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Semileptonic B decays, latest $|V_{xb}|$

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In this talk, the status of the long-standing $|V_{cb}|$ and $|V_{ub}|$ puzzle is briefly reviewed. A personal selection of recent new results are discussed, focusing on the theoretical challenges arising when aiming at the cleanest and most precise determinations of these CKM elements.

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[*] Speaker

1. Introduction: A long-standing puzzle

Semileptonic beauty decays, mediated through the weak interaction, are the prime candidates to determine the CKM-matrix elements $|V_{cb}|$ and $|V_{ub}|$. These decays are either observed as exclusive modes, where the B decays to one fixed final states (in general $D^{(*)}$ for the $b \to c$ decay and $\pi(\rho)$ from the $b \to u$ transition), or as inclusive modes, where all possible final states are considered. In both cases, the challenge in describing these decays lies in the determining the non-perturbative objects that parametrize the fundamental mismatch between the theoretical quark and observable hadron transitions.

For exclusive modes, these are the form factors of the $B \to D^{(*)}$ and $B \to \pi$ transition. On the other hand, the inclusive decays are described using the Heavy-Quark Expansion (HQE), where especially the precision determination of $|V_{cb}|$ is a beautiful example of the sophistication of this method. In these transitions, the non-perturbative objects, the HQE matrix elements are determined from global fits to the data [1–4].

The determinations of both $|V_{cb}|$ and $|V_{ub}|$ differ when extracted from inclusive versus exclusive decays, the origin of this difference is a long-standing puzzle in particle physics, see e.g. [5–7].

In this talk, a personal selection of the latest determinations of $|V_{xb}|$ is presented with focus on the theoretical aspects and challenges, with most attention on the inclusive determinations.

These proceedings are outlined as follows. We start discussing the challenge of determining $|V_{cb}|$, focusing on new determinations in the inclusive sector. In Sec. 3, we continue with $|V_{ub}|$, while in Sec. 4 we discussion ratio measurements of V_{ub}/V_{cb} in the exclusive modes. We end with a short outlook and conclusion.

2. The challenge of $|V_{cb}|$

2.1 Inclusive $|V_{cb}|$

The determination of inclusive $|V_{cb}|$ follows from $\bar{B}(p_B) \to X_c(p_X)\ell(p_\ell)\bar{\nu}_\ell(p_\nu)$, where X_c presents the sum over all final states with a c quark and $q \equiv p_\ell + p_\nu$. The decay can be described in the Heavy-Quark Expansion (HQE), which is set up by splitting the momentum of the b quark as $p_b = m_b v + k$, where v is the velocity of the B meson and k is a residual momentum. Using then the optical theorem, we can set up a local Operator Product Expansion (OPE), which schematically gives

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \cdots \qquad d\Gamma_i = \Sigma_k C_i^k \langle \bar{B}|O_i^{(k)}|\bar{B}\rangle \,, \tag{1}$$

where $C_i^{(k)}$ are pertubatively calculable Wilson coefficients. The $\langle \cdots \rangle$ are non-pertubative forward-matrix elements that contain strings of covariant derivatives depending on the order in the $1/m_b$ expansion. For Γ_2, two HQE elements enter

$$2M_B\mu_\pi^2 = -\langle B|\bar{b}_v iD_\mu iD^\mu b_v|B\rangle \,, \qquad 2M_B\mu_G^2 = -\langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\rangle \,, \tag{2}$$

while Γ_3 has 2 elements, Γ_4 has 9 and Γ_5 has 18 parameters (see [8–10]). The HQE elements are current most precisely obtained from measurements of moments of kinematical distributions of the spectrum (see also e.g. [11] for progress on the lattice). The available moments are lepton energy

E_ℓ, hadronic invariant mass M_X and dilepton invariant mass q^2 moments, obtained by integrating the full spectrum with a lower lepton energy or q^2 cut. Schematically, we have

$$\langle M^n \rangle = \frac{\int_{E_\ell > E_{\rm cut}} dM M^n \frac{d\Gamma}{dM}}{\int_{E_\ell > E_{\rm cut}} dM \frac{d\Gamma}{dM}} \,, \tag{3}$$

for each moment M^n, and similar for the q^2-cut. Experimentally, several moments up to $n = 3, 4$ with several energy cuts are available. Most recently, also the q^2 moments with corresponding q^2 cuts were measured by both the Belle [12] and Belle II collaboration [13].

From the measurements, both the HQE parameters and $|V_{cb}|$ can be extracted. The current-state-of-the-art extractions include α_s^3 corrections to the total rate [14]:

$$|V_{cb}|^{E_\ell, M_X}_{\rm incl} = (42.16 \pm 0.51) \cdot 10^{-3} \,, \qquad |V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.63) \cdot 10^{-3} \,, \tag{4}$$

using only the E_ℓ, M_X measurements [2] or q^2 measurements [3], respectively. More recently, also a first analysis was presented combining all available moments, yielding similar results [4].

Including even higher orders in the HQE expansion has the challenge that the number or parameters grows really fast. Here the q^2 moments, which are reparametrization invariant quantities, have the advantage that they depend on a reduced set of parameters [9] opening the way for a fully data-driven extraction of HQE elements up to $1/m_b^4$ [15]. In [3], the q^2 moments were used for the first time to also extract the $1/m_b^4$ elements, which are important to check the convergence of the HQE. The extracted $|V_{cb}|$ value given in (4) includes these terms and contains an additional 0.23 due to missing higher-orders. The latter have been recently identified for q^2 moments [10] and are currently implemented in an open source package Kolya [16]. The possible effects of New Physics (NP) on the moments of the spectrum were briefly discussed [17]. A simultaneous fit of the hadronic parameters and possible NP interactions is currently in progress.

Finally, we briefly mention the possibilities to measure also inclusive lepton-flavor ratios

$$R_{e/\mu} \equiv \frac{\Gamma(B \to X_c e \bar{\nu}_e)}{\Gamma(B \to X_c \mu \bar{\nu}_\mu)} \,. \tag{5}$$

A recent Belle II result, with a cut on the lepton energy gives $R_{e/\mu} = 1.007 \pm 0.021$ [18]. Using the framework described above and the HQE elements extracted from data, we can also calculate the SM predition for this ratio, where the difference from 1 is caused purely by the mass effects of the muon. Including up to $1/m_b^3$-terms, gives $R_{e/\mu} = 1.006 \pm 0.001$ [19], in perfect agreement with the experimental measurement.

2.2 Exclusive V_{cb}

The $B \to D$ and $B \to D^*$ transitions give direct access to $|V_{cb}|$. The theoretical description of these decays depends on form factors that describe the non-pertubative input. These can be obtained from Lattice QCD (LQCD), Light-cone Sumrules (LCSR) and/or data. In this, the knowledge of the q^2 dependence is crucial, as the theoretical methods do not directly determine the form factors at the physically relevant kinematics. In general, this requires thus an extrapolation of the high-q^2 region (for LQCD) or negative q^2 region (LCSR) using a form factor paramatrization.

We point out that historically the Caprini-Lellouch-Neubert (CLN) [20] parametrization is often used as this gives an easy parameterization based on HQE. However, with the current level of precision, this framework should at least be adapted and the current consensus is that this framework should no longer be used [7]. An update of the framework, including higher-order corrections at the level of $1/m_c^2$ was performed in [21]. With the data available at that time, they find

$$|V_{cb}|_{\rm excl} = (40.3 \pm 0.8) \times 10^{-3} \,, \tag{6}$$

which is in general much larger than the values extracted using the CLN parametrization. Alternatively, there is the Boyd-Grindstein-Lebed (BGL) parametrization [22, 23], which is model independent and makes use of dispersive bounds.

Due to the current precision, determining the $B \to D^*$ form factors at non-zero recoil is crucial. The Fermilab/MILC collaboration first measured this, where a interesting tension between the slope of the lattice and the experimental data was found [24]. In the recent Belle determination from $B \to D^*$ decays [25], several form factor parametrizations (BGL, CLN, with and without shape information) were applied to the Fermilab/MILC data, leading to a variety of $|V_{cb}|$ extractions. Not using the shape information obtained from LQCD, they find $|v_{cb}|_{\rm excl} = (40.6 \pm 0.9) \times 10^{-3}$. More recently, HPQCD [26] and JLQCD [27] presented form factors at non-zero recoil. Combining these determinations, the results from $B \to D$ and $B_s \to D_s^{(*)}$ using the BGL parameterization for the form factors recently resulted in [6]

$$|V_{cb}|_{\rm excl} = (40.69 \pm 0.49) \times 10^{-3} \,. \tag{7}$$

Comparing with the inclusive average of (4), they find a 1.9σ tension. We conclude that more experimental and lattice data is needed to solve the V_{cb} puzzle and to reduce the tensions between the different exclusive results.

3. The challenge of $|V_{ub}|$

In the remainder, we briefly describe the V_{ub} puzzle, focusing on recent determinations both in inclusive and exclusive decays. A more detailed review can be found in e.g. [5, 6].

3.1 Exclusive V_{ub}

The V_{ub} exclusive determination is mainly driven by the determination from $B \to \pi \ell \nu_\ell$, where only one form factor plays are role. There are several LQCD determinations available, the most recent from the JLQCD collaboration reads $|V_{ub}|_{\rm excl,JLQCD} = (3.93 \pm 0.41) \cdot 10^{-3}$ [28]. This value is in agreement with determinations from Fermilab/MILC [29], RBC/UKQCD [30], although with larger uncertainties. A recent review, combining these lattice results finds [6]:

$$|V_{ub}|_{\rm excl} = (3.75 \pm 0.20) \cdot 10^{-3} \,. \tag{8}$$

This is in excellent agreement the value obtained by including also LCSR results [31].

3.2 Inclusive V_{ub}

The story of inclusive V_{ub} is much more challenging as for V_{cb}. Due to the large experimental $b \to c$ background, that has to be removed, the local OPE used for the $b \to c$ decays no longer works. Instead, a switch to a light-cone OPE is required which introduces a dependence on the non-perturbative shape function of the B meson. There are several methods/models available to deal with these non-pertubative effects. Most commonly used are GGOU [32], DGE [33], ADFF [34] and BLNP [35, 36]. Specifically highlighted was the recent determination from Belle [37]

$$|V_{ub}|_{\text{incl}} = (4.10 \pm 0.28) \cdot 10^{-3} , \tag{9}$$

which takes the arithmetic average of all four determinations. This results is in good agreement with the arithmetic average of all the measurements available in the HFLAV review [38]. Comparing with the exclusive determination in (**??**), we find a tension at the level of the 1.5σ.

From the theoretical point, not all four frameworks stand at the same level. Therefore, the inclusive determinations need to be scrutinized. Specifically, an update of the BLNP method is long overdue and currently in progress.

The $B \to X_u \ell \nu$ differential decay rate can be separate (in the appropriate region of phase space)

$$d\Gamma = H \otimes J \otimes S , \tag{10}$$

where H is the hard scattering kernel of $O(m_b)$, J is the universal Jet function at $O(\sqrt{m_b \Lambda_{\text{QCD}}})$ and S is the Shape function at $O(\Lambda_{\text{QCD}})$. Moments of the Shape function can be linked to the HQE parameters of the $b \to c$ transition as described in Sec. 2, where currently only HQE parameters up to $1/m_b^2$ are used. In the update, higher moments will be included, as well as a more flexible parameterization of the Shape function to allow for a conservative systematic uncertainty on the theoretical prediction. Finally, also the known α_s^2 corrections that are currently not yet implemented, will be included, reducing largely the scale dependence of the predictions and thus reducing the theoretical uncertainty. Stay tuned for an update on the model and new $|V_{ub}|_{\text{incl}}$ determinations.

4. Ratios of exclusive V_{ub}/V_{cb}

Finally, it is interesting to discuss also the ratios of CKM elements from exclusive decays. A recent LHCb measurement of branching ratios for the decays $B_s^0 \to \{K^-, D_s^-\}\mu^+\nu_\mu$, allowed to determine the ratio V_{ub}/V_{cb} in two bins of q^2 for the first time in a B_s decay [39]. As in any exclusive decay, this determination requires inputs for the form factors, which are know both from LQCD as LCSRs. Recently, a consistent description of the form factors in the full q^2 range was obtained [40] by combining data from the RBC/UKQCD-lattice collaboration [41] and Light-cone Sumrules [42]. Applying these results to a recent LHCb measurement, leads to [40]

$$\left|\frac{V_{ub}}{V_{cb}}\right|_{q^2<7\ \text{GeV}^2} = 0.0681 \pm 0.0040 , \qquad \left|\frac{V_{ub}}{V_{cb}}\right|_{q^2>7\ \text{GeV}^2} = 0.0801 \pm 0.0047 \tag{11}$$

which are mutually compatible only at the 1.9σ level.

5. Outlook

The study of V_{xb} is and remains a fascinating topic, with rapid progress in the last years. An impressive precision has been obtained, clearly marking the high-precision era of flavour physics. Yet at the same time, tensions between the different determinations (or different LQCD form factor results) persist and we have to stay tuned for new data and updated theory predictions that will hopefully shine new light on the V_{xb} puzzle. In this endeavour, the close collaboration between experiment and theory has proven to be necessary.

References

[1] P. Gambino and C. Schwanda, *Inclusive semileptonic fits, heavy quark masses, and* V_{cb}, *Phys. Rev. D* **89** (2014) 014022, [`1307.4551`].

[2] M. Bordone, B. Capdevila and P. Gambino, *Three loop calculations and inclusive Vcb*, *Phys. Lett. B* **822** (2021) 136679, [`2107.00604`].

[3] F. Bernlochner, M. Fael, K. Olschewsky, E. Persson, R. van Tonder, K. K. Vos et al., *First extraction of inclusive* V_{cb} *from* q^2 *moments*, *JHEP* **10** (2022) 068, [`2205.10274`].

[4] G. Finauri and P. Gambino, *The* q^2 *moments in inclusive semileptonic B decays*, *JHEP* **02** (2024) 206, [`2310.20324`].

[5] M. Fael, M. Prim and K. K. Vos, *Inclusive* $B \to X_c \ell \bar{\nu}_\ell$ *and* $B \to X_u \ell \bar{\nu}_\ell$ *decays: current status and future prospects*, .

[6] F. U. Bernlochner, M. T. Prim and K. K. Vos, $|v_{ub}|$ *and* $|v_{cb}|$ *from exclusive semileptonic decays*, *The European Physical Journal Special Topics* (Feb., 2024) .

[7] P. Gambino et al., *Challenges in semileptonic B decays*, *Eur. Phys. J. C* **80** (2020) 966, [`2006.07287`].

[8] T. Mannel, S. Turczyk and N. Uraltsev, *Higher Order Power Corrections in Inclusive B Decays*, *JHEP* **11** (2010) 109, [`1009.4622`].

[9] T. Mannel and K. K. Vos, *Reparametrization Invariance and Partial Re-Summations of the Heavy Quark Expansion*, *JHEP* **06** (2018) 115, [`1802.09409`].

[10] T. Mannel, I. S. Milutin and K. K. Vos, *Inclusive semileptonic* $b \to c\ell\bar{\nu}$ *decays to order* $1/m_b^5$, *JHEP* **02** (2024) 226, [`2311.12002`].

[11] A. Barone, S. Hashimoto, A. Jüttner, T. Kaneko and R. Kellermann, *Approaches to inclusive semileptonic* $B_{(s)}$*-meson decays from Lattice QCD*, *JHEP* **07** (2023) 145, [`2305.14092`].

[12] BELLE collaboration, R. van Tonder et al., *Measurements of* q^2 *Moments of Inclusive* $B \to X_c \ell^+ \nu_\ell$ *Decays with Hadronic Tagging*, *Phys. Rev. D* **104** (2021) 112011, [`2109.01685`].

[13] Belle-II collaboration, F. Abudinén et al., *Measurement of lepton mass squared moments in $B \to Xc\ell\nu\,\bar{}\ell$ decays with the Belle II experiment*, Phys. Rev. D **107** (2023) 072002, [2205.06372].

[14] M. Fael, K. Schönwald and M. Steinhauser, *A first glance to the kinematic moments of $B \to X_c\ell\nu$ at third order*, JHEP **08** (2022) 039, [2205.03410].

[15] M. Fael, T. Mannel and K. Keri Vos, *V_{cb} determination from inclusive $b \to c$ decays: an alternative method*, JHEP **02** (2019) 177, [1812.07472].

[16] "Kolya package for inclusive decays." https://gitlab.com/vcb-inclusive/kolya.

[17] M. Fael, M. Rahimi and K. K. Vos, *New physics contributions to moments of inclusive $b \to c$ semileptonic decays*, JHEP **02** (2023) 086, [2208.04282].

[18] Belle-II collaboration, L. Aggarwal et al., *Test of Light-Lepton Universality in the Rates of Inclusive Semileptonic B-Meson Decays at Belle II*, Phys. Rev. Lett. **131** (2023) 051804, [2301.08266].

[19] M. Rahimi and K. K. Vos, *Standard Model predictions for lepton flavour universality ratios of inclusive semileptonic B decays*, JHEP **11** (2022) 007, [2207.03432].

[20] I. Caprini, L. Lellouch and M. Neubert, *Dispersive bounds on the shape of anti-B —> D(*) lepton anti-neutrino form-factors*, Nucl. Phys. B **530** (1998) 153–181, [hep-ph/9712417].

[21] M. Bordone, M. Jung and D. van Dyk, *Theory determination of $\bar{B} \to D^{(*)}\ell^-\bar{\nu}$ form factors at $O(1/m_c^2)$*, Eur. Phys. J. C **80** (2020) 74, [1908.09398].

[22] C. G. Boyd, B. Grinstein and R. F. Lebed, *Model independent determinations of anti-B —> D (lepton), D* (lepton) anti-neutrino form-factors*, Nucl. Phys. B **461** (1996) 493–511, [hep-ph/9508211].

[23] C. G. Boyd, B. Grinstein and R. F. Lebed, *Precision corrections to dispersive bounds on form-factors*, Phys. Rev. D **56** (1997) 6895–6911, [hep-ph/9705252].

[24] Fermilab Lattice, MILC, Fermilab Lattice, MILC collaboration, A. Bazavov et al., *Semileptonic form factors for $B \to D^*\ell\nu$ at nonzero recoil from $2 + 1$-flavor lattice QCD: Fermilab Lattice and MILC Collaborations*, Eur. Phys. J. C **82** (2022) 1141, [2105.14019]. [Erratum: Eur.Phys.J.C 83, 21 (2023)].

[25] Belle collaboration, M. T. Prim et al., *Measurement of differential distributions of $B \to D^*\ell\nu\,\bar{}\ell$ and implications on $|Vcb|$*, Phys. Rev. D **108** (2023) 012002, [2301.07529].

[26] J. Harrison and C. T. H. Davies, *$B \to D^*$ vector, axial-vector and tensor form factors for the full q^2 range from lattice QCD*, 2304.03137.

[27] JLQCD collaboration, Y. Aoki, B. Colquhoun, H. Fukaya, S. Hashimoto, T. Kaneko, R. Kellermann et al., *$B \to D^*\ell\nu_\ell$ semileptonic form factors from lattice QCD with Möbius domain-wall quarks*, 2306.05657.

[28] JLQCD Collaboration collaboration, B. Colquhoun, S. Hashimoto, T. Kaneko and J. Koponen, *Form factors of $b \to \pi\ell\nu$ and a determination of $|V_{ub}|$ with möbius domain-wall fermions*, Phys. Rev. D **106** (Sep, 2022) 054502.

[29] Fermilab Lattice, MILC collaboration, J. A. Bailey et al., *$|V_{ub}|$ from $B \to \pi\ell\nu$ decays and (2+1)-flavor lattice QCD*, Phys. Rev. D **92** (2015) 014024, [`1503.07839`].

[30] J. M. Flynn, T. Izubuchi, T. Kawanai, C. Lehner, A. Soni, R. S. Van de Water et al., *$B \to \pi\ell\nu$ and $B_s \to K\ell\nu$ form factors and $|V_{ub}|$ from 2+1-flavor lattice QCD with domain-wall light quarks and relativistic heavy quarks*, Phys. Rev. D **91** (2015) 074510, [`1501.05373`].

[31] D. Leljak, B. Melić and D. van Dyk, *The $\overline{B} \to \pi$ form factors from QCD and their impact on $|V_{ub}|$*, JHEP **07** (2021) 036, [`2102.07233`].

[32] P. Gambino, P. Giordano, G. Ossola and N. Uraltsev, *Inclusive semileptonic B decays and the determination of |V(ub)|*, JHEP **10** (2007) 058, [`0707.2493`].

[33] J. R. Andersen and E. Gardi, *Inclusive spectra in charmless semileptonic B decays by dressed gluon exponentiation*, JHEP **01** (2006) 097, [`hep-ph/0509360`].

[34] U. Aglietti, G. Ferrera and G. Ricciardi, *Semi-Inclusive B Decays and a Model for Soft-Gluon Effects*, Nucl. Phys. B **768** (2007) 85–115, [`hep-ph/0608047`].

[35] S. W. Bosch, B. O. Lange, M. Neubert and G. Paz, *Proposal for a precision measurement of |V(ub)|*, Phys. Rev. Lett. **93** (2004) 221801, [`hep-ph/0403223`].

[36] S. W. Bosch, B. O. Lange, M. Neubert and G. Paz, *Factorization and shape function effects in inclusive B meson decays*, Nucl. Phys. B **699** (2004) 335–386, [`hep-ph/0402094`].

[37] Belle collaboration, L. Cao et al., *Measurements of Partial Branching Fractions of Inclusive $B \to X_u\, \ell^+\, \nu_\ell$ Decays with Hadronic Tagging*, Phys. Rev. D **104** (2021) 012008, [`2102.00020`].

[38] HFLAV collaboration, Y. S. Amhis et al., *Averages of b-hadron, c-hadron, and τ-lepton properties as of 2021*, Phys. Rev. D **107** (2023) 052008, [`2206.07501`].

[39] LHCb collaboration, R. Aaij et al., *First observation of the decay $B_s^0 \to K^-\mu^+\nu_\mu$ and Measurement of $|V_{ub}|/|V_{cb}|$*, Phys. Rev. Lett. **126** (2021) 081804, [`2012.05143`].

[40] C. Bolognani, D. van Dyk and K. K. Vos, *New determination of $|V_{ub}/V_{cb}|$ from $B_s^0 \to \{K^-, D_s^-\}\mu^+\nu$*, JHEP **11** (2023) 082, [`2308.04347`].

[41] RBC/UKQCD collaboration, J. M. Flynn, R. C. Hill, A. Jüttner, A. Soni, J. T. Tsang and O. Witzel, *Exclusive semileptonic $Bs{\to}K\ell\nu$ decays on the lattice*, Phys. Rev. D **107** (2023) 114512, [`2303.11280`].

[42] A. Khodjamirian and A. V. Rusov, *$B_s \to K\ell\nu_\ell$ and $B_{(s)} \to \pi(K)\ell^+\ell^-$ decays at large recoil and CKM matrix elements*, JHEP **08** (2017) 112, [`1703.04765`].