

# Evolution of a Non-Isotropic SN remnant in a Turbulent Background Medium

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We examine how the characteristics of Supernova remnant (SNR) shocks evolve in a non-uniform background with a non-isotropic ejection using the MHD code of Athena++. As an example, the remnant SN1006 is modelled using various scenarios. The turbulent density and magnetic field in the background interstellar medium (ISM) are assumed to have a Kolmogorov spectrum. Using the spherical harmonic function  $Y_{l,m}(\theta, \phi)$  as a basis, we model the non-isotropic ejection also by a Kolmogorov-like spectrum in (l,m). We consider cases where the ejecta speed in one direction differs significantly from other directions by adding a Gaussian profile to the ejection profile. Simulation results are compared to cases of an isotropic supernova explosion in both a uniform and non-uniform background. We find that the morphology of the SNR shock looks noticeably different at earlier times for different scenarios. In later times, ~ 1,000 years, the differences become smaller. Synthetic synchrotron radiation maps for different scenarios are obtained and compared to observations.

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## 1. Introduction

Supernova Remnant shocks are long believed to be the acceleration sites of galactic cosmic rays [2–6]. Earlier models have assumed a hydrodynamic code. However, since the interstellar medium is permeated with galactic magnetic field, a proper modeling of SNR shock requires MHD code. While an easy approach is to assume a uniform background field, the presence of turbulence in the interstellar magnetic field can lead to morphological variations of the SNR shock [9]. Other simulations that model SNRs with a turbulent background are [7] and [8]. However, these simulations does not consider the effects of a non isotropic ejection, which can arise naturally due to the presence of strong stellar magnetic field and various instabilities. In a completely different setting, coronal mass ejections at the Sun, which also drive shock waves and accelerate particles, show clearly non-uniform velocity profile [10]. Therefore a natural question one can ask is what is the effect of a non-isotropic eruption on the morphology of the SNR shock?

At late phases, when the swept mass is larger than the driver mass, the effect of a non-isotropic ejection should be small. However, at younger age, fewer than 1,000 years, there can be clear effects of a non-isotropic ejection. This paper examines the effect of a non-isotropic ejection of SNR shock. We use the Athena++ MHD to model the background ISM and the SNR shock. [1]

#### 2. Isotropic SNR in a Uniform Background

The first case considered is an isotropic SNR in a uniform background. We assume the ejected mass of the remnant has a radial speed given by

$$v_r = \frac{(v_{avg} - v_0)}{\frac{3(r_{out}^4 - r_{in}^4)}{4(r_{out}^3 - r_{in}^3)} - r_{in}} (r - r_{in}) + v_0$$
(1)

Following [9], we set the SNR ejected mass to be  $1.0 \text{ M}_{\odot}$ . It is also assumed that the kinetic energy of the SNR makes up 95% of the total energy, and the gas energy makes up 5% of the total energy. The black circular outline in the figures above is the shock of the SNR.

#### 3. Isotropic SNR in a Turbulent Background

Next, we consider the case of the same isotropic SNR, but now the remnant is expanding in a turbulent background. Both the density and the magnetic field background have a Kolmogorov-like



**Figure 1:** SNR 1006 shock at time 1,125 years showing the shock and initial ejected mass on the left and on the right the shock and the entropy.

spectrum given by

$$\mathbf{B} = \mathbf{B}_{0} + \mathbf{b}$$

$$\mathbf{B}_{0} = B_{0}[\cos(\beta)\hat{x} + \sin(\beta)\hat{y}]$$

$$\mathbf{b} = \sum_{n=1}^{N} A(k_{n})[\cos(\alpha_{n})\hat{x'} + \sin(\alpha_{n})\hat{y'}]\exp(i(k_{n}z' + \gamma_{n}))$$

$$P(k_{n}) = \frac{\Delta V}{1 + (k_{n}L_{c})^{11/3}}$$

$$\Delta V = 4\pi k_{n}^{2}\Delta k_{n}$$

$$A^{2}(k_{n}) = \frac{\sigma_{B}^{2}P(k_{n})}{\sum_{n}^{N}P(k_{n})}$$
(2)

The primed coordinates are related to the unprimed coordinates with the following

$$\begin{pmatrix} x'\\ y'\\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta_n \cos\phi_n & \cos\theta_n \sin\phi_n & -\sin\theta_n\\ -\sin\phi_n & \cos\phi_n & 0\\ \sin\theta_n \cos\phi_n & \sin\theta_n \sin\phi_n & \cos\theta_n \end{pmatrix} \begin{pmatrix} x\\ y\\ z \end{pmatrix}$$
$$\begin{pmatrix} x\\ y\\ z \end{pmatrix}$$
$$\begin{pmatrix} x\\ y\\ z \end{pmatrix} = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \end{pmatrix} \begin{pmatrix} \cos\theta_n \cos\phi_n & -\sin\phi_n & \sin\theta_n \cos\phi_n\\ \cos\theta_n \sin\phi_n & \cos\phi_n & \sin\theta_n \sin\phi_n\\ -\sin\theta_n & 0 & \cos\theta_n \end{pmatrix}$$

## 4. Non-Isotropic SNR in a Turbulent Background

Next, we consider the case of a non-isotropic SNR. This time the remnant will have angular dependence. The functions describing the remnant's velocity and density may now be expanded in

(3)



**Figure 2:** Density and Synchrotron radio emission of a spherically symmetric explosion with ejecta velocity having linear radial dependence with a turbulent density and magnetic field background.  $\sigma_B$  and  $\sigma_\rho$  of the background have value 0.08. Snapshot time is 1,125 years.

terms of the spherical harmonics.

$$v(r,\theta,\phi) = v_r(r)[1+\zeta(\theta,\phi)]$$

$$\langle v(r,\theta,\phi) \rangle = \frac{3}{4\pi (r_{out}^3 - r_{in}^3)} \iiint v(r,\theta,\phi) r^2 \sin\theta dr d\theta d\phi = v_{avg}$$

$$\rho(\theta,\phi) = \rho_0[1+c\zeta(\theta,\phi)]$$

$$\langle \rho(\theta,\phi) \rangle = \frac{1}{4\pi} \iint_{N} \rho(\theta,\phi) \sin \theta d\theta d\phi = \rho_0$$

$$\zeta(\theta,\phi) = \sum_{l=1}^{N} \sum_{m=-l}^{l} A(l,m) * Y_l^m(\theta,\phi)$$

$$A(l,m) = \left[4\pi\sigma^{2} * \Phi(l,m)\right]^{1/2} * \left[\sum_{l=1}^{N} \sum_{m=-l}^{l} \Phi(l,m)\right]^{-1/2}$$

The energy and pressure of the ejected mass is then expressed as

$$E(\theta, \phi) = \frac{1}{2}\rho(\theta, \phi)[v(r, \theta, \phi)]^{2}$$

$$E = E_{KE} + E_{th}$$

$$E_{KE} = 0.95E(\theta, \phi)$$

$$E_{th} = 0.05E(\theta, \phi)$$

$$P(\theta, \phi) = (\gamma - 1)E_{th}$$

$$\langle E(r, \theta, \phi) \rangle = \frac{3}{4\pi(r_{out}^{3} - r_{in}^{3})} \iiint E(r, \theta, \phi)r^{2}\sin\theta dr d\theta d\phi$$
(4)

Using a Kolmogorov-like spectrum to model the remnant, we have the following for  $\Phi(l, m)$ 

$$\Phi(l,m) = \frac{1}{(1+l^{5/6})(1+|m|^{5/6})}$$



**Figure 3:** Non-uniform explosion with turbulent background, showing the density on the left and shock and entropy on the right. Snapshot time is 1,125 years.



**Figure 4:** Non-uniform explosion with turbulent background, showing the magnetic field on the left and synchrotron emission map on the right. Snapshot time is 1,125 years.

#### 4.1 Gaussian Model

Finally, we consider the non-isotropic SNR in a turbulent background, but now we include the following to the velocity

$$v(r,\theta,\phi) = v_r(r) \left[ 1 + \zeta(\theta,\phi) + \frac{A}{\sqrt{2\pi\sigma^2}} exp(-(\theta-\theta_0)/2\sigma^2) \right]$$
(5)



**Figure 5:** SNR 1006 with  $\Phi$  (5/6) spectrum for the density and velocity. Explosion in turbulent background. The standard deviation chosen is  $\sigma = 0.05$ . The plots correspond to time 1,125 years and show the shock and initial ejected mass on the left and on the right the shock and the entropy.

#### 5. Discussion and Conclusion

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