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The uniqueness of the Parker equation solution

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The Parker equation is widely used to describe the modulation of cosmic rays in the heliosphere. In this article, we present a test of the uniqueness of the solution of the Parker equation for 1D and 2D models of heliospheric modulation. Namely for the 1D B-p model and Geliosphere 2D model. The dependence of solution uniqueness on the selected model's input parameters is presented and discussed.

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1. Introduction

The Parker equation is for decades widely used to describe the modulation of cosmic rays in the heliosphere. The equation, depending on how complex the physical description includes, include also not directly measured, but estimated input parameters. The correctness of estimation of these parameters is in some cases verified by comparison of model results with measurements. This is a strategy assuming that the solution of an equation is unique against non-measured model parameters. In this article, we present a test of the uniqueness of the solution of the Parker equation for 1D and 2D models of heliospheric modulation. Namely for the 1D B-p model [1] and Geliosphere 2D model. The proposed method to test the uniqueness of the Parker equation solution consists of a scan of the parametric space of used models, which creates a library of simulated spectra covering the whole parametric space, and a comparison of libraries with measured spectra. Obtained libraries are compared with measured spectra to identify if there are regions in parametric space that give similar solutions. The tool named CudaHelioCommander developed and used for this research is presented in Chapter 2.

2. CudaHelioCommander tool

CudaHelioCommander is a desktop application tool developed using C# and Windows Presentation Foundation (WPF). We chose desktop application over web application because of the ability to work with offline data. Data can be stored on external storage or directly in the memory of the computer from which the user accesses the application. As part of the target operating system, we chose the Windows platform, as it is one of the most widespread and most used operating system in the world. This application was specifically designed to fulfill the requirements outlined in the research paper [2], which focuses on parallelizing cosmic rays modulation in heliosphere models using GPUs using command line interface. The main purpose of CudaHelioCommander is to provide a user-friendly interface for executing simulations. Its primary objective is to serve as a comprehensive software solution for working with heliospheric computations, enabling users to easily interact with and manipulate various aspects of these simulations. The tool is designed to provide users with a software solution for working with heliospheric computations. It enables users to easily interact and manipulate with various aspects of heliospheric simulations. It performs several calculations and offers visualizations to aid in the analysis of the computed data. The screenshot from the tool is shown in figure 1. For the purpose of this research paper, we published lightweight version of the CudaHelioCommander available on [3].

Many researchers download the resulting data to their external devices and share them among themselves. For this reason, the system is designed to support browsing of offline data directly from the computer's memory or external devices. The summary of what the tool does is following:

1. **Calculation of Root Mean Square (RMS) and Maximum Deviation:** The tool calculates the root mean square (RMS) and the maximum deviation of the computed data. These calculations provide statistical measures to evaluate the accuracy and reliability of the implemented computational models.

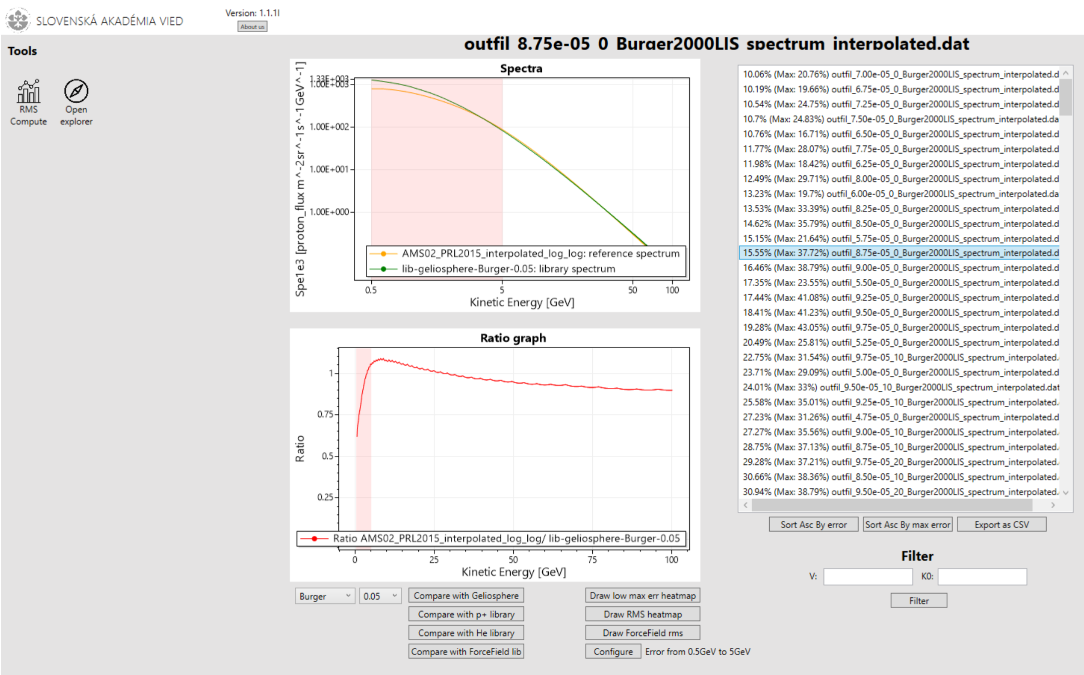

Figure 1: Screenshot of the CudaHelioCommander tool's GUI

2. **Graph Visualization:** The tool enables users to create graphs based on the computed data. Commonly generated graphs include particle energy distribution, particle count, and other related parameters. These graphs offer a visual representation of the computed results and provide researchers with insights into the behavior and characteristics of the simulated particles.

3. **Heatmap Visualization:** In addition to graphs, the tool supports the generation of heatmaps. Heatmaps are visual representations that use color gradients to display the intensity or density of certain data values. Users can create heatmaps to visualize patterns, correlations, or variances in the computed data, enhancing their understanding of the simulation results.

4. **Data Comparison and data visualization:** The tool facilitates the comparison of output files from different models. Users can analyze the level of deviation between the computed results of various models, which helps assess the correctness and accuracy of the implemented models. The tool allows users to visualize the comparison results in the form of graphs. The tool also offers visualization capabilities for spectra data. It can load spectra data from log files and present them in a graphical format. This visualization aids researchers in assessing the progress and quality of the computations, as well as comparing the computed spectra with reference values obtained from other calculations or experimental measurements.

5. **Interactive User Interface:** The tool provides an interactive user interface, allowing users to manipulate and explore the rendered data. Users can zoom in, pan, and select specific data points or regions of interest in the graphs and heatmaps. Such interactivity enhances the user experience and enables a detailed analysis of the computed data.

6. **Data export:** The CSV format is used for simple data and JSON for complex data export. JSON is mainly used to transfer data between web servers and a web client, such as a browser. Compared to XML, it is more readable and in the case of using the JavaScript programming language, no additional functionality is required to process data in this format, as the language contains its own function for processing this format.

Initially, the system was designed for scientific researchers involved in heliospheric simulations at the Slovak Academy of Sciences. The lightweight version of the tool has been made publicly available and is now accessible to any user conducting research on the heliosphere or other interested parties.

3. Evaluation on 1D B-p model comparison with PAMELA data

The usefulness of the system was demonstrated with the verification of computational models by determining the RMS error against the approximated Force field spectra, which were obtained from neutron monitor measurements and PAMELA experiment measurements [4]. We tested all the added functionalities of the system as well as the external tool itself in practice. The tool made it possible to run grid-param calculation, read online data from the server and find the most similar spectra between the set of simulated spectra and the imported spectrum from the experiment by determining the RMS errors.

To evaluate the accuracy of the programmed models, we chose to use the 1D B-p model with a map of parameters for solar wind speed V from $300 km/s$ to $700 km/s$ in steps of $20 km/s$, diffusion coefficients K_0 from $1.5 * 10^{22} cm^2/s$ by $1 * 10^{23} cm^2/s$ with a step of $0.5 * 10^{22} cm^2/s$, with a time step of 50 seconds and a number of injected particles of 160 million. As a result, we should get a total of 372 calculations.

For verification, we used data, proton spectrum, from the PAMELA experiment, from the period from 07/07/2006 to 07/26/2006, which is the first published spectrum of the experiment. This is referred to as S1 and in the article [4]. The data are approximated by a Force field spectrum with a modulation potential of 559MV.

From the measurements we expect that the smallest error, i.e. the smallest difference between the measurement and one of the spectra from the B-p model, will be the combination of parameters $K0 = 3.6 * 10^{22} cm^2/s$ and $V = 429 km/s$. In the tool we can compute the RMS errors percentage, which are displayed directly in the graphical interface. The error in this case is the RMS error η_{RMS}, which indicates the difference between the spectrum from the B-p method and the imported spectrum, which usually comes from measurements. We determine the RMS error according to the following relation 1.

$$\eta_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \eta_i^2} \tag{1}$$

The following relationship 2 applies to the relationship for η_i.

$$\eta_i = \frac{J_{B-p}(T_i) - J_{experiment}(T_i)}{J_{experiment(T_i)}} \tag{2}$$

where T_i is the kinetic energy of the i-th bin, $J_{B-p}(T_i)$ is the intensity for T_i obtained by simulation in the B-p model and $J_{experiment}(T_i)$ is the intensity from the experiment i-th bin. Since the modulation of cosmic rays is highest at low energies, we calculate the RMS error in the energy range 0.1GeV to 2GeV. Based on the value, we can arrange them in ascending order as well as in descending order. In addition, we have implemented a heat map functionality that visually distinguishes the degree of deviation by color. The output is shown in the figure 2.

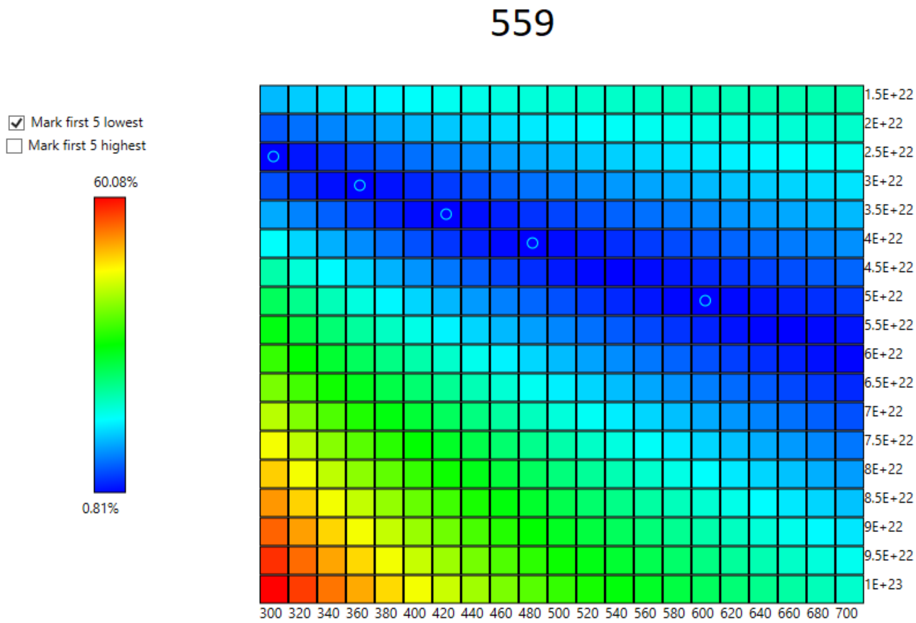

Figure 2: Heat map of 1D B-p model spectra deviations from Force field proton spectrum with a modulation potential of 559MV.

As we can see from the figure 2, the smallest deviations represent the blue color and the combination of simulation parameters with the smallest deviations are shown in the table 1. Our expected combination of switches meets the criteria for the parameters $K_0 = 3.5 * 10^{22} cm^2/s$ and $V = 400 km/s$. We verified that our implementation of the B-p method on the graphics card is correct when compared to loaded experimental data.

K_0 [cm^2/s]	V [km/s]	Deviation [%]
3e22	360	0,81%
5e22	600	0,82%
4e22	480	0,83%
3,5e22	420	0,84%
2,5e22	300	0,84%

Table 1: Table of parameters with the smallest deviation from the Force Field spectrum with a potential of 559MV

These results show an important feature of not only one-dimensional solutions of the B-p and F-p methods. The map shows that several spectra from the B-p method are similar to the spectra from the measurements. For several combinations of solar wind speed and diffusion coefficient parameters, we obtain a very similar spectrum. All spectra from the blue region are similar to the measurement. Since we know from the measurements of the speed of the solar wind what its value was, we can choose from the blue area the value K_0, which belongs to the given measurement. This physically states that if the spectrum from the B-p or F-p model coincides within a small error (e.g. less than 1 percent difference in intensities between 0.1GeV to 2GeV) with the measurement, then the model does not necessarily describe the parameters present during the measurement. In terms of one percent error, the same spectrum is obtained using several combinations of input parameters V and K_0.

4. Geliosphere 2D model comparison with selected AMS-02 daily spectrum

We used the Geliosphere library to evaluate η_{RMS} between Geliosphere simulated spectra and AMS-02 selected daily spectra [5]. The comparison was done for Geliosphere library consisting of 1944 spectra, covering a scan of parameter space with 3 dimensions, diffusion coefficient K_0 (36 values, between $1.0 * 10^{-5}$ and $9.75 * 10^{-5}$ AU^2/s) x tilt angle (9 values, between 0 and 80 degrees) x ratio of the perpendicular diffusion component to the parallel diffusion component R_d (6 values, between 0.05 and 0.3). The comparison was done for selected ranges of energies. In the figure 3 we present a comparison for energies between 0.5 and 5.0GeV. As first, we selected AMS-02 daily spectrum for 12. January 2014. Selection criteria were to have a spectrum from the period where AMS-02 and PAMELA experiments measure in parallel (i.e. May 2011 till January 2014), from the period without geomagnetic storm and the period covered by Geliosphere 2D model current release. The other criterion here was a positive solar period (qA>0). The evaluation for negative solar periods will double parametric space and was left for future work.

The K_0 vs. R_d maps presented in figure 3 show the situation for tilt angles 10,30,50 and 70 degrees. The ranges of η_{RMS} vary in scanned parameter space between relatively small values at a level of 10 percent to big values in order of hundred percent. From the presented figure we could see, that in maps is the region where η_{RMS} values are small with similar values (marked by violet color). It means that there are many spectra from Geliosphere library similar to AMS-02 daily spectrum from 12. January 2014. In other words, we could fit AMS-02 spectrum by Geliosphere model with many combinations of input parameters, namely with many combinations of K_0 and R_d. The tilt angle for January 2014 was 66.3 degrees. Map for tilt angle 70 degrees shows that we could expect similar Geliosphere solution for K_0 and R_d combinations along the violet region of map, for example for combinations $K_0 = 6.25 * 10^{-5} AU^2/s$ and $R_d = 0.1$; $K_0 = 3.5 * 10^{-5} AU^2/s$ and $R_d = 0.2$ (one close to default Geliosphere settings); and for $2.5 * 10^{-5} AU^2/s$ and $R_d = 0.3$. The dots in grayscale on the maps show 100 smallest η_{RMS}, where most dark (black) show the smallest values. Two of these three combinations of parameters used in Geliosphere for evaluation of proton intensities in years 2013 and 2014 for rigidities 1.08 and 5.125GV (energies 0.492GeV and 4.272GeV) are shown in the figure 4. One could see that time evolution of intensities at two selected energies have very similar patterns and values. This further proves that the solution of FPE from the point of view of the combination of input parameters K_0 and R_d is not unique. Small differences between presented intensities could be further tuned by the selection of K_0 and R_d from denser parametric space.

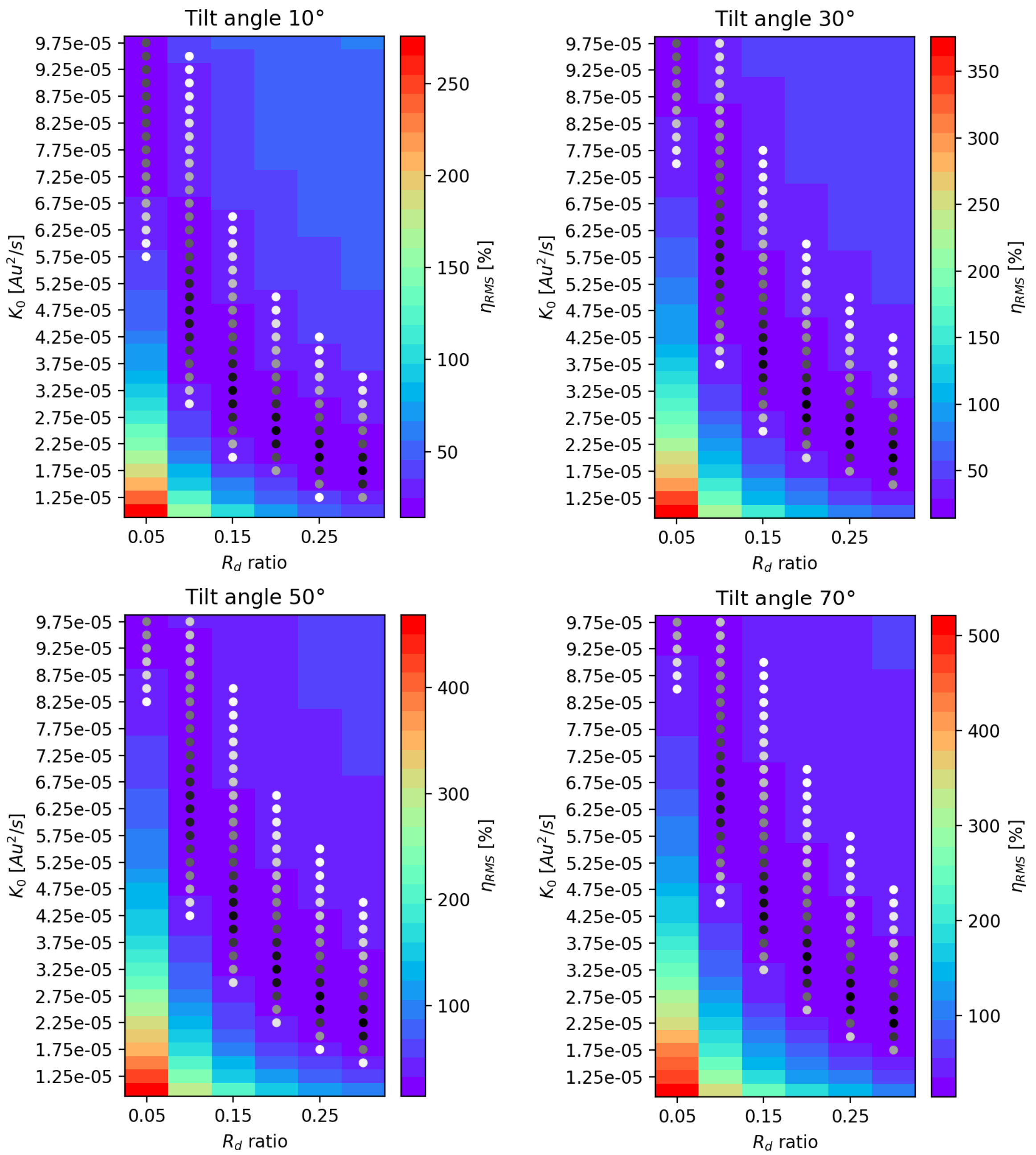

Figure 3: η_{RMS} at K_0 vs. R_d map for Geliosphere library comparison with AMS-02 daily spectrum from 12. January 2014. The situation is shown for tilt angles 10,30,50 and 70 degrees.

5. Conclusion

We show that the scan of parametric space of Parker equation solutions by the Geliosphere 2D model has a region where η_{RMS} values are small with similar values. This show that, there are many spectra from Geliosphere 2D library similar to the selected AMS-02 daily spectrum. In other words, we could fit the AMS-02 spectrum by Geliosphere 2D model with many combinations of

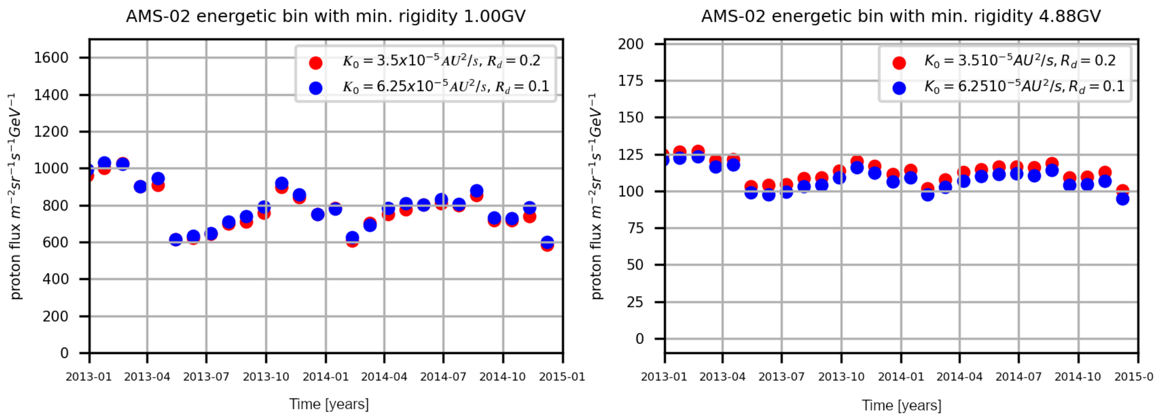

Figure 4: Comparison of Geliosphere simulations with two combinations of input parameters K_0 and R_d for two selected energies in years 2013 and 2014.

input parameters, namely with numerous combinations of K_0 and R_d. The solution of the Parker equation isn't unique as demonstrated for 2 combinations of parameters used for evaluation of proton intensities in the Geliosphere 2D model in years 2013 and 2014 for rigidities 1.08 and 5.125GV.

6. Acknowledgment

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