

## Ion Heating Mechanism & Cosmic Ray Production in Collisionless Shocks

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We investigate a novel collisionless shock jump condition that constrains the cosmic-ray (CR) energy density. The injection process and the subsequent acceleration of CRs in the SNR shocks are closely related to the formation process of the collisionless shocks. The shock formation is caused by wave-particle interactions. Since the wave-particle interactions result in the energy exchange between electromagnetic fields and charged particles, the randomization of particles around the shock may occur at the rate given by the scalar product of the electric field and current. The randomization can be quantified by the entropy production. We find that order-of-magnitude estimates of the entropy production with reasonable strength of the electromagnetic fields in the SNR constrain the amount of the CR nuclei and ion temperatures. The constrained amount of the CR nuclei can be sufficient to explain the Galactic CRs. The ion temperature becomes half of the case without CRs. Future observations by *XRISM* and *Athena* can distinguish whether the SNR shock accelerates the CRs or not from the ion temperature observations.

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## 1. Introduction

Collisionless shocks of supernova remnants (SNRs) are invoked as the main sources of the Galactic cosmic rays (CRs). However, the production process of CRs is still an open question despite numerous studies reported. The amount of CRs in the shock is related to the thermal energy, i.e., the ion temperature. Once we determine the downstream ion temperature, we can derive the CR amount from the shock energy budget arguments. We recently proposed a novel collisionless shock model in which the ion temperatures are estimated by modeling the entropy production at the shock transition [1]. We review the model in this paper.

## 2. Model

In this model, we assume that a part of shock kinetic energy is consumed for the generation of the CRs and the amplification of the magnetic field. The generated magnetic field is assumed to be disturbed (not an ordered field). In this model, we consider the randomization of the particles incoming from the far upstream region at the shock transition region. The randomization is quantified by the entropy. We consider the parallel shock or the case of negligibly small magnetic field strength at the far upstream region.

Conservation laws of total mass and momentum flux can be written as

$$\rho_0 v_0 = \rho_2 v_2, \quad (1)$$

$$\rho_0 v_0^2 + P_0 + F_{\text{esc}} = \rho_2 v_2^2 + P_2 + \frac{\delta B^2}{4\pi} + P_{\text{cr}}, \quad (2)$$

where the symbols  $\rho$ ,  $P$ , and  $v$  have their usual meaning. The generated (turbulent) magnetic-field strength is  $\delta B$ . We regard that the field with  $\delta B$  has a coherent length scale (injection scale of turbulence) much larger than the Larmor radius of the thermal particles with a velocity of  $\sim v_0$  and that the turbulence cascades to the smaller scale. The disturbances associated with the field are assumed to randomize the thermal particles by the wave-particle interactions. The net momentum flux of escaping CRs is  $F_{\text{esc}} \lesssim \rho_0 v_0^3 / 3c$  which is negligibly small ( $F_{\text{esc}} = 0$ ). From these conservation laws, we can derive the relation between the compression ratio  $r_c \equiv \rho_2 / \rho_0$  and the jump of pressure (internal energy)  $x_c \equiv P_2 / P_0 = \varepsilon_2 / \varepsilon_0$  as

$$r_c = \left[ 1 + \frac{1 - x_c}{\gamma \mathcal{M}_s^2} - \xi_B - \xi_{\text{cr}} \right]^{-1}, \quad (3)$$

or

$$x_{c,j} = 1 + \gamma \mathcal{M}_s^2 \left( 1 - \frac{1}{r_c} - \xi_B - \xi_{\text{cr}} \right), \quad (4)$$

where  $\xi_B \equiv \delta B^2 / (4\pi \rho_0 v_0^2)$ ,  $\xi_{\text{cr}} \equiv P_{\text{cr}} / (\rho_0 v_0^2)$ , and  $\mathcal{M}_s = v_0 / \sqrt{\gamma P_0 / \rho_0}$  is the sonic Mach number. Thus, once another relation between  $r_c$  and  $x_{c,j}$  is found, we can derive the shock jump condition with given  $\xi_B$  and  $\xi_{\text{cr}}$ . In this model, the entropy production of the thermal particles is modeled explicitly that determines the fraction of downstream thermal energy. If the thermal energy fraction is smaller than the incoming kinetic energy upstream, the remaining energy is regarded to be

consumed for the nonthermal components ( $\xi_{\text{cr}}$  and  $\xi_{\text{B}}$ ). Note that in the collisional, adiabatic shock, especially the strong shock limit, we can derive the downstream temperature as  $kT_2 = (3/16)\bar{m}v_0^2$  or  $(3/2)kT_2 = (1/2)\bar{m}v_0'^2$ , where  $\bar{m}$  is the mean molecular mass and  $v_0'$  is the upstream velocity measured at the downstream rest. This relation indicates that the coherent upstream motion of each particles in the fluid is completely randomized at the downstream region due to the particle-particle collisions, corresponding to the maximum entropy production. In our model, the entropy production is not enough, and therefore a part of shock kinetic energy can be divided into  $\delta B$  and  $P_{\text{cr}}$ .

The entropy per unit mass is defined as

$$ds = \frac{1}{M} \frac{d\tilde{Q}}{kT}, \quad (5)$$

where  $M$  is the mass of fluid parcel and  $d\tilde{Q}$  is the energy transferred from electromagnetic fields to the *internal energy* due to the shock transition. Note that  $d\tilde{Q} = dU + PdV$  indicates only the increment of the internal energy rather than the total kinetic energy of the thermal particles (a sum of the bulk motion and the random motion). Substituting  $d\tilde{Q} = dU + PdV$  to the equation (5), and using the relation of  $d\varepsilon = d(\rho e) = e d\rho + \rho de$ , where  $e \equiv U/M$ , we can derive the change of the internal energy per unit volume as

$$\frac{d\varepsilon}{\varepsilon} = \gamma \frac{d\rho}{\rho} + (\gamma - 1)\bar{m} ds. \quad (6)$$

Note that we have presumed that  $N_j$  is constant during the shock transition. Thus, we obtain the entropy jump before and after the shock transition,  $\Delta s = s_2 - s_0$  as

$$(\gamma - 1)\bar{m}\Delta s = \ln\left(\frac{\varepsilon_2}{\varepsilon_0}\right) - \gamma \ln\left(\frac{\rho_2}{\rho_0}\right) = \ln x_c - \gamma \ln r_c. \quad (7)$$

Then, the jump conditions are derived by estimating  $\Delta s$  independently from the equation (7). Since the SNR shock is expected to be formed by the wave-particle interactions, the transferred energy in total  $\Delta\tilde{Q}$  may be around  $\sim \mathbf{J} \cdot \mathbf{E}\Delta t$ , where  $\mathbf{J}$  is the electric current of species  $j$ . The electric field measured in the comoving frame of the ions is  $\mathbf{E}$ .  $\Delta t$  is a time taking the shock transition. We estimate each value as  $J \sim qN\langle\tilde{v}\rangle$ ,  $E \equiv |\mathbf{E}| \sim (\langle\tilde{v}\rangle/c)\delta B$ , and  $\Delta t \sim mc/q\delta B$ , where  $q$  is the typical electric charge of the particles in the fluid, and  $\langle\tilde{v}\rangle = v_0 + \sqrt{2kT_0/\bar{m}}$  is the mean kinetic velocity, respectively. The transition time scale is assumed to be comparable with an inverse of the cyclotron frequency. In a hybrid simulation solving the particle acceleration (e.g., [2]), the shock jump seems to occur at a very small length scale despite a significant amplification of turbulent magnetic fields at the ‘upstream’ region (it may correspond to a shock precursor region in our situation). We regard that the randomization of particles resulting in the shock transition mainly occurs at such a very small length scale. Thus, we assume the entropy production due to the shock transition as

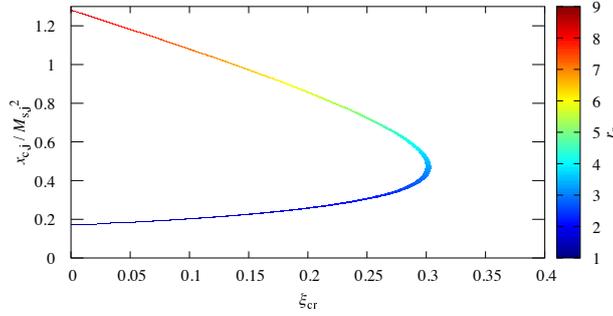
$$\Delta s = \frac{1}{M} \frac{JE\Delta t}{kT} = \frac{1}{M} \frac{qN\langle\tilde{v}\rangle\langle\tilde{v}\rangle}{kT_2} \delta B \frac{\bar{m}c}{q\delta B} = \frac{\langle\tilde{v}\rangle^2 r_c}{kT_0 x_c}, \quad (8)$$

where we suppose  $T \sim T_2$ . Substituting the equation (8) to the equation (7), we obtain the relation between  $r_c$  and  $x_c$  as

$$f \equiv \frac{x_c}{r_c} [\ln x_c - \gamma \ln r_c] - \gamma (\gamma - 1) \left(\frac{\langle\tilde{v}\rangle}{v_0}\right)^2 \mathcal{M}_s^2 = 0. \quad (9)$$

We solve this equation setting  $P_{\text{cr}}$ ,  $\delta B$  and  $\mathcal{M}_s = v_0/\sqrt{\gamma P_0/\rho_0}$  with the equation (3) to derive  $x_c$  in the case of the proton by regarding that the most abundant ions form the shock structure. Then, the compression ratio  $r_c$  is derived from the equation (3) by using the derived  $x_c$ . When we specify the energy flux conservation laws of CRs and  $\delta B$ , the collisionless shock is fully described. However, the diffusion coefficient of CRs, the magnetic field amplification, and their relation have not been understood yet (e.g.,[3–5]). In this paper, we consider the most efficiently accelerating CR shock feasible in which all of CRs is confined around the shock (there is no escaping CRs). In such a situation, the CR pressure is a practical function of  $\delta B$  because of the energy budget of the shock. The upstream kinetic energy is divided into the thermal energy, the magnetic field, and the CRs. The fraction of the thermal energy is given by the entropy production. The fraction of the magnetic field is treated as a free parameter. Thus, the remaining energy is divided into the CRs.

### 3. Results and Discussion



**Figure 1:** Solutions of  $f = 0$  with fixed  $1/\sqrt{\xi_B} = 3$  and  $\mathcal{M}_s = 197$  for the proton ( $\bar{m} = m_p$ ). The horizontal axis shows the CR fraction  $\xi_{\text{cr}}$  and the vertical axis shows the pressure jump  $x_c/M_s^2$ . The color indicates the compression ratio  $r_c$ . This figure is taken from [1].

Figure 1 shows the sets of  $\xi_{\text{cr}}$ ,  $x_c$ , and  $r_c$  satisfying  $f = 0$ . The function  $f(x_c)$  shows two solutions for a given  $\delta B$  depending on  $P_{\text{cr}}$ . The one solution describes the allowed shock jump and the other describe the energy gain: Let us consider the solutions around  $\xi_{\text{cr}} = 0$  for simplicity. We will refer to the solution giving  $x_c/M_s^2 \approx 0.17$  and  $r_c \approx 1.27$  as ‘solution A’, while we will refer to the other solution giving  $x_c/M_s^2 \approx 1.28$  and  $r_c \approx 8.31$  as ‘solution B’. The resultant temperature ( $T_2/T_0 = x_c/r_c \approx 0.1mv_0^2/\gamma kT_0$ ) is almost the same as each other because each solution is derived from the same  $\Delta\tilde{Q}$ . This means that the speed of particles’ random motion is almost the same as each solution. On the other hand, the difference in the compression ratios indicates that the speed of particles’ bulk motion is significantly different from each other. In a collisional shock in the strong shock limit, the downstream temperature satisfies  $(3/2)kT_2 = mv_0'^2/2$ . This might mean that since our shock consumes its energy for the generation of the nonthermal components, the random motion speed measured in the downstream rest frame  $\tilde{v}'_R \equiv \sqrt{3kT_{j,2}/m_j}$  should be equal or smaller than  $v'_0 = v_0 - v_2$  for the solution representing the shock transition (i.e.  $\tilde{v}'_R/v'_0 \leq 1$ ). Solution A gives the speed as  $\tilde{v}'_R/v'_0 \approx 2.3$ , while solution B gives  $\tilde{v}'_R/v'_0 \approx 0.6$ . Hence, solution B

may correspond to the shock transition. Solution A should be rejected because it does not satisfy the energy flux conservation law.

When  $\xi_{\text{cr}}$  becomes large, the two solutions approach with each other, coinciding at  $\xi_{\text{cr}} \approx 0.3$  (multiple roots), and finally, the solution vanishes. The multiple roots ( $\xi_{\text{cr}} = 0.3$ ) give  $\tilde{v}'_{\text{R}}/v'_0 \approx 0.7$  and  $\Delta s_j/\Delta s_{j,\text{ncr}} \approx 0.93$ . Thus, the multiple roots may represent the shock transition giving the maximum  $P_{\text{cr}}$  feasible in our shock model. In this article, we set the maximum  $\xi_{\text{cr}}$  to compare the no CR cases with the case of extremely efficient CR acceleration. The maximum  $\xi_{\text{cr}}$  is derived from the multiple roots of  $f = 0$  with given  $\xi_{\text{B}}$ .

For the case of  $v_0 = 4000 \text{ km s}^{-1}$  and  $T_0 = 3 \times 10^4 \text{ K}$  with given  $1/\sqrt{\xi_{\text{B}}} = 3$ , we obtain the maximum acceptable CR production  $\xi_{\text{cr}} \approx 0.3$ ,  $r_c \approx 3.29$ , and  $kT_{\text{p},2} \approx 14.4 \text{ keV}$ . Note that in the usual collisional shock case, we obtain  $r_c = 4.00$  and  $kT_{\text{p},2} = 31.3 \text{ keV}$ . The prediction of such smaller ion temperature can be tested by future X-ray observations such as *XRISM* [6] and *Athena* [7] (see, [1] for details). The fraction of the CRs  $\xi_{\text{cr}} = 0.3$  seems to be reasonable for the SNR shocks as sources of Galactic CRs. From the subtraction of the energy fluxes of the thermal particles at the far upstream and downstream, we can regard that roughly 50 % of the upstream energy flux is transferred to the nonthermal components. The fraction of magnetic pressure  $1/\sqrt{\xi_{\text{B}}} = 3$  corresponds to magnetic-field strength of  $\delta B \approx 611 \mu\text{G} (v_0/4000 \text{ km s}^{-1})(n_{\text{p},0}/1 \text{ cm}^{-3})^{1/2}$  which is consistent with estimated strength from X-ray observations of young SNRs (e.g., [8–10]). Thus, our parameter choice of  $1/\sqrt{\xi_{\text{B}}} = 3$  can be reasonable to adopt our model to the young SNR shocks.

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