

Self-confinement of low-energy cosmic rays around supernova remnants

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Cosmic rays below 10 GeV are important for the dynamics of the interstellar medium. However, modelling of their propagation around potential sources such as supernova remnants is complicated due to their dynamical backreaction. Due to their overdensity close to the acceleration site they produce turbulence upon which scatter, thereby confining themselves in these regions. This highly non-linear problem can be approached by numerically solving the coupled transport equations of cosmic rays and magnetic turbulence. In this work we will focus on supernovae exploding in those phases of the interstellar medium, where these effects are the most relevant, the warm ionised and warm neutral medium. Interestingly we find, that the self-generated turbulence of low energy cosmic rays suppresses the diffusion coefficient by up to two orders of magnitude for several tens of kiloyears. This results in a non negligible grammage accumulated around the source and has to be accounted for in cosmic ray precision fitting.

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1. Introduction

Supernova remnants (SNRs) have long been suspected [2] as the most likely candidate for the bulk of Galactic cosmic rays (CRs). However, to the deflection of CRs by magnetic turbulence, the identification of any individual object as a source of the locally observed CRs is still elusive. Using CR fluxes we can only draw conclusions about large scale properties of CR transport. Recently evidence for acceleration of hadronic CRs has been suggested by gamma-ray data [1, 9]. Additionally gamma ray observations of dense molecular clouds around supernova remnants, likely caused by pion production of escaping cosmic rays, found indications for locally suppressed diffusion around supernova remnants [?].

In the last couple of years, phenomenological models of CR transport have shown that CRs excite plasma instabilities which produce turbulence, upon which CRs scatter (see [16] for a recent review). Close to their acceleration sites, the gradient formed by the overdensity of these particles introduces an additional term in the dispersion relation of plasma waves, leading to a growth of those waves travelling in the direction of the gradient [12]. CR propagation, in this case, turns out to be a highly non-linear problem which is difficult to solve analytically. Cases in which the CR density is low enough to ignore the non-linearity are called test-particle cases.

To simplify the problem and focus on the escape and propagation of CRs, Malkov et. al. [14] introduced the concept of a CR cloud. The CR cloud model considers the coupled dynamics of CRs and magnetic turbulence *after* acceleration and escape of CRs from the source. Most of the previous studies using this approach have focused on the energy range above 10 GeV while the peak of the CR energy density is at lower energies than that. CR at these energies are important for the dynamics of the ISM, since these particles are energetic and abundant enough to penetrate and ionise the interior of molecular clouds, they determine the ionisation rate which ultimately controls not only the chemistry but also the coupling of gas and magnetic fields in star forming regions [8, 19, 21, 22].

Answering the above questions thus requires a better modelling of self-confinement of CRs below 10 GeV around SNRs. However, low-energy CRs undergo significant energy losses which the previously used setups could not cover (see e.g [23]) due to the choice of numerical methods used. Here, we will propose an extended framework which will be able to take into account the energy losses which allows us to model the self-confinement of CRs below 10 GeV. Additionally, we will improve on the spectral transport of magnetic turbulence.

2. Model

We model the transport of cosmic rays in the vicinity of their source in the 1D flux tube approximation, which is valid for distances smaller than the coherence length $L_c = 100$ pc, as perpendicular transport is suppressed compared to parallel transport. We simulate the transition to 3D as a free escape boundary and neglect perpendicular transport. The propagation of CR protons in a 1D flux tube is described by the usual transport equation [4, 10]:

$$\frac{\partial f}{\partial t} + v_A \frac{\partial f}{\partial z} - \frac{\partial}{\partial z} \left[\kappa(z, p, t) \frac{\partial f}{\partial z} \right] - \frac{dv_A}{dz} \frac{p}{3} \frac{\partial f}{\partial p} + \frac{1}{p^2} \frac{\partial}{\partial p} \left[p^2 \left(\frac{dp}{dt} \right) f \right] = Q_p(z, p, t), \quad (1)$$

where $f = f(z, p, t)$ is the phase space density of protons. The second term on the left hand side of eq. (1) describes advection at the (local) Alfvén speed v_A , the third one diffusion with the parallel diffusion coefficient $\kappa(z, p, t)$. Momentum losses by adiabatic expansion are accounted for by the fourth term and other energy loss processes, e.g. due to ionisation or pion production, by the last term. The term $Q_p(z, p, t)$ on the right hand side describes sources of CRs.

The diffusion of CRs is mediated by scattering processes on the turbulent magnetic field with turbulence spectral power $W = W(z, k, t)$. We consider Alfvén waves of wavelength $(2\pi)/k$ as the main scattering centres for CRs with Larmor radius $r_L(p) = 1/k$, that is the interactions are resonant. In this case the diffusion coefficient is given by [3]:

$$\kappa(p, z, t) = \frac{\kappa_B(p)4/\pi}{kW(k, z, t)}, \quad (2)$$

where $\kappa_B(p) = r_L(p)c\beta/3$ is the Bohm diffusion coefficient. This couples eq. (1) to the transport equation for the turbulence spectral power [6]:

$$\frac{\partial W}{\partial t} + \frac{\partial}{\partial z}(v_A W) = (\Gamma_{\text{CR}}(f) - \Gamma_D(W))W + \Gamma_D(W_{\text{BG}})W_{\text{BG}} + \frac{\partial}{\partial k} \left[\kappa_{kk}(W) \frac{\partial W}{\partial k} \right]. \quad (3)$$

Here, $\Gamma_{\text{CR}}(f)$ describes the growth of turbulence due to the resonant streaming instability, and Γ_D are damping terms. The turbulent cascade is described by a diffusion in wavenumber k , in which eddies of similar size interact. In Kraichnan turbulence phenomenology the diffusion coefficient in k is given by [18]:

$$\kappa_{kk}(W) = c_k v_A k^4 W, \quad (4)$$

where $c_k = 0.052$ [7]. Since the continuous injection of turbulence spectral power at large scales and its dissipation at small scales is neglected in this work, this requires the introduction of the background compensating term $\Gamma_D(W_{\text{BG}})W_{\text{BG}}$. Protons at lower energies undergo energy loss processes, which is accounted for by the last term on the left hand side of eq. (1). The dominant processes are Coulomb losses, given by eq. (4.22) of Ref. [15], ionisation, given by eq. 4.32 of Ref. [15], where we corrected for a one order of magnitude to small factor in front of the logarithm and pion production, given by eq. (34) of Ref. [11]. If the drift velocity of CRs $v_d = -D\partial \ln(f)/\partial z$ is larger than the local Alfvén speed v_A , CRs transfer energy to the waves. The resulting growth rate of W can be shown to be [24]

$$\Gamma_{\text{CR}}(f) = -\frac{4\pi}{3} \frac{c |v_A|}{kW(k)U_0} \beta(p) p^4 \frac{\partial f}{\partial z}, \quad (5)$$

where $U_0 = B_0^2/8\pi$ is the energy density of the background magnetic field.

The phase of the interstellar medium in which the SNR expands has important implications. Here will focus on the warm ionised medium (WIM) and warm neutral medium (WNM) phases, since for hot ionised medium the extension of the SNR at the escape time of CR is too large to induce the streaming instability and all other phases have a small filling fraction.

The plasma waves upon which the cosmic rays scatter are damped by a variety of processes, the most important ones for this work are ion neutral damping, Farmer Goldreich damping and non linear Landau damping.

Ion neutral damping In partially ionised plasmas the ions that form the wave encounter collisions with neutral particles. If the frequency of collisions is much larger than the wave frequency, the neutrals are well coupled. The neutral mass density must then be taken into account in the Alfvén speed which makes it smaller than in the case with ions only:

$$v_A = \frac{B}{\sqrt{4\pi\mu m_p n_{i,n}}}. \quad (6)$$

On the other hand, if the collision frequency is much smaller than the wave frequency the neutrals will not co-oscillate and instead dampen the wave. If the ion-to-neutral mass density is small, $\epsilon \ll 1$, the damping rate is approximately given by

$$\Gamma_D^{\text{in}} \approx \frac{\omega_k^2 v_{\text{in}}}{2 [\omega_k^2 + (1 + \epsilon)^2 v_{\text{in}}^2]} \quad \text{with} \quad \epsilon = \frac{m_i n_i}{m_n n_n}, \quad v_{\text{in}} = \frac{m_n}{m_i + m_n} \langle \sigma_{\text{mt}} v \rangle_{\text{in}} n_n. \quad (7)$$

Former Goldreich damping External magnetic turbulence produced by supernova remnants at large scales cascades to smaller scales anisotropically. If self generated turbulence interacts with this this oppositely directed turbulence it gets damped with

$$\Gamma_D^{\text{FG}} = \left(\frac{v_{\text{turb}}^3}{L_{\text{inj}} r_L v_A} \right)^{1/2}. \quad (8)$$

If ion neutral damping of the external turbulence is effective, there exists a lower limit in wave length for Farmer Goldreich damping, explained in [?].

Non-linear Landau damping In regions of increased turbulence, the beat of two Alfvén waves can transfer energy to the background plasma by non-linear Landau damping with damping rate [17]

$$\Gamma_D^{\text{NLLD}} = \sqrt{\frac{\pi}{2} \left(\frac{k_B T}{\mu m_p} \right) \frac{W(k)}{r_L^2(p)}}, \quad (9)$$

where k_B is the Boltzmann constant, T the temperature of the thermal background and μm_p the effective mass.

In the model of a CR cloud particles are initially confined within the SNR and at low energies only escape once the shock dissipates at the end of the Sedov-Taylor phase [23]. The initial phase space density can be described with a step like initial condition:

$$f(z, p) = \begin{cases} f_0(p) & \text{if } z \leq R_{\text{SNR}}(t_{\text{rad}}). \\ 0 & \text{if } z > R_{\text{SNR}}(t_{\text{rad}}). \end{cases} \quad (10)$$

where we take the momentum dependence to be power-law with index -4.2 [13], resulting from modified first order Fermi acceleration at the shock and subsequent escape from the acceleration region. The overall normalisation is set by the requirement that the total CR energy initially contained in the flux tube be $\xi_{\text{CR}} \simeq 10\%$ of the supernova ejecta kinetic energy E_{SNR} . The turbulence spectrum is assumed to be the same everywhere initially and follow the background turbulence spectrum defined by a Kraichnan diffusion coefficient

$$\kappa_0(p) = \frac{\kappa_B(p) 4/\pi}{k W_{\text{BG}}(k)} = 0.03 \frac{\text{pc}^2}{\text{yr}} \left(\frac{p}{10 \text{ GeV}/c} \right)^{1/2} \beta, \quad (11)$$

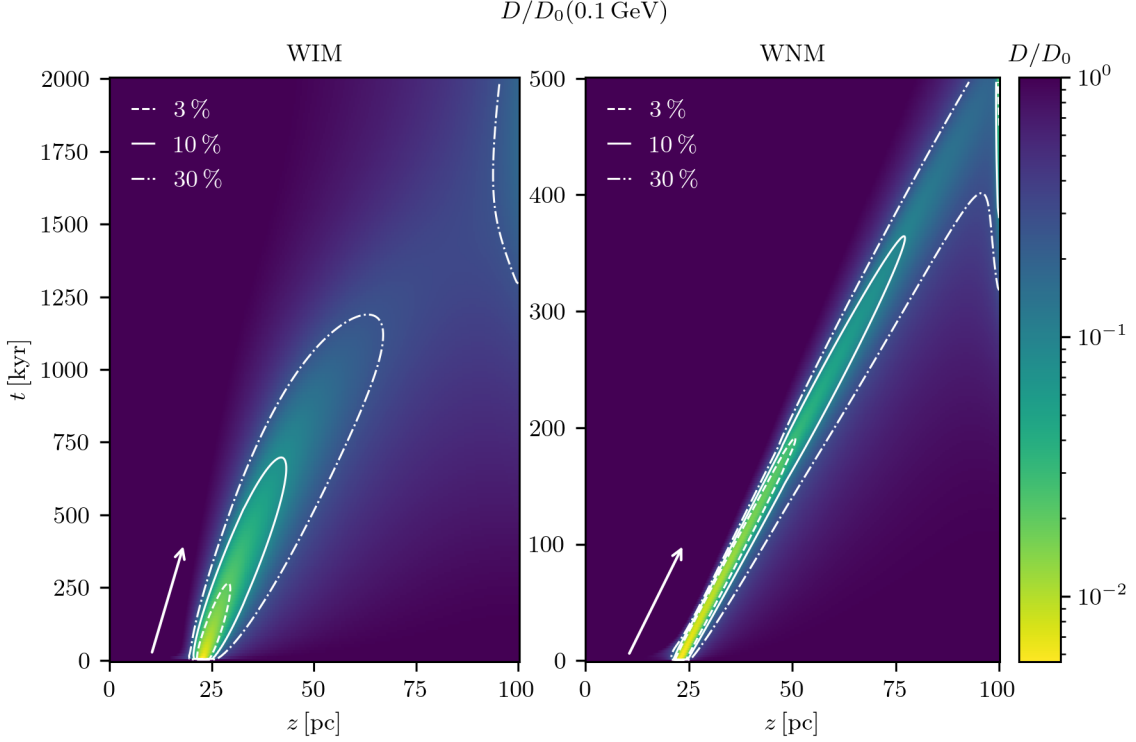


Figure 1: Diffusion coefficient D with respect to its background value D_0 as a function of time and space at 100 MeV. The left panel shows the WIM case and the right one the WNM case. The contour lines mark different suppression values and the white arrows the local Alfvén speed. In both cases the suppression is strongest around the edge of the cosmic ray cloud, where the gradient in phase space density is highest. Over time both the particles and the turbulence propagates outwards. The suppression at the boundary is over-estimated here due to the free escape boundary.

with $k = 1/r_L(p)$ as in eq. (2). Inside the SNR a higher turbulence level is expected, but this has very little impact on the calculations and is therefore omitted here [20].

3. Results

The diffusion coefficient as a function of space and time is shown in Figure 1. After the shock dissipates and the cosmic rays escape from the supernova remnant, the initially strong gradients induces a large suppression of the diffusion coefficient at the edge of the SNR at 23 pc, which is reaching up to two orders of magnitude in both the WIM and WNM cases initially. Due to the small diffusion coefficient, the gradient of cosmic rays is preserved longer compared to the test-particle case, hence leading to a self preserving process. The maximum level of turbulence is reached when the growth rate is balanced by the damping processes. This leads to a very long duration of an outwards propagating region of enhanced turbulence. Within this time turbulence is continuously produced and dissipated on timescales significantly shorter than the duration of the region, hence the level of turbulence represents the changing balance of the decreasing gradient with the damping processes. The Alfvén speed in the WNM is higher than in the WIM, causing the turbulent region

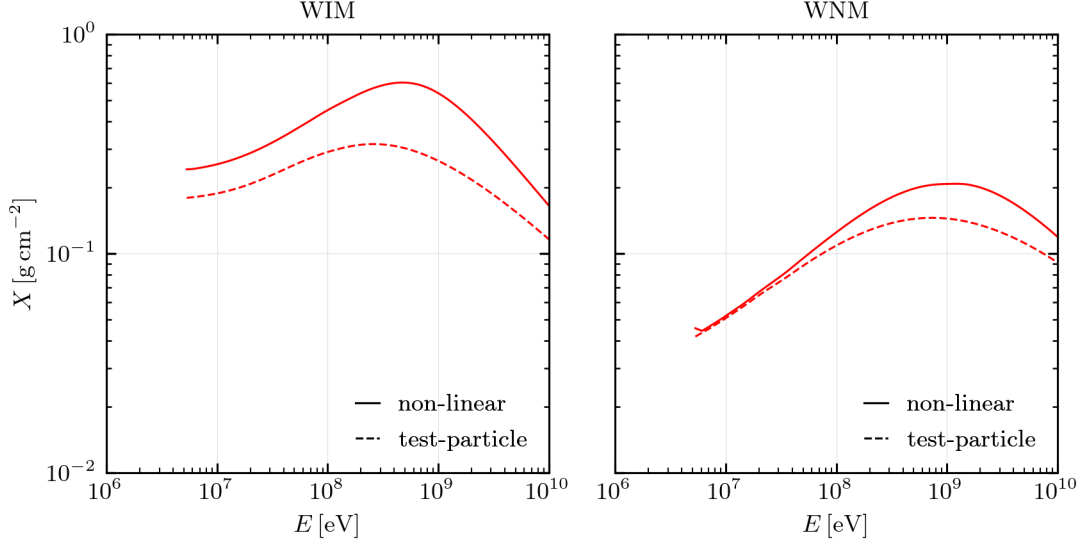


Figure 2: Grammage X accumulated within the simulation domain as a function of kinetic energy E . Left panel: WIM, right panel: WNM. The test-particle solution is marked with a dashed line and the result of the non-linear computation with a solid line. At 10 GeV the grammage agrees with the results of [23]. For WIM the grammage at low energies is increased by up to a factor of 3. For the WNM the increase is smaller.

to be advected close to the region where the flux tube approximation fails at $z = L_c$ within 0.5 Myr. Together with the increased ion-neutral damping this leads to a faster dissipation of turbulence than in the WIM, where a suppression of one order of magnitude lasts up to 0.7 Myr.

The grammage accumulated during the propagation within the flux tube can be calculated similar to the method described by D’Angelo et al. [5]. First we need to define the single particle grammage:

$$X_{1p}(p, t) = \int_0^t \rho v_0(p, t') dt'. \quad (12)$$

Then we need the escape flux of particles, given by:

$$\Phi(p, t) = -D(p, z, t) \left. \frac{\partial f(p, z, t)}{\partial z} \right|_{z=L}. \quad (13)$$

Combining the single particle grammage and the escape flux, we find the average grammage accumulated by an escaped particle to be

$$\langle X(p) \rangle = \frac{\int_0^\infty \Phi(p, t) X_{1p}(p, t) dt}{\int_0^\infty \Phi(p, t) dt}. \quad (14)$$

The results as a function of kinetic energy are shown in Figure 2. The dashed lines show the test-particle case and the solid lines show the non-linear result. Due to the long duration of the suppression of the diffusion coefficient around supernova remnants, the source grammage is increased by up to a factor of 3 compared to the test particle case for the WIM, where the suppression lasts longer. At low energies advection dominates over diffusion, leading to particles being advected

to the boundary. Due to the energy loss being proportional to $E^{-1/2}$, particles will spend less time at low energies, where they travel with $v \ll c$. For the WNM, where the advection speed is larger, this leads to the observed slope with the test-particle and non-linear solutions being nearly identical. In the WIM, where diffusion is still relevant, the grammage becomes energy independent at low energies due to the particles accumulating basically no grammage anymore. At high energies the increase in the diffusion coefficients leads to faster particle escape and a decline in grammage. The maximum of the source grammage is at around 1 GeV, reaching 0.8 g/cm^2 , which is roughly 8% [?].

4. Discussion and Conclusion

In this paper we have studied the propagation of low-energy CRs around SNR. The analytical treatment of this problem is difficult due to the non-linearity of the problem caused by cosmic rays generating turbulence upon which they scatter. We extended previous models based on the simplification of a cosmic ray cloud in a $1D$ flux tube to energies below 10 GeV, where the bulk of the cosmic ray energy density is. We find that the diffusion coefficient at 100 MeV can be suppressed by more than an order of magnitude for 0.7 Myr in the WIM and 0.3 Myr in the WNM. This is significantly longer than found at higher energies. Hence, the source grammage is increased up to a factor of 3 compared to the test particle case, reaching up to 8% of the total grammage of cosmic rays at these energies. Hence, this might be important for precision cosmic ray fitting. Additionally, the long duration and extend of the low diffusion region may impact global cosmic ray propagation. We refer the reader to Ref. [?] for more details.

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