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On the Flat Proton Spectra at Interplanetary Shocks

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Spacecraft observations of interplanetary shocks have revealed significant deviations in energetic particle spectra from the diffusive shock acceleration (DSA) theory predictions. Within almost two decades of particle energy, spanning about seven e-folds upstream, the particle flux is almost energy independent. Although at and behind the shock, it falls off as ϵ^{-1} (as predicted by DSA for reasonably strong shocks), the flux decreases with the coordinate close to the shock upstream progressively steeper at lower energies, which leads to a flat energy distribution. Within a standard DSA solution under a fixed turbulence spectrum, pre-existing or self-excited by accelerated particles, a flat particle spectrum over an extended upstream area means that the particle diffusivity must be energy-independent, contrary to most transport models. We propose a resolution of this paradox by invoking a strongly nonlinear solution upstream under a self-driven but short-scale turbulence, in which the particle diffusivity increases with energy as $\propto \epsilon^{3/2}$, but also decays with the wave energy as $1/E_{\rm w}$, which compensate for the $\epsilon^{3/2}$ rise. The main difference with the traditional DSA is that the wave-particle interaction is nonresonant, and the turbulence is not saturated at the Bohm level (that would require $\delta B \sim B_0$ turbulence saturation amplitude). A steep, energy-dependent final drop in the particle flux far ahead of the shock to its background level in the solar wind is likely due to a quick particle escape upstream caused by turbulence deficiency.

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1. Introduction

The diffusive shock acceleration (DSA) mechanism is known to be universal and robust. The particle spectrum behind the shock is a power-law in momentum $\propto p^{-q}$, with an index $q = 3r/(r-1)$, that depends solely on the shock compression, r. The simplicity of the DSA is, however, deceptive. This paper considers the DSA disagreement with the spectra observed *in situ,* at interplanetary shocks, e.g., [4, 5, 11]. According to these observations, the particle flux often *flattens upstream,* whereas the downstream part agrees with the DSA. Since the disagreement is partial, it helps identify the DSA elements responsible. Recall that within the DSA, particles gain energy when they cross and recross the shock by scattering off magnetic perturbations of whatever origin. Central to the DSA paradigm is a *simultaneous* growth of wave amplitudes and their lengths during the acceleration. Scattering is most efficient when the wave-particle resonance condition, $kr_{\rm g}(p) \sim 1$, is maintained throughout the wave and particle spectra up to at least the maximum particle momentum, $p_{\rm max}$. Here, $r_{\rm g}$ is the particle gyroradius, $r_g = cp/eB_0$, and k is the wave number of resonant Alfven waves. However, the fastest-growing waves may evolve by mechanisms not necessarily aimed at achieving the most efficient particle scattering, implied in most DSA models. This paper explores alternative particle transport regimes that lead to the observed flat spectra upstream. We obtain them as a modified steady-state analytic DSA solution. Note that flat spectra have been obtained as transients using a numerical model in [10].

2. Observational Hints

Figure 1 demonstrates disagreements with the "standard" DSA upstream but the agreement is close enough downstream, where the particle flux decreases as ϵ^{-1} with energy, which is consistent with the DSA if the shock compression ratio $r \approx 4$. Immediately on the upstream side, the low-energy part of the spectrum decays more steeply with distance from the shock, which is also qualitatively consistent with the DSA, if the particle diffusivity grows with energy, as often implied in DSA treatments. Further ahead, the disagreements are evident.

We first consider the DSA steady-state solution for a given particle diffusivity $\kappa(\epsilon, z)$ to understand them. Assuming that the spectrum is ϵ^{-s} at and behind the shock, the DSA solution upstream arises from a balance between convective and diffusive particle fluxes. In the shock reference frame, moving at a speed u, on its upstream ($z < 0$) side, we thus can write:

$$\kappa(\epsilon, z)\frac{\partial F}{\partial z} - uF = 0. \tag{1}$$

To compare this DSA solution with the data, we use here the particle flux, $F(\epsilon)\,d\epsilon = f(p)\,vp^2dp/4\pi$, with energy normalization, instead of the ordinary particle distribution, f, normalized to $4\pi p^2 dp$. The DSA solution downstream, with freshly injected particles of intensity Q, is $F = Q\epsilon^{-s}$. Here $s = (r+2)/2(r-1) = q/2 - 1$. The data indicate that $s \approx 1$.

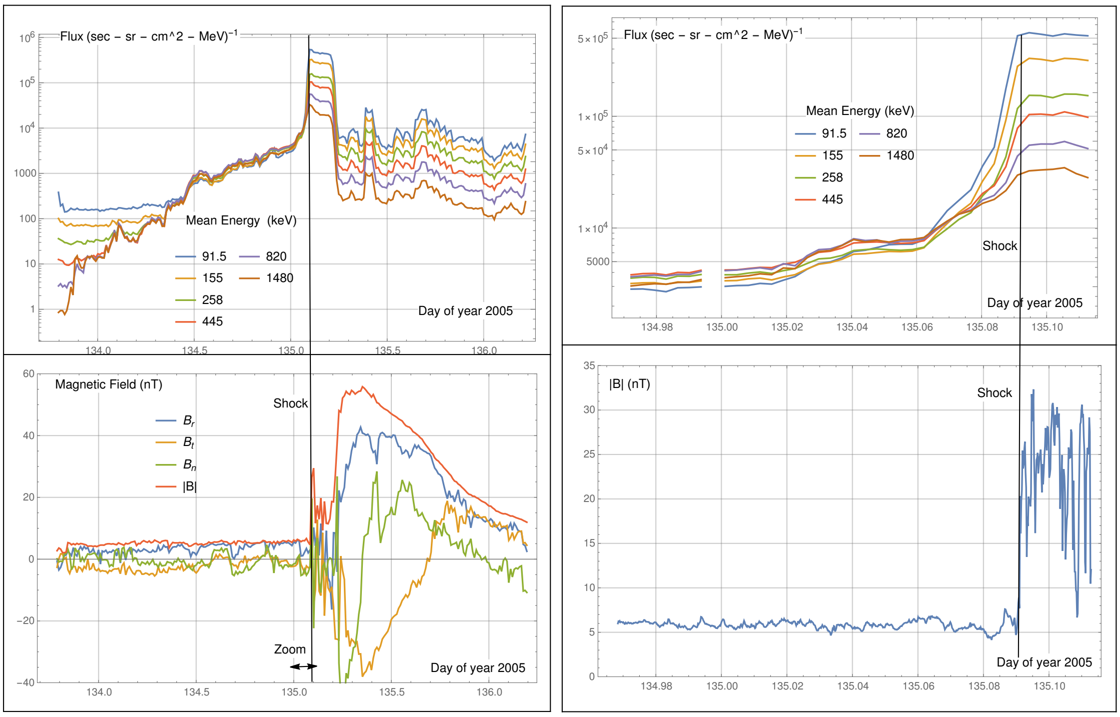

Figure 1: Top panels: observed particle spectra. Bottom panels: magnetic fields. The right panels represent a zoom into the near shock area upstream, marked on the left with arrows.

Far upstream, the flux in Fig.1 becomes *constant*, $F \approx F_\infty(\epsilon)$. Since $F_\infty(\epsilon) \ll F(0,\epsilon)$, we have neglected it in eq.(1) for now. A complete solution then is:

$$F = \begin{cases} Q\epsilon^{-s} \exp\left[-u\int_z^0 \frac{dz}{\kappa(\epsilon,z)}\right], & z<0 \\ Q\epsilon^{-s}, & z \ge 0 \end{cases} \tag{2}$$

Immediately behind the shock, the observed spectrum remains nearly constant (see zoom in Fig.1), which justifies the above solution at $z \ge 0$.

As suggested by Fig.1, the spectrum flattens for $z < z_0$, which requires

$$u\int_{z_0}^0 \frac{dz}{\kappa(\epsilon,z)} = s\ln\frac{\epsilon_0}{\epsilon}, \tag{3}$$

where $\epsilon_0 > \epsilon$ is related to z_0, beyond which the spectrum is observed to be flat and which is about the same for all ϵ. Furthermore, the spectral flatness requires κ to be ϵ-independent at $z < z_0$. An ordinary DSA assumption is that the particle cyclotron resonance with waves supports their diffusion, so $k \sim \omega_c/v$, where v is the particle velocity and the wave frequency $\omega \ll \omega_c$. Hence, for the diffusion coefficient, we have, $\kappa \sim \kappa_{\|} \propto v^3/\delta B_k^2$, e.g., [6]. This relation links z with ϵ in eq.(3) and requires the z- averaged $\overline{\delta B_k^2} \propto k^{-3}\ln(k/k_0)$ within $z_0 < z < 0$, where $k_0 = \omega_c/\sqrt{2\epsilon_0/m}$. Furthermore, if we ignore the logarithm factor, κ becomes energy independent for $z < z_0$ if $\delta B_k^2(z) \propto k^{-3}$. However, it is still inconsistent with a $k^{-1.51}$wave spectrum observed associated with the flat particle flux upstream [11].

Turbulence spectra possibly generated by accelerated particles include: k^{-2} corresponding to the resonant Bohm diffusion of nonrelativistic particles [6] (for $r = 4$), $k^{-5/3}$ (Kolmogorov spectrum), and $k^{-3/2}$ (Iroshnikov-Kraichnan, or IK, almost perfectly agreeing with the observed spectrum). However, the k^{-2}-like spectrum readily emerges from a Fourier decomposition of a periodic system of magnetic discontinuities, which corresponds to a sequence of shocks. Such structures have been observed ahead of bow-, interplanetary and cometary shocks (see [13] for an update). They are often called shocklets or shocktrains and can be easily constructed analytically [8]. Weak shocks can be found in the magnetic data in Fig.1. As we cannot apply the above-mentioned linear resonance condition to the particle interaction with these nonlinear structures [7], we explore an alternative approach below.

Let us assume that shocklets are generated in the shock precursor by one of the instabilities driven by accelerated particles. As it was recently shown [9], the particle pressure gradient may efficiently drive an ensemble of shocks that later evolves into an Alfven spectrum $k^{-3/2}$. The wave-particle interaction is likely dominated by a nonresonant $kr_g \gg 1$ condition, but the phases of the short-scale magnetic perturbations are randomized during the cascade. The nonresonant small-angle scattering frequency is then easily calculated to be $\nu = \Delta\vartheta^2/2t \sim \left(lv/r_g^2\right)\left(\delta B_l/B_0\right)^2$, where l is the turbulence correlation length. Here we assumed that the angle $\Delta\vartheta$ is accumulated from $vt/l \gg 1$ uncorrelated deflections, each of which at an angle $\sim \left(l/r_g\right)\left(\delta B_l/B_0\right)$. Note that unlike the standard quasi-linear derivation that we implied in the resonant spectrum with $\kappa_\parallel \propto v^3/\delta B_k^2$ earlier, the length scale l in the spectral density δB_l is not related to the particle velocity, v. Thus, we arrive at the following expression for $\kappa_\parallel$:

$$\kappa_\parallel = \frac{v^2}{3\nu} \simeq \frac{v^3}{l\omega_c^2}\left(\frac{B_0}{\delta B_l}\right)^2$$

Since l is fixed now, $\kappa_\parallel \sim \epsilon^{3/2}\omega_c^2$, instead of $\kappa_\parallel \propto \epsilon^{3/4}$ for the resonant particle diffusion in the IK turbulence. This change in the diffusivity scaling is central to the flat spectra formation.

Apart from the nonresonant diffusion and other DSA modifications introduced below, we essentially follow Bell's approach to the κ suppression by Alfven waves [1] upstream. Let us introduce a dimensionless wave spectral density, $E_{\rm w}$, related to the rms magnetic field fluctuations, $\left\langle \delta B^2 \right\rangle$, and the background field, B_0, as follows:

$$\frac{\left\langle \delta B^2 \right\rangle}{B_0^2} = \int E_{\rm w}\left(k\right) d\ln k = \int E_{\rm w}\left(p\right) d\ln p.$$

The last relation, implying an approximate resonance $k\rho_g\left(p\right) \sim 1$ [12], is not an accurate cyclotron resonance condition and is implausible in the nonresonant case. Nevertheless, it is physically justifiable to assume that the nonlinear suppression of $\kappa\left(\epsilon\right)$ depends most strongly on the particle flux with the same ϵ. Therefore, as in Bell's approach, we can relate $\kappa \propto \kappa_0\left(\epsilon\right)/F\left(\epsilon\right)$. For, we introduce a partial pressure of energetic particles, normalized, similarly to $E_{\rm w}$, to $d\ln p$:

$$P\left(p\right) = \frac{8\pi}{3\rho V_A^2} v p^4 f, \tag{4}$$

Here f is the ordinary particle distribution function normalized to $p^2 dp$, v and V_A are the particle and Alfven speeds, respectively, and ρ is the plasma density. Balancing the wave pondermotive pressure with that of the energetic particles, e.g., [2], we have

$$\left(M_A - 1\right)\frac{\partial E_{\rm w}}{\partial z} = \frac{\partial P}{\partial z}, \tag{5}$$

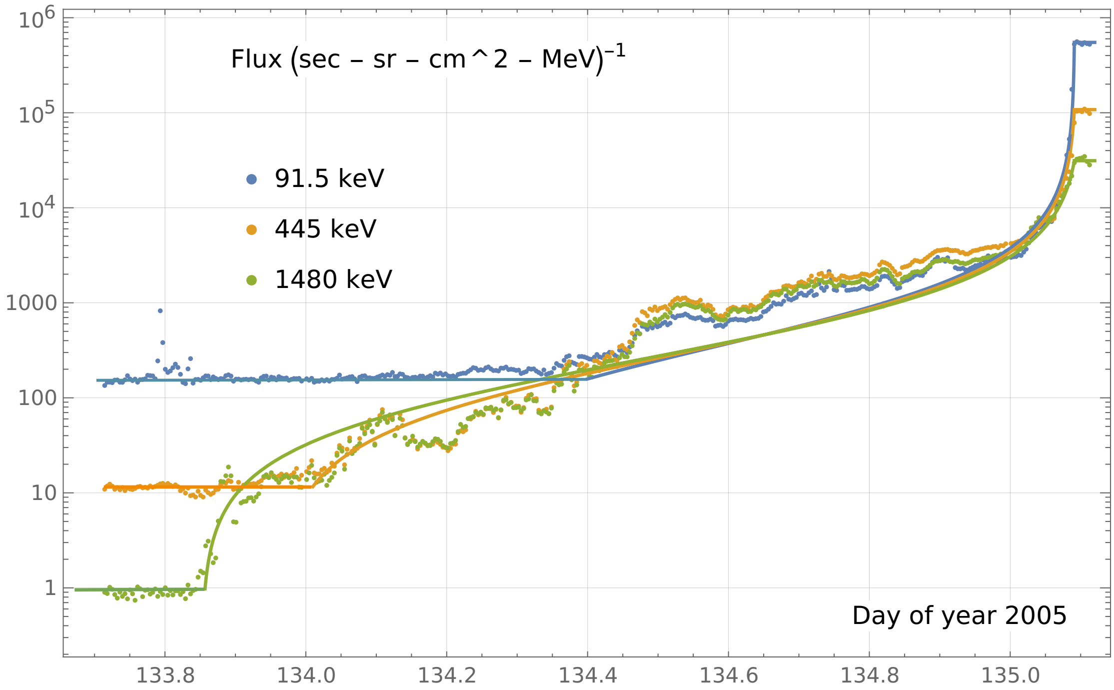

Figure 2: Fit of the data shown in Fig.1 to eq.(13).

where $M_A = u/V_A$. Physically, P and $E_{\rm w}$ are completely independent of each other far upstream from the shock, which we account for by introducing an integration constant, $\psi(p)$:

$$E_{\rm w} = \frac{64\pi^2\sqrt{2m}\epsilon^{3/2}}{3(M_A-1)\rho V_A^2}(F+\psi). \tag{6}$$

Note, that $\psi(p)$ is very small compared to F and can generally be neglected, except it becomes comparable with F far upstream where it ensures a correct transition of F to its far-upstream value.

3. Acceleration Model

Now we turn to a more general than eq.(1) but still ordinary DSA equation

$$u\frac{\partial f}{\partial z} - \frac{\partial}{\partial z}\kappa_\parallel \cos^2\vartheta_{Bn}\frac{\partial f}{\partial z} = -\frac{p}{3}\Delta u\frac{\partial f_0}{\partial p}\delta(z) + Q_0(p)\delta(z). \tag{7}$$

It includes on its r.h.s. the acceleration and injection terms, where the velocity jump $\Delta u \equiv u - u_{\rm d}$, with u_d being the downstream flow speed. Unlike eq.(1), this equation is not constrained by vanishing its l.h.s. at $z = \infty$. It also includes shock wave obliquity, $\vartheta_{nB} > 0$, but assuming that $\kappa_\parallel = \kappa_0/E_{\rm w}$ with $E_{\rm w} \lesssim 1$ we dropped the cross field transport term with $\kappa_\perp \approx \kappa_0 E_{\rm w}$.

Integrating eq.(7) once, we can write on the upstream side:

$$E_{\rm w}^{-1}\cos^2\vartheta_{Bn}\kappa_0\frac{\partial f}{\partial z} - uf(z) = \Phi(p) \tag{8}$$

The integration constant $\Phi(p)$ is thus a minus the total z- independent particle flux (diffusive plus convective) on the l.h.s. If Φ is known, we may write the full solution and see how the particle distribution at the shock, $f_0(p) \propto p^{-4}$, precipitously flattens upstream to $f \propto p^{-2}$, that is to $F(\epsilon) \approx const$. In the traditional DSA framework, $\Phi(p)$ is specified using the far-upstream convective flux $\Phi = \Phi_\infty = -uf(-\infty, p) \equiv -uf_\infty(p)$, provided that $\partial f/\partial z \to 0$, $E_{\rm w} \neq 0$ at $z \to -\infty$. Its relation to $f_0(p)$ is obtained by integrating eq.(7) across the discontinuity at $z = 0$ and matching the particle fluxes across it. However, the data shown in Fig.1, point to the following problems with identifying $\Phi(p)$ in this way.

Far upstream, a gradual decay of $f(p)$ with the distance suddenly flattens to $z-$ independent $f_\infty(p)$ starting from some $z = -z_\infty(p)$. The derivative $\partial f/\partial z$ can be regarded discontinuous at $z = -z_\infty$, thus turning the diffusive particle flux to zero at this point. Hence, the breakpoint at $z = -z_\infty$ (jump of $\partial f/\partial z$) effectively represents a particle sink, similar to the source of injected particles $+Q\delta(z)$, except with the opposite sign. A plausible assumption is that upon diffusing to the point $z = -z_\infty$ against the plasma flow, particles injected and continuously accelerated at the shock front promptly escape toward $-\infty$ other than diffusively. This observation questions typical DSA analyses that do not include the particle sink at $z = -z_\infty$. Curiously enough, it is reminiscent of the "free-escape boundary" invoked in Monte Carlo DSA simulations [3], except its position was assumed energy independent.

As the particle distribution is constant outside $(-z_\infty, 0)$, the boundary value problem for eq.(7) should be formulated for $z \in (-z_\infty, 0)$. We, therefore, set $f = f_\infty(p)$ as the left boundary condition at $z = -z_\infty$. Both f_∞ and z_∞ can be extracted from the data in all energy channels to calibrate the acceleration model.

As a second boundary condition for eq.(7), it is natural to set $f = f_0(p)$ at $z = 0$, which can also be extracted from the data. This strategy is also motivated by poorly known u and u_d needed to determine Φ from the shock matching conditions, mentioned above. Upon relating the wave energy density $E_{\rm w}(z, \epsilon)$ in eq.(8) to the particle flux F by eq.(6), eq.(8) can be rewritten in the following form:

$$K \frac{\kappa_0(\epsilon)\,\epsilon^{-3/2}}{F + \psi(\epsilon)} \cos^2\vartheta_{Bn}(z) \frac{\partial F}{\partial z} - F = \Psi(\epsilon), \tag{9}$$

where we have introduced the notation

$$K \equiv \frac{3\rho V_A \left(1 - M_A^{-1}\right)}{64\pi^2\sqrt{2m}} \text{ and } \Psi(\epsilon) = 2m\frac{\epsilon}{u}\Phi.$$

Eq.(9) thus contains two integration “constants”: $\psi(\epsilon)$ and $\Psi(\epsilon)$, which are not determined. They emerged from the integration of the wave production balance in eq.(5) and the convection-diffusion equation, eq.(7), respectively. As we discussed above, Ψ can in principle, be expressed through the particle injection rate Q and the flow speeds u and u_d:

$$\Psi(\epsilon) = -\frac{\epsilon^{-s+1}}{s+1}\frac{\partial}{\partial \epsilon}\epsilon^s F_0, \tag{10}$$

where we converted the power-law index q, introduced earlier for $f_0(p)$, to its equivalent for $F_0 = F(\epsilon, z = 0)$, $s = q/2 - 1$. As s is poorly known from the data, we do not impose the above relation but determine Ψ from fitting the solution given below to the data.

Returning to ψ, near $z = -z_\infty$, F is way below its values in the rest of the shock precursor. By eq.(5), the same conclusion is true for E_{w}. Hence, the integration constant $\psi(p)$ in eq.(6) must be small compared to F, except near $z = -z_\infty$. Therefore, we treat Ψ and ψ as free parameters and use F_0 and F_∞ as more reliable input parameters for the model. We will approach this problem more systematically in a longer publication, which should further scrutinize our model.

After introducing a new "coordinate" ζ in place of z with the dimensionality $1/[F]$

$$\zeta = \frac{\epsilon^{3/2}}{K\kappa_0(\epsilon)} \int_0^z \frac{dz}{\cos^2 \vartheta_{Bn}(z)}, \tag{11}$$

and rewriting eq.(9) as

$$\frac{1}{F+\psi} \frac{\partial F}{\partial \zeta} - F = \Psi, \tag{12}$$

we obtain its solution, that unconditionally satisfies the boundary condition $F(0,\epsilon) = F_0(\epsilon)$:

$$F = (\Psi - \psi)\left[\left(1 + \frac{\Psi - \psi}{F_0 + \psi}\right) e^{(\psi - \Psi)\zeta} - 1\right]^{-1} - \psi. \tag{13}$$

This solution is shown in Fig.2 for three different energy channels and some power-law representations of Ψ , ψ and a suitable choice of the scaling constant K in eq.(11) to properly convert from the coordinate dependence of F to the time-series given in Fig.2 and satisfy the condition $F(-z_\infty) = F_\infty$ Since for nonresonant wave-particle interactions $\kappa_0 \propto \epsilon^{3/2}$ the normalized distance ζ upstream does not depend on energy, which is crucial for the flatness of $F(\epsilon)$. As seen from its series expansion for $|(\psi - \Psi)\zeta| \ll 1$, valid in the most of the shock precursor,

$$F \simeq \frac{F_0 + \psi}{1 - (F_0 + \psi)\zeta} - \psi, \tag{14}$$

the uncertain quantity Ψ drops, while ψ is small compared to F_0. According to the above formula, it is also small compared to F in the most of the upstream medium. Moreover, for ζ to the left (upstream) of a narrow interval $-1/F_0 < \zeta < 0$, where $2\times10^{-6} < 1/F_0(\epsilon) < 3\times10^{-5}$, the solution can be approximated as $F \approx -1/\zeta$, as long as it exceeds $\psi \sim 10^2$, which is completely flat, as observed. It is curious to note that the approximate version in eq.(14), while not being accurate at large ζ, agrees with the data more closely in the rest of the shock precursor, Fig.3. To fit the data everywhere upstream, it is sufficient to insert power-law dependencies on ϵ for Ψ and ψ in eq.(13) to match the boundary conditions. We defer a discussion of the systematic determination of these model parameters to a longer paper.

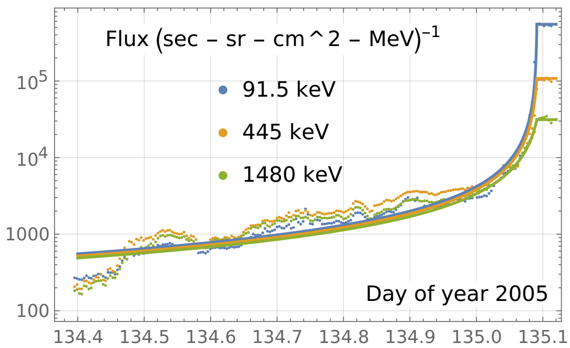

Figure 3: The same as in Fig.2, but using an approximate formula in eq.(14),

Conclusions We have proposed the following two modifications to the DSA theory to explain the flat spectra observed ahead of several interplanetary shocks

- Dependence of particle diffusivity κ on the particle flux F (nonlinear particle transport) through the scattering wave intensity
- Short-scale magnetic perturbations that are generated by, but not resonant with, accelerated particles

In the resulting DSA solution, the particle diffusivity increases with energy as $\propto \epsilon^{3/2}$, simultaneously decaying with the wave energy as $E_{\rm w}^{-1} \propto \epsilon^{-3/2} F^{-1}$, thus turning ϵ- independent almost everywhere in the shock precursor.

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References

[1] Bell, A. R., 1978. *MNRAS*, 182:147.

[2] Drury, L. O., 1983. *Reports on Progress in Physics*, 46:973.

[3] Ellison, D. C., Baring, M. G., and Jones, F. C., 1996. *Astrophys. J.* , 473:1029.

[4] Lario, D., Berger, L., et al., 2018. *Journal of Physics: Conference Series*, 1100(1):012014.

[5] Lario, D., Richardson, I. G., et al., 2022. *The Astrophysical Journal*, 925(2):198.

[6] Lee, M. A., 1982. , 87:5063.

[7] Malkov, M. A. and Diamond, P. H., 2006. *Astrophys. J.* , 642:244.

[8] Malkov, M. A., Kennel, C. F., et al., 1991. *Physics of Fluids B*, 3:1407.

[9] Malkov, M. A. and Moskalenko, I. V., 2021. *The Astrophysical Journal*, 911(2):151.

[10] Ng, C., Reames, D., and Tylka, A., 2003. *The Astrophysical Journal*, 591(1):461.

[11] Perri, S., Prete, G., et al., 2023. *arXiv e-prints*, arXiv:2301.05454.

[12] Skilling, J., 1975. *MNRAS*, 172:557.

[13] Trotta, D., Hietala, H., et al., 2023. *MNRAS*, 520(1):437.