Signals of a New Gauge Boson from IceCube and Muon $g \rightarrow 2$

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Gauging the muon lepton flavor minus the tau lepton flavor number, which is a global symmetry of the SM, introduces a new gauge boson $Z'$ upon symmetry breaking. Interestingly, it offers an economical solution to the long-standing $g_\mu - 2$ anomaly, confirmed and strengthened by recent measurements at Fermilab. Here, we revisit the impact of such a $Z'$ on the spectrum of high-energy astrophysical neutrinos, as measured by the IceCube experiment. This spectrum has been observed to exhibit a dip-like feature at sub-GeV energies, which could plausibly arise from the physics of the sources themselves, but could also be the consequence of high-energy neutrinos resonantly scattering with the cosmic neutrino background, mediated by a $Z'$ with a mass on the order of $m_{Z'} \sim 10 \text{ MeV}$. In this study, we calculate the impact of such a $Z'$ on the high-energy neutrino spectrum for a variety of model parameters and source distributions. For couplings that can resolve the $g_\mu - 2$ anomaly, we find that this model could self-consistently produce a spectral feature that is consistent with IceCube’s measurement, in particular if the neutrinos observed by IceCube predominantly originate from high-redshift sources. We also briefly discuss a possible scenario where $Z'$ could act as a portal to a dark sector.
1. Introduction

Measurements of the anomalous muon’s magnetic moment, $a_\mu \equiv (g_\mu - 2)/2$, performed at Fermilab [5, 7] and at the Brookhaven National Laboratory [10] have yielded an experimental average of $a_\mu^{\text{exp}} = 116592061(41) \times 10^{-11}$. Comparing this to the value predicted by the Standard Model based on dispersion relations [8], one obtains

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 251(59) \times 10^{-11},$$  \hspace{1cm} (1)

constituting a $4.2\sigma$ discrepancy. In the years ahead, we expect the experimental uncertainties associated with this measurement to be reduced considerably. In tandem, studies of the hadronic contributions to $a_\mu$ using lattice QCD techniques [9, 11], which currently hint at a lower significance for this discrepancy, promise to substantially refine the Standard Model prediction for this quantity.

The simplest scenario which could potentially resolve this discrepancy is a class of models that introduce a new particle with an MeV-scale mass that couples to muons with a strength on the order of $g_\mu \sim 10^{-4}$ [13, 24]. The Feynman diagrams that provides the leading contribution is shown on the left sketch of Figure 1.

![Feynman diagrams](image)

**Figure 1:** Feynman diagrams providing the leading contribution to the muon’s magnetic moment (left) and the scattering among neutrinos (right) due to the new gauge boson $Z'$. 
Such a particle could have significant implications for astrophysics and cosmology, opening up the possibility that such a state could be constrained or studied using astrophysical probes [15, 17]. In particular, it could cause high-energy neutrinos to appreciably scatter with the cosmic neutrino background (see diagram on the right in Figure 1), impacting the propagation of high-energy neutrinos across cosmological distance scales. Such interactions could induce spectral features that would be measurable at large-volume neutrino telescopes such as IceCube [12, 14, 18, 20].

The IceCube neutrino observatory, which was completed in 2010, consists of an approximately cubic kilometer of Antarctic ice, with over 5000 digital optical modules distributed throughout its volume. This array of detectors is sensitive to essentially two types of events: muon tracks and cascades or showers. The IceCube Collaboration has reported the detection of an approximately isotropic flux of astrophysical neutrinos, spanning energies between several TeV and several PeV [3, 4]. In this study, we focus on the 6-year dataset of shower events presented in Ref. [3], as such events allow for the most direct measurement of the underlying neutrino spectrum. We will consider how this spectrum might be altered in models which include a MeV-scale gauge boson with couplings motivated by the $g_\mu - 2$ anomaly.

The rest of the manuscript is structured as follows. In Sec. 2 we review the model under consideration and its impact on the propagation and spectrum of high-energy neutrinos. In Sec. 3 we briefly discuss the statistical method we follow and present our results. Finally, in Sec. 4, we summarize our results and discuss directions for future research. This document is heavily based on the paper [1], to which we will constantly refer the reader for more details.

2. Model and Spectral features

There are many gauge symmetries beyond those of the Standard Model that could be invoked in order to obtain a new gauge boson [22]. The simplest scenario corresponds to a $U(1)$ symmetry which does not require the introduction of new chiral fermions to cancel gauge anomalies. There are just a few of such symmetries, and, in the light of the very stringent constraints that have been placed on the couplings of a light $Z'$ to electrons or light quarks, the only anomaly-free $U(1)$ that could potentially explain the observed $g_\mu - 2$ anomaly is one that gauges the quantity $L_\mu - L_\tau$ [16]. After the spontaneous breaking of this symmetry, the Lagrangian for this model is given by

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} Z'^{\alpha\beta} Z'^{\alpha\beta} + \frac{m_{Z'}^2}{2} Z'^{\alpha} Z'^{\alpha} + Z'^{\mu} J_{\mu - \tau}^a, \quad (2)$$

where $\mathcal{L}_{\text{SM}}$ is the Standard Model Lagrangian, $Z'^{\alpha\beta} \equiv \partial_\alpha Z'_\beta - \partial_\beta Z'_\alpha$ is the field strength tensor, and $m_{Z'}$ is the mass of the new gauge boson. If no new states charged under this symmetry exist, the $\mu - \tau$ current is given by

$$J_{\mu - \tau}^a = g_{Z'} (\bar{\mu} y^{\alpha} \mu + \bar{\nu}_\mu y^{\alpha} P_L \nu_\mu - \bar{\tau} y^{\alpha} \tau - \bar{\nu}_\tau y^{\alpha} P_L \nu_\tau), \quad (3)$$

where $g_{Z'}$ is the new gauge coupling and $P_L = (1 - \gamma_5)/2$.

The contribution of $Z'$ to the muon’s magnetic moment (at leading order in terms of powers of $g_{Z'}$ [19]) could accommodate the measured value of the muon’s magnetic moment for $m_{Z'} \sim 10 - 300$ MeV; below this range, such $Z$’s are ruled out by cosmological considerations [15], while
larger masses are excluded by laboratory constraints [23]. Such considerations set the relevant region in the parameter space \( \{m_{Z'}, g_{Z'} \} \) associated to \( Z' \).

The coupling of \( Z' \) to muons (as well as tauon and neutrinos) is given by the \( g_\mu - 2 \) anomaly, which leads to a value around \( g_{Z'} \sim 10^{-4} \). Such new gauge boson leads to a significant cross section for neutrino-neutrino scattering given by

\[
\sigma(v_i\bar{v}_j \rightarrow \nu\bar{\nu}) = \frac{2g_{Z'}^2 \left( U_{\mu i}^* U_{\mu j} - U_{\tau i}^* U_{\tau j} \right)^2}{3\pi \left[ (s - m_{Z'}^2)^2 + m_{Z'}^2 \Gamma_{Z'}^2 \right]},
\]

where \( U_{\alpha i} \) is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, and where Greek (Latin) indices denote flavor (mass) eigenstates. Interestingly, in the presence of such an interaction, the scattering of high-energy neutrinos with the cosmic neutrino background could induce potentially observable features in the astrophysical spectrum of such particles that could be observed by neutrino telescopes such as IceCube. For the case of \( m_{Z'} \ll 2m_\mu \), the \( Z' \) will decay almost entirely into neutrino-antineutrino pairs of muon or tau flavor. The neutrino spectrum that reaches Earth can be calculated by solving the following set of coupled differential equations [14]:

\[
-(1 + z) \frac{H(z)}{c} \frac{d\tilde{n}_i}{dz} = J_i(E_0, z) - \tilde{n}_i \sum_j \langle n_{\nu j}(z) \sigma_{ij}(E_0, z) \rangle + P_i \int_{E_0}^{\infty} dE' \sum_{j,k} \tilde{n}_k \left( n_{\nu j}(z) \frac{d\sigma_{kj}}{dE_0}(E', z) \right),
\]

where

\[
\tilde{n}_i \equiv \frac{dN_i}{dE} (E_0, z) \quad \text{and} \quad P_i \equiv \sum_j \text{Br}(Z' \rightarrow \nu_i \nu_j),
\]

and \( N_i \) is the comoving number density of neutrinos in the \( i \)-th mass eigenstate. Note that if the \( Z' \) can only decay into neutrinos, then \( \sum_i P_i = 1 \) and the \( P_i \)'s are uniquely determined by the neutrino mass-mixing parameters. The Hubble rate in the redshift range of interest is given by \( H(z) \approx H_0 \sqrt{\Omega_\Lambda + \Omega_M (1 + z)^3} \) and throughout this paper, we adopt the best-fit cosmological parameters as reported by the Planck Collaboration [6].

The quantity, \( E_0 \), is the neutrino energy as measured at Earth. In absence of scattering, this is related to the energy at the source, \( E \), according to \( E = (1 + z)E_0 \). \( n_{\nu j}(z) \) is the number density of neutrinos of mass eigenstate, \( j \), in the cosmic neutrino background. The function \( J_i(E_0, z) \) describes the spectrum and redshift distribution of the injected high-energy neutrinos. The second term on the right-hand side of Eq. (5) accounts for the disappearance of high-energy neutrinos resulting from their scattering with the cosmic neutrino background, while the rightmost term describes the neutrinos that are produced in those scattering events. More details on each of the terms can be found in [1]. This set of three coupled equations is computationally very expensive to solve for. Once again, we refer the interested reader to reference [1] for more details on the computational methods used.

The high-energy neutrino spectrum is resonantly attenuated when the total energy in the center-of-momentum frame is approximately equal to the mass of the gauge boson, \( m_{Z'} \approx E_{CM} \approx \sqrt{2m_{\nu,i}E_\nu} \). This produces an absorption feature in the observed spectrum at an energy given by

\[
E_\nu \approx \frac{m_{Z'}^2}{2m_{\nu,i} (1 + z_{abs})} \approx 1 \text{ PeV} \times \left( \frac{m_{Z'}}{10 \text{ MeV}} \right)^2 \left( \frac{0.05 \text{ eV}}{m_{\nu,i}} \right) \left( \frac{1}{1 + z_{abs}} \right),
\]
where $m_{\nu,i}$ is the mass of the $i$th neutrino species and $z_{\text{abs}}$ is the redshift at which the scattering takes place. In Figure 2 we provide a sketch of the expected spectral feature induced by such resonant scattering. Essentially, we expect a depletion of the neutrino flux at energies close to the resonant value and an increase of such flux at the immediate lower energy bins due to the down-scattering of neutrinos to lower energies.

3. Statistical analysis and Results

In the previous section we showed how to compute the expected spectrum of high-energy neutrinos at Earth. We fit the cascade data reported in [3] via a likelihood defined for a baseline model in absence of new physics, $L_0$, and including it, $L_m$. In particular, we choose the single power-law model as the benchmark scenario. The best-fit power-law for the neutrino source term, $J(E_0,z) \propto E_0^{-\gamma}$, yields $\gamma = 2.65$.

We acknowledge that this power-law parameterization does not provide a particularly good fit to the measured spectrum, but we use it as it is the customarily adopted benchmark in both experimental and theoretical analyses of IceCube data. In particular, the spectrum appears to be suppressed at energies between $E_\nu \sim 0.2 - 1$ PeV. While this spectral feature could plausibly have something to do with the nature of the sources themselves, we will take this apparent “dip” in the spectrum to motivate models in which neutrinos in this energy range are significantly attenuated by the scattering induced by a new gauge boson. The improvement or worsening of the fit is quantified by the test statistics $TS \equiv -2\ln\left(L_m/L_0\right)$. Under a Gaussian hypothesis for the flux variable, the above test statistics $TS$ reduces to the difference of the familiar chi-squared functions, i.e. $TS = \Delta \chi^2 = \chi^2 - \chi^2_0$, for the model including new physics minus the benchmark astrophysics-only one.

Due to limited statistics of IceCube data, one might question the hypotheses of Gaussianity and independence of flux variables. In reference [1], we provide an analysis of IceCube data assuming a Poissonian distribution of counts in each bins. We refer the reader to the discussion reported in this work for more details. Let us just note that the results are not modified significantly, so in this manuscript we will just report the results assuming Gaussianity.

We will first report the results for an idealized source distribution corresponding to a Dirac delta, where all the sources are located at the same value of redshift $z_0$:

$$J(E_0,z) \propto E_0^{-\gamma} \delta(z - z_0).$$

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Spectral feature induced by the resonant scattering between high energy neutrinos and neutrinos from the cosmological neutrino background and mediated by the new gauge boson $Z'$.}
\end{figure}
**Figure 3:** The spectrum of high-energy neutrinos with a gauge boson with \( m_{Z'} = 13 \text{ MeV} \) and \( g_{Z'} = 5 \times 10^{-4} \) (left panel) and fixing \( z_0 = 6 \) and varying \( m_{Z'} \) (right panel). Results are shown for the maximum allowed total neutrino mass and for the case of the normal mass hierarchy. We take the injected spectral index to be \( \gamma = 2.5 \), and fix the normalization to obtain the best overall fit to the IceCube data.

In Fig. 3, we show the spectrum that results in this case for \( m_{Z'} = 13 \text{ MeV} \) and \( g_{Z'} = 5 \times 10^{-4} \), for four choices of \( z_0 \) (left panel) and for different values of \( m_{Z'} \), for the case of \( z_0 = 6 \) (right panel). We have adopted the normal neutrino mass hierarchy and have set the sum of the three neutrino masses to the maximum (0.12 eV) value allowed by oscillation data and cosmology [6]. We have further set the injected spectral index to \( \gamma = 2.5 \) and fixed the normalization in each case in order to obtain the best overall fit to the IceCube data.

In the case shown in the left frame, notice that the overall magnitude of the resulting attenuation is larger and extends to lower energies for neutrinos that originate from higher redshift sources. Also, one can identify in this figure small bump-like features which result from neutrinos being produced in \( Z' \) mediated scattering events. In the plot shown in the right frame, one can clearly see the effect of varying the gauge boson mass, that is shifting the absorption feature to higher energies for larger values of the mass. The effect of varying other parameters of the model (neutrino hierarchy, gauge coupling constant etc.) are reported in [1].

In Fig. 4 (left panel), we compare the value of the \( \chi^2 \) obtained for the benchmark model without new physics (for the case of \( z_0 = 1 \)) to the best-fit found without a new gauge boson, where we see that the quality of the fit can be improved substantially for \( m_{Z'} \approx 4 - 8 \text{ MeV} \).

Note that the preferred value of the gauge mass is below 10 MeV, which is in tension with cosmological constraints \( (N_{\text{eff}} \, [15]) \). This can be relaxed, however, if the neutrinos originate predominantly from high-redshift sources (middle panel in Fig. 4), allowing good fits to be obtained for larger values of \( m_{Z'} \). In Fig. 4 (right panel), we also report the results obtained for a realistic source distribution. In particular, we consider a source distributions which traces the observed rate of star formation. We see that the preferred mass band is now more diffuse and the resulting spectral feature can be more pronounced. Also, it and can appear in the energy range favored by IceCube for larger values of \( m_{Z'} \) (resulting in less tension with cosmological constraints).

Finally, let us mention that we also studied the case where the new gauge boson is also coupled to an extra light state that do not carry any SM charge. In this case, the scattering of high-energy neutrinos with the cosmic neutrino background could additionally result in the production of light dark sector states. The attenuation feature can now be more pronounced, providing a better fit to the
Figure 4: The improvement in the fit to the high-energy neutrino spectrum as a function of $m_{Z'}$ and $g_{Z'}$, for the case of $z_0 = 1$ (left), $z_0 = 6$ (middle) and SFR redshift distribution (right), and relative to the best-fit power-law without any new gauge boson. Results are shown for the maximum allowed total neutrino mass and for the case of the normal mass hierarchy.

IceCube data, although at the expense of an additional parameter. Once again, we refer the reader to reference [1] for more details on this scenario.

4. Conclusions

Motivated by the measured value of $g_{\mu-2}$, we have considered models with a broken $U(1)_{L_{\mu}-L_{e}}$ gauge symmetry, giving rise to a gauge boson that couples to muons, taus, and their respective neutrinos. Such a gauge boson, with a mass in the range of $m_{Z'} \sim 10 \sim 200$ MeV and with a coupling on the order of $g_{Z'} \sim (3-8) \times 10^{-4}$, could solve the $g_{\mu-2}$ discrepancy while remaining consistent with all constraints and mediate the resonant scattering between high-energy neutrinos and neutrinos from the cosmological neutrino background, inducing spectral features that could be potentially observed by IceCube. The new gauge boson $Z'$ can improve the fit to the IceCube data by $\sim 2\sigma$ for $m_{Z'} \sim O(10)$ MeV, while lower masses are in tension with cosmological measurements of $N_{\text{eff}}$. A reliable detection is still limited by experimental and theoretical uncertainties ($g - 2$ value, neutrino masses, IceCube spectrum [2] and $N_{\text{eff}}$ value [21]), to be substantially reduced in the near future.

References


