Stochasticity of cosmic rays from supernova remnants and the ionisation rates in molecular clouds

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Cosmic rays are the only agent able to penetrate into the interior of dense molecular clouds. Depositing (part of) their energy through ionisation, cosmic rays play an essential role in determining the physical and chemical evolution of star-forming regions. To a first approximation their effect can be quantified by the cosmic-ray induced ionisation rate. Interestingly, theoretical estimates of the ionisation rate assuming the cosmic-ray spectra observed in the local interstellar medium result in an ionisation rate that is one to two orders of magnitude below the values inferred from observations. However, due to the discrete nature of sources, the local spectra of MeV cosmic rays are in general not representative for the spectra elsewhere in the Galaxy. Such stochasticity effects have the potential of reconciling modelled ionisation rates with measured ones. Here, we model the distribution of low-energy cosmic-ray spectra expected from a statistical population of supernova remnants in the Milky Way. The corresponding distribution for the ionisation rate is derived and confronted with data. We find that the stochastic uncertainty helps with explaining the surprisingly high ionisation rates observed in many molecular clouds.

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1. Introduction

Supernova remnants (SNRs) are considered to be the standard sources of galactic cosmic rays (CRs). They accelerate particles to a power law spectrum which subsequently get injected into the interstellar medium. In the following, we will argue that if the sources of cosmic rays are SNRs then for purposes of the transport they can be considered as burst-like and point-like, meaning they inject the accelerated particles at one point in time and in space.

As known from measurements of nuclear ratios of stable and unstable species, CRs of GeV energies spend a few Megayears in the extended halo around the Galactic disk before escaping. This is very long compared to the time scales over which particles can escape from SNRs, that is some ten thousands of years at most. Sources of GeV particle can therefore safely be approximated as burst-like. Also, on the timescales of a few Megayears, CRs traverse kiloparsec distances across the disk. If the sources are the results of supernova explosions, their sizes are likely limited to a few tens of parsecs, again much smaller than the propagation distances. The sources can therefore also assumed to be point-like.

The propagated spectrum of CRs from a single point-like and burst-like source depends very sensitively on its age and distance. It might seem that this would lead to a spectrum with many features. However, if enough sources contribute, by (some variant of) the central limit theorem, such spectral features will become very small on average.

To estimate if enough sources contribute, let's assume that sources occur as frequently as implied by the Galactic supernova rate, that is three per hundred years. The source density rate σ is then given by the rate divided by the area of the Galactic disk: $\sigma = (3 \times 10^{-2}\,\text{year}^{-1})/(\pi(15\,\text{kpc})^2)$. The number of sources that occur within a distance of, say five kiloparsecs over a time of some ten Megayears is then $\sigma(10\,\text{Myr})\,\pi(5\,\text{kpc})^2 \simeq 3 \times 10^4$. The density of CRs observed at any one point in the disk is therefore contributed by a large number (some ten thousand) of sources and no sizeable spectral features should be observable [1]. (A rough estimate would be that if 1, 100 or 10 000 sources contributed, the fluctuations should be order 1, 10% and 1%, respectively.)

The situation can be markedly different at other energies were the distances and times over which CRs can propagate get limited, for instance by energy losses. A well-known example is the transport of CR electrons and positrons at hundreds of GeV and beyond where radiative losses limit the number of sources contributing to a few, the details depending on various model parameters (e.g. [2]).

Here, we will be studying the transport of CR protons and electrons at MeV energies where ionisation losses are the dominant transport process. While the fluxes of these particles at Earth are significantly suppressed due to solar modulation, this energy range has recently been studied by the Voyager probes which having left the heliosphere should not be affected by solar modulation. MeV CRs are also responsible for the ionisation of molecular clouds. The CR-induced ionisation rate ζ_{CR} can be inferred from the observed ratios of molecular absorption lines. It has recently been noticed, however, that the inferred ionisation rates are on average one to two orders of magnitude above the values predicted from the Voyager data. This problem has been dubbed the "ionisation problem". In addition there is a markable spread in the ionisation rates for a given column depth of molecular clouds.

In the following, we argue that the fluctuations of CR density between different positions in

the Galaxy are responsible for the spread. If in addition, the source rate is locally enhanced, for instance due to molecular clouds more likely to be located in a spiral arm, then the enhanced level of ionisation can be explained.

2. Methodology

The intensity of CRs at position $\mathbf{r}$ and kinetic energy E for a set of N_s sources at positions $\vec{r}_i$ and times t_i is given as

$$J = \frac{v}{4\pi} \sum_{i=1}^{N_s} \mathcal{G}(\mathbf{r}, E; \mathbf{r}_i, t - t_i),$$ (1)

where $\mathcal{G}$ denotes the Green's function of the transport equation for the spectral density ψ, e.g. [3],

$$\frac{\partial \psi}{\partial t} + \frac{\partial}{\partial z}(u\psi) - D\nabla^2\psi + \frac{\partial}{\partial E}\left(\dot{E}\psi\right) = q(\mathbf{r}, E, t).$$ (2)

The spectral density ψ denotes the number of particles per unit volume and unit kinetic energy. $D = D(E)$ is the homogeneous and isotropic diffusion coefficient, $u = u(z)$ denotes the advection speed which we assume to be oriented in the $\pm z$ direction above and below the disk. Energy losses enter through the loss rate $\dot{E}$.

The source density rate q is assumed to factorise into a spatio-temporal and a spectral part, that is

$$q(\mathbf{r}, t, E) = \rho(\mathbf{r}, t)Q(E).$$ (3)

Note that the distribution ρ is usually assumed to be continuous which as argued above is a good approximation, e.g. for GeV CRs. At MeV energies, however, where ionisation losses limit the transport the point-like nature needs to be taken into account, that is

$$q(r, z, E, t) = \sum_{i=1}^{N_s} Q(E)\delta(\mathbf{r} - \mathbf{r}_i)\delta(t - t_i).$$ (4)

We assume a free boundary condition at $z = \pm H$.

For the diffusion coefficient $D(E)$, we have adopted the following form,

$$D(E) = D_0 \beta \gamma^{\delta}.$$ (5)

Here, γ and β are the particle's Lorentz factor and speed, respectively. This form guarantees an energy-independent mean free path at low energies, motivated by theoretical studies and observations in the solar system [4, 5], while showing the power law behaviour at higher energies indicated by measurements of stable secondary-to-primary ratios. We have adopted $\delta = 0.63$ [6] and chosen D_0 such that $D(E = 10\,\text{GeV}) \simeq 5 \times 10^{28}\,\text{cm}^2\,\text{s}^{-1}$ for both CR protons and electrons.

For the advection speed we have adopted the dependence $u(z) = u_0\,\text{sgn}(z)$ [7], with $u_0 = 16\,\text{km}\,\text{s}^{-1}$. Adiabatic energy losses contribute to the loss rate $\dot{E}$ as $\dot{E}_{ad} = pvu_0/(3h)$ where $h = 150\,\text{pc}$ is the half-thickness of the Galactic disk. Other energy losses are ionisation, inelastic nuclear interactions and bremsstrahlung. We adopt a mean gas density of $n_H = 0.9\,\text{cm}^{-3}$, in agreement with the observed surface mass density of $2\,\text{mg}\,\text{cm}^{-2}$ [8].

The source spectrum of individual SNRs is modelled as

$$Q(E) = \frac{\xi_{CR} E_{SNR}}{(mc^2)^2 \Lambda \beta} \left(\frac{p}{mc}\right)^{2-\alpha} \tag{6}$$

For the acceleration efficiencies ξ_{CR} of CR protons and electrons we have chosen $\xi_{CR}^p \simeq 8.7\%$ and $\xi_{CR}^e \simeq 0.55\%$, respectively, in order to match the observed data at high energy. The function Λ is chosen to guarantee the normalisation with respect to the total kinetic energy of the supernova explosion, $E_{SNR} = 10^{51}$ erg. For the spectral index, we have adopted $\alpha = 4.23$ which is compatible with the fit of observational data for high energy CR protons [6]. The half-height of the CR halo is set to $H = 4$ kpc.

We have solved for the Green's function of the CR transport equation by numerically solving eq. (2) for individual sources. Their positions were drawn from the spiral distribution as modelled in Ref. [9] and we assumed their ages to be uniformly distributed with a galactic rate of 3 per century. For a particular distribution of sources, the intensity at a particular position was then computed according to eq. (1). In order to study the dependence on the local source density, we consider two scenarios: a molecular cloud at the solar position (LOC) and one in a nearby spiral arm, specifically at a galacto-centric distance of 6.5 kpc (SPA).

To compute the ionisation rate, one needs to consider the transport of CRs into the molecular cloud. In the literature, both ballistic and diffusive transport are considered [10–13]. Which one is realised, likely depends on the ratio of the correlation length of magnetised turbulence to the spatial extent of the cloud. In the following, we consider both scenarios. The ionisation rate is then computed as a convolution of the CR intensities with the ionisation kernel. We refer the interested reader to Ref. [14] for more details.

3. Results

In Fig. 1 we show the intensities of CR protons and electrons for the LOC and SPA positions and indicate the stochastic uncertainties. The top panels show the proton spectra, the bottom panels the electron spectra. The panels on the left refer to the LOC position, the panels on the right to the SPA position. At each energy, there is a distribution of intensities due to the stochastic nature of the sources. The black solid lines indicate the medians of these distributions, the dotted black lines indicate the expectation values and the shaded bands indicate the ranges between the 2.5 %, and the 97.5 % quantiles. We also show measurements from Voyager 1 [15] and from AMS-02 [16, 17] for comparison.

At high energies, the medians and expectation values agree and there is very little stochastic uncertainty. This is in line with the arguments laid out in Sec. 1. At the LOC position (left column of Fig. 1), the model predictions approach the AMS-02 data at the highest energies considered; note that the measured intensities below tens of GeV are markedly affected by solar modulation which we have not attempted to model.

Below hundreds of MeV, only a small number of sources contribute because of energy losses and hence the stochastic uncertainties are sizeable. For protons (top panels of Fig. 1), the intensities show a clear maximum. For the median, this maximum occurs just below hundred MeV. At lower energies the intensities are dominated in fact by a single source and follow an $E^{-0.5}$ power law,
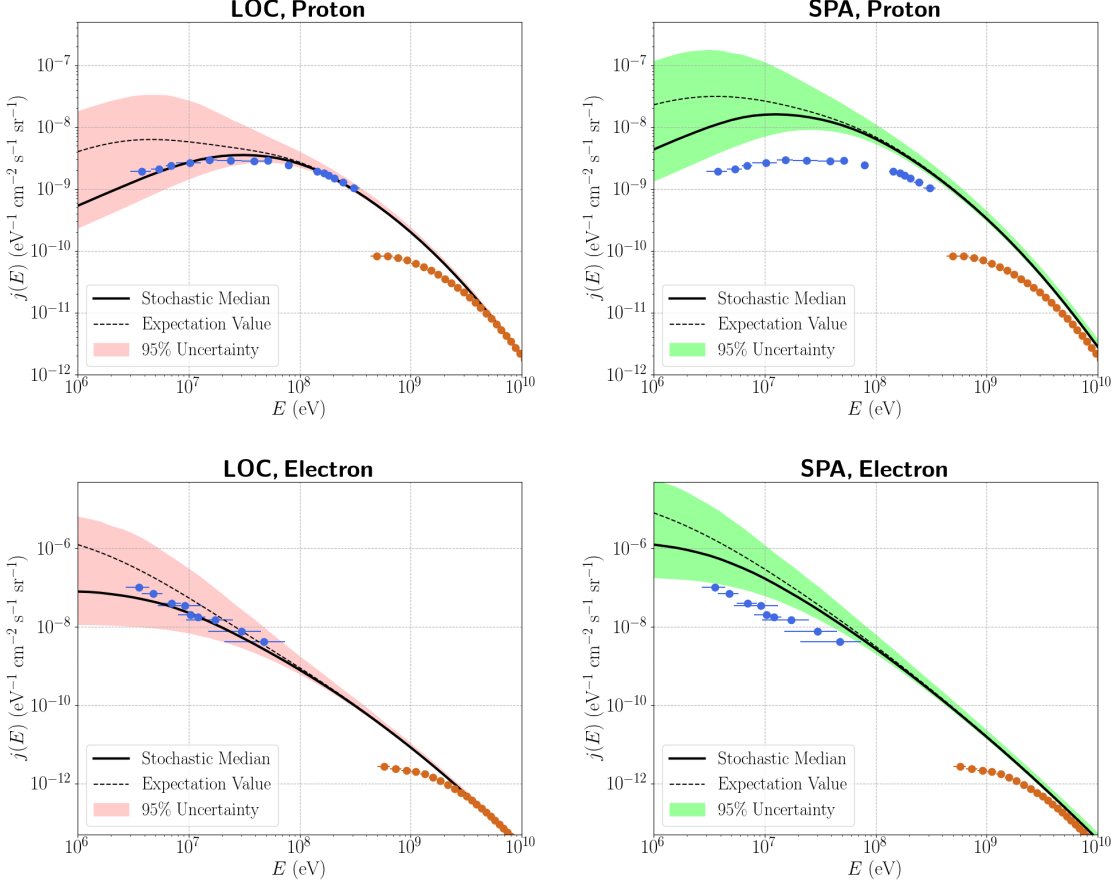
Figure 1: Intensities and corresponding stochastic fluctuations of CR protons (upper panels) and electrons (lower panels) in comparison with data from Voyager 1 [15] (blue) and AMS-02 [16, 17] (orange). Results are presented for an observer in the local ISM (the LOC position, left column) and in a spiral arm (the SPA position, right column). The dotted and solid black curves are respectively the expectation values and the median of the intensities. The shaded red or green regions are the 95% uncertainty ranges of the intensities.

as can be explained by inspection of an analytic approximation to the Green's function. Overall, the median is in good agreement with the Voyager data. Remaining discrepancies could be due to variations in the gas density which would broaden the maximum, rendering the proton spectrum around tens of MeV almost energy-independent. In contrast, the expectation value appears not representative of the distribution. This is due to the long power law tail of the non-Gaussian distribution of intensities. For electrons, the behaviour is qualitatively the same as for protons, but the maximum of the spectra appears at lower energies than considered here. Again, the median for the LOC position is in reasonable agreement with Voyager data while the expectation value is not.

At the SPA position (right panels of Fig. 1), the intensities are enhanced by up to a factor of five due to the higher local source density. The effect is biggest at energies below a few hundred MeV. Due to the short energy loss length, intensities at these energies are particularly sensitive to the local source density. Of course, neither the model median nor the stochastic uncertainty bands at the SPA position are in agreement with the Voyager data which have not been taken at the SPA

position.
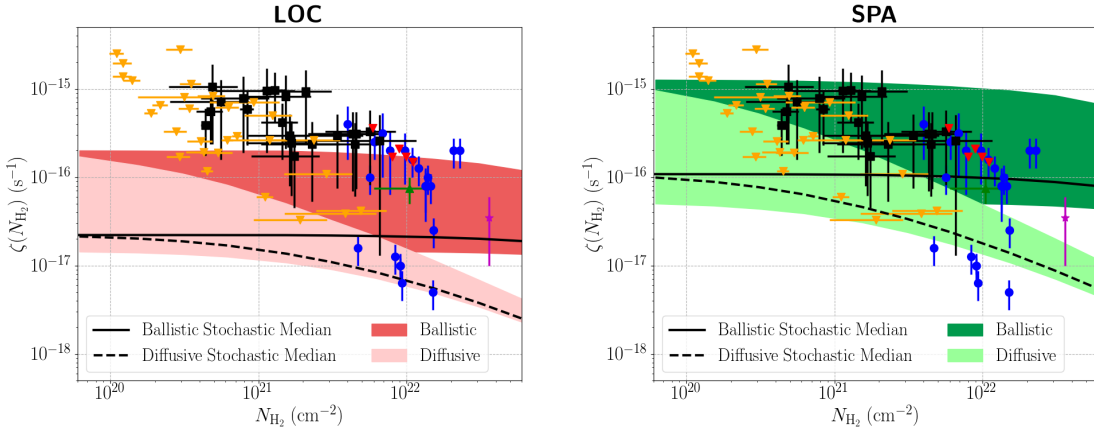
Figure 2: Stochastic fluctuations of the ionisation rate for the local ISM (left) and for the chosen point in a spiral arm (right). The dashed and solid black lines correspond to the median intensities predicted from the diffusive and ballistic model. Data for the ionisation rate are from [18] (filled blue circles), [19] (green triangle), [20] (red triangles are upper limits), [21] (asterisk), and [22] (black squares are data points while inverted yellow triangles are upper limits).

In Fig. 2, we show the ionisation rates due to CR protons and electrons at the LOC position (left panel) and at the SPA positions (right panel), considering both ballistic transport (solid black lines and dark coloured bands) and diffusive transport (dashed black lines and dark coloured bands). For comparison, we also show the inferred ionisation rates and upper limits from observations of molecular absorption lines [18–22]. The stochastic uncertainty bands represent the variation of ionisation rates experienced by a population of molecular clouds. The 95 % uncertainty bands span about one order of magnitude at column densities of $10^{20}\,\mathrm{cm^{-2}}$. For diffusive transport, the uncertainty is significantly reduced at very high column densities. At the LOC position, the uncertainty band covers part of the scatter in the data, but is lower on average than the data. At the SPA position, the ionisation rates are enhanced up to a factor of five which improves the agreement with data. It is noteworthy, that the two possible modes of transport into the molecular clouds, that is ballistic and diffusive motion, lead to disparate predictions for the ionisation rates at column depths of $10^{22}\,\mathrm{cm^{-2}}$ and above. It is tempting to associate a certain degree of bi-modiality in the observed distribution of inonisation rates, but this will require further study.

4. Summary and conclusion

We have discussed the *ionisation problem*, that is the underprediction of the obeserved CR ionisations rates from molecular clouds. We have argued that two effects improve the agreement between models and observations: First, the stochastic spread in predicted ionisation rates due to the point-like and burst-like nature of sources of CRs, that is supernova remnants. And second the higher rate of sources in spiral arms where also the occurrence of molecular clouds is likely enhanced. We have investigated these effects by modelling the transport of CRs from an ensemble of CR sources in a Monte Carlo fashion which amounts to adding up the Green's functions of the

transport equation. We have found that this introduces a significant spread in CR intensities below a GeV or so. If positions in a spiral arm are considered (SPA position), the intensities are enhanced by a factor up to five at MeV energies. The ionisation rates depend sensitively on the position of the molecular clouds, with predicted ionisation rates for molecular clouds at the position of the solar system (LOC position) being in some tension with observations. For clouds in the spiral arm (SPA position), the rates are enhanced by a factor up to five which allows for much better agreement with the data. In addition, the spread afforded by the stochastic nature of sources largely matches the scatter in the observed ionisation rates.

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