile Neutrino Sector: Matter Effects and Mass Ordering

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Future long-baseline experiments will be able to probe hitherto unexplored regions of sterile neutrino parameter space for masses ranging from meV to eV. We present an analytic calculation of the neutrino conversion probability \( P(\nu_\mu \rightarrow \nu_e) \) in the presence of sterile neutrinos, with exact dependence on \( \Delta m^2_{41} \) and matter effects. We further express the neutrino conversion probability as a sum of terms of the form \( \sin(x)/x \), thus allowing a physical understanding of matter effects and their possible resonance-like behavior. We focus on the identification of sterile mass ordering (sign of \( \Delta m^2_{41} \)) at DUNE. The conversion probability obtained reveals the complex interplay between sterile and matter contributions. We perform numerical calculations of DUNE’s sensitivity to sterile mass ordering over a broad range of sterile neutrino masses. Our analytic expressions enable us to explain the dependence of this sensitivity on \( \Delta m^2_{41} \) values for all mass ordering combinations.
1. Introduction

Future long-baseline experiments will be able to probe the low mass regions (meV to eV) of sterile neutrino parameter space [1]. In these proceedings, we probe the effects of such low mass sterile neutrinos, we present the conversion probability $P_{\mu e}$ in the presence of sterile neutrinos, with exact dependence on $\Delta m_{41}^2$, and matter effects, as a summation of terms of the form $\sin(x)/x$ [2].

We explore how the complex interplay between the sterile contribution and matter effects depends on the possible active and sterile mass ordering (SMO) combinations (signs of $\Delta m_{31}^2$ and $\Delta m_{41}^2$). Focusing on identification of SMO, i.e., the sign of $\Delta m_{41}^2$, we find that our analytic expressions enable us to explain the key features of the sensitivity plots for long-baseline experiments like DUNE [3, 4].

2. The Sterile Contribution to the Neutrino Conversion Probability

The neutrino oscillation Hamiltonian in the presence of sterile neutrinos can be expressed as

$$\mathcal{H}_{3+1} = \frac{1}{2E_\nu} \mathbf{U} \cdot \text{diag} \left[ \left( 0, \Delta m_{21}^2, \Delta m_{31}^2, \Delta m_{41}^2 \right) \right] \cdot \mathbf{U}^\dagger + \text{diag} \left[ P_\nu + V_n, V_n, V_n, 0 \right] ,$$

in the flavor basis. Here, $V_\mu$ and $V_\tau$ are the charged-current and neutral-current potentials, and the $3 + 1$ PMNS matrix $\mathbf{U}$ is expressed in terms of three independent phases ($\delta_{13}$, $\delta_{24}$, $\delta_{34}$) and six independent rotation angles ($\theta_{12}$, $\theta_{13}$, $\theta_{23}$, $\theta_{14}$, $\theta_{24}$, $\theta_{34}$). We define a few dimensionless quantities: $\alpha \equiv \frac{\Delta m_{31}^2}{\Delta m_{41}^2}$, $R \equiv \frac{\Delta m_{41}^2}{\Delta m_{31}^2}$, $A_{\mu e, n} \equiv 2E_\nu V_{\mu e, n}/\Delta m_{31}^2$, and $\Delta \equiv \Delta m_{31}^2 L/(4E_\nu)$. Defining a book-keeping parameter $\lambda \equiv 0.2$, and expressing in terms of $\lambda$, we may express: $\alpha \sim O(\lambda^2)$, $s_{13} \sim O(\lambda)$, and $s_{14}$, $s_{24}$, $s_{34} \sim O(\lambda)$, where $s_{ij} \equiv \sin(\theta_{ij})$. By taking an approximation for the Earth’s crust, $A_n \approx -A_e/2$, we express the sterile contribution to the conversion probability $P_{\mu e}$ as:

$$P_{\mu e}^{(\text{sterile})} = 4s_{13}s_{14}s_{24}s_{34} \frac{\sin ([A_e - 1] \Delta)}{A_e - 1} \left[ \sin(\delta_{24}^x)P_{24}^x + \cos(\delta_{24}^x)P_{24}^c \right]$$

$$+ 4s_{13}s_{14}s_{24}s_{34}s_{23}c_{23} \frac{\sin ([A_e - 1] \Delta)}{A_e - 1} \left[ \sin(\delta_{34}^c)P_{34}^x + \cos(\delta_{34}^c)P_{34}^c \right] + O(\lambda^4) ,$$

where $\delta_{ij}^* = \delta_{ij} + \delta_{ij}$. The terms $P_{24,34}^{x,c}$ can be expressed in the $\sin(x)/x$ form, for example

$$P_{24}^{x} = R \left[ \frac{1}{2} A_e c_{23} + (R - 1) \left( s_{23}^2 + 1 \right) \right] \frac{\sin \left( (R - 1 + \frac{A_e}{2}) \Delta \right) \sin \left( (R - \frac{A_e}{2}) \Delta \right)}{R - 1 + \frac{A_e}{2}}$$

$$+ c_{23}^2 R \sin \left( (R - 1 - \frac{A_e}{2}) \Delta \right) \frac{\sin \left( (R + \frac{A_e}{2}) \Delta \right)}{R + \frac{A_e}{2}} .$$

The full expressions for the other term can be found in [2]. This $\sin(x)/x$ form brings out the non-intuitive interplay between the sterile term $R$ and the matter contribution $A_e$. In the limit of $R \rightarrow 1 - A_e/2$, $R \rightarrow A_e/2$, and $R \rightarrow -A_e/2$, we may obtain possible resonance-like behavior.


In Table 1, we show how the different mass orderings (signs of $\Delta m_{31}^2$ and $\Delta m_{41}^2$) and neutrino/antineutrino ($\nu/\bar{\nu}$) channel combinations regulate the sterile-matter interplay. This is because
Table 1: Modifications in the probabilities $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ due to the interplay between $\Delta m^2_{31}$ and matter effect [2]. Here, N (I) corresponds to Normal (Inverted) ordering in the active sector, i.e. $\Delta m^2_{31} > 0$ ($\Delta m^2_{31} < 0$), whereas, Ns (Is) corresponds to the same in sterile sector. Dash (‘—’) denotes the absence of significant enhancement due to matter effects. The single tick (‘.’) denotes a small enhancement and double ticks (‘✓✓’) denote a large enhancement due to possible resonance-like behaviors.

different mass ordering combinations and different channels lead to different signs of $A_e$ and $R$. Therefore, the sensitivity to SMO for a long-baseline neutrino experiment like DUNE would depend on the signs of $\Delta m^2_{31}$ and $\Delta m^2_{41}$, values of $|\Delta m^2_{31}|$ and $\nu/\bar{\nu}$ channel.

To numerically calculate the sensitivity of DUNE to the SMO, the General Long Baseline Experiment Simulator (GLoBES) package [5, 6] is used. The detector specifications are listed in Table 2. We simulate the data by using the input (“true”) values of the parameters

\[ \theta_{12} = 33.56^\circ, \theta_{13} = 8.46^\circ, \theta_{23} = 45^\circ, \delta_{13} = -90^\circ, |\Delta m^2_{31}| = 2.5 \times 10^{-3} \text{ eV}^2, \alpha = 0.03 \]
\[ \theta_{14} = 5^\circ, \theta_{24} = 10^\circ, \theta_{34} = 0^\circ, \delta_{24} = 0^\circ, \delta_{34} = 0^\circ, \]

and try to fit the data with alternative values of these parameters, corresponding to the opposite SMO. The quantity that evaluates the sensitivity of DUNE to SMO is defined as $\Delta \chi^2_{\text{SMO}} \equiv \chi^2(\text{test}) - \chi^2(\text{true})$, where the $\chi^2$ value is obtained by using the GLoBES package [5, 6]. We also perform minimization of $\chi^2(\text{test})$ by varying over the fitting parameters to include real-world effects of parameter uncertainties. The range of variation for the neutrino mixing parameters is

\[ \theta_{23} = [40^\circ, 50^\circ], \delta_{13} = [-180^\circ, 0^\circ], \theta_{14} = [0, \theta_{14}^{\text{max}}], \theta_{24} = [0^\circ, 55^\circ], \delta_{24} = [-180^\circ, 180^\circ], \Delta m^2_{31} = \Delta m^2_{41}(\text{true}) \pm 15\% \]  

For $\theta_{14}^{\text{max}}$ value, see [2] and references therein [7, 8]. In Fig. 1, we plot the sensitivity to SMO ($\Delta \chi^2_{\text{SMO}}$) as a function of $|\Delta m^2_{41}|$, for all mass ordering combinations and for both $\nu$ and $\bar{\nu}$ as well

<table>
<thead>
<tr>
<th>Detector details</th>
<th>Normalization error</th>
<th>Energy calibration error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline = 1300 km, 40 kton, LArTPC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Runtime (yr) = 3.5 $\nu$ + 3.5 $\bar{\nu}$</td>
<td>$\nu_e : 5%$</td>
<td>$\nu_e : 10%$</td>
</tr>
<tr>
<td>$\epsilon_{\text{app}} = 80%$, $\epsilon_{\text{dis}} = 95%$</td>
<td></td>
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</tr>
<tr>
<td>$R_e = 0.15/\sqrt{E_{\nu}(\text{GeV})}$, $R_\mu = 0.20/\sqrt{E_{\nu}(\text{GeV})}$</td>
<td>$\nu_\mu : 5%$</td>
<td>$\nu_\mu : 10%$</td>
</tr>
</tbody>
</table>

Table 2: Details of detector configurations, efficiencies, resolutions, and systematic uncertainties for DUNE. Here, $\epsilon_{\text{app}}$ and $\epsilon_{\text{dis}}$ are signal efficiencies for $\nu_e^C$ and $\nu_\mu^C$ respectively; $R_e$ and $R_\mu$ are energy resolutions for $\nu_e^C$ and $\nu_\mu^C$ events respectively. Runtime of 1 year corresponds to $1.47 \times 10^{21}$ POT (protons on target).
as the combined channel. Some of the key features are listed in the figure itself, for more details, see [2]. In particular, we observe that the interplay between $\Delta m^2_{41}$ and matter effects has led to an increased sensitivity for

- the $\bar{\nu}$ channel: for N-Ns combination, at $|R| > 1$ and for I-Is combination, at $|R| < 1$,

- the $\nu$ channel: for I-Is combination, at $|R| > 1$ and for N-Is combination, at $|R| < 1$, note that this also leads to a noticeable overall increase in the combined ($\nu + \bar{\nu}$) sensitivity.

Furthermore, we also see an expected drop in sensitivity at $\Delta m^2_{41} = \Delta m^2_{31}$. Thus, we observe that our analytic expressions are able to explain the key features of the sensitivity plot.

4. Conclusion

The analytic expression obtained [2] allows us to probe the complex intertwined nature of the sterile term and matter contributions, even when the active neutrino contributions cannot be disentangled easily. Moreover, by focusing on the signs and values of $R$ and $A_\epsilon$, we are able to explain the features of the sensitivity to SMO plots at future long-baseline neutrino experiments like DUNE.

References

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