The Fitting Procedure for Longitudinal Shower Profiles Observed with the Fluorescence Detector of the Pierre Auger Observatory

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The Pierre Auger Observatory uses fluorescence telescopes in conjunction with ground level particle detectors to measure high-energy cosmic rays and reconstruct, with greater precision, their arrival direction, their energy and the depth of shower maximum. The depth of shower maximum is important to infer cosmic ray mass composition. The fluorescence detector is capable of directly measuring the longitudinal shower development, which is used to reconstruct the cosmic ray energy and the atmospheric depth of shower maximum. However, given the limited field of view of the fluorescence detector, the shower profile is not always fully contained within the detector observation volume. Therefore, considerations need to be taken in order to reconstruct some events. In this contribution we will describe the method that the Pierre Auger Collaboration uses to reconstruct the longitudinal profiles of showers and present the details of its performance, namely its resolution and systematic uncertainties.
1. Introduction

At lower energies Auger showers land close to a fluorescence detector and only a small fraction of their profile ends up within the field of view. In these cases the estimation of the calorimetric energy (the integral of the energy deposit) requires a significant extrapolation of the fit function beyond the range of the measurements. HEAT is a set of three fluorescence telescopes installed next to the Coihueco fluorescence detectors (FD) site. HEAT telescopes have a field of view (FoV) ranging from 30$^\circ$ to 60$^\circ$ in elevation, while the Coihueco FoV ranges from 1.6$^\circ$ up to 30$^\circ$. The HeCo system (HEAT and Coihueco telescope) helps to extend the FoV, but for lower energies the HeCo FoV is still not large enough. Therefore, “long” track lengths are not common at low energies, and we are forced to introduce constrains to fit shower development profiles in order to reduce the $X_{\text{max}}$ and energy reconstruction biases.

2. The Gaisser-Hillas function

The Gaisser-Hillas (GH) function is used to fit the shower development profile. Originally, the GH function was expressed in terms of $(X_0, \lambda)$ [1]:

$$f_{\text{GH}}^{X_0, \lambda}(X) = \frac{(dE/dX)_{\text{max}}}{\lambda} \left( \frac{X - X_0}{X_{\text{max}} - X_0} \right)^{\frac{X_{\text{max}} - X_0}{\lambda}} \exp \left( \frac{X_{\text{max}} - X}{\lambda} \right),$$  \hspace{1cm} (1)

which has four parameters: the maximum energy deposit, $(dE/dX)_{\text{max}}$, the depth at which this maximum is reached, $X_{\text{max}}$, and shape parameters $X_0$ and $\lambda$. A mathematically equivalent representation of the GH function can be written in terms of parameters $R$ and $L$ [2]:

$$f_{\text{GH}}^{R,L}(X) = \frac{(dE/dX)_{\text{max}}}{L} \left( 1 + \frac{R(X - X_{\text{max}})}{L} \right)^{-2} \exp \left( \frac{-(X - X_{\text{max}})}{RL} \right),$$  \hspace{1cm} (2)

where

$$R = \sqrt{\lambda/|X'_0|}, \quad L = \sqrt{|X'_0|\lambda} \quad \text{and} \quad X'_0 = X_0 - X_{\text{max}}. \hspace{1cm} (3)$$

The properties of the coefficients $(X_0, \lambda)$ and $(R, L)$ are different. Below we explore these differences and will show that it is more convenient to use the $f_{\text{GH}}^{R,L}$ to fit the shower profile, applying constraints to the coefficients $(R, L)$.

2.1 Properties of the coefficients $(X_0, \lambda)$ and $(R, L)$

![Figure 1: Shape of the GH function when changing only $X_0$, $\lambda$, $R$ and $L$ respectively. The units for $dE/dX$ is [PeV/(g/cm$^2$)].](image-url)
The Fitting Procedure for Longitudinal Shower Profiles

José Bellido

Figure 1 shows how the shape of the GH function changes when varying \((X_0, \lambda)\) or \((R, L)\). The width of the shower profile increases for smaller values of \(X_0\), but also for larger values of \(\lambda\). This gives rise to a strong correlation between \(X_0\) and \(\lambda\). Figure 2 shows shower profile fit correlations using the GH functions \(f_{GH}^{X_0}\) and \(f_{GH}^{R,L}\). When performing unconstrained shower profile fits, we only considered profiles longer than 600 g/cm² and with \(X_{\text{max}}\) within the FoV. The observed correlation between \(R\) and \(L\) is negligible compared to the correlation between \(X_0\) and \(\lambda\).

\[ \text{Figure 2: Correlation between reconstructed } X_0 \text{ and } \lambda \text{ (left) and between reconstructed } R \text{ and } L \text{ (middle). Change in calorimetric energy as a function of changing } L \text{ or } R \text{ (right).} \]

The right panel in Figure 2 shows the variation of the GH function integral (i.e. the calorimetric energy) over reasonable ranges of \(R\) and \(L\). \(E_{\text{cal}}\) is directly proportional to \(L\) \([2]\). The impact of changing \(R\) over a reasonable range is negligible, less than 0.3%.

2.2 Correlation of the coefficients \((X_0, \lambda)\) or \((R, L)\) with Energy and with \(X_{\text{max}}\)

\[ \text{Figure 3: Unconstrained fit results of real data: mean values for } X_0, \lambda, L \text{ and } R \text{ as a function of energy. The solid lines correspond to linear fits. The dashed lines correspond to the central values of the constraints used in a constrained shower profile fit.} \]

\[ \text{Figure 4: Unconstrained fit results of CONEX shower (Sibyll2.3d, } E = 10^{19} \text{ eV) profiles for iron (blue), protons (red) and gamma-rays (green). Correlation between reconstructed } X_0, \lambda, L \text{ and } R \text{ with } X_{\text{max}}. \]

Figures 3 and 4 show the correlation of the Gaisser-Hillas coefficients with energy and \(X_{\text{max}}\) respectively. There is a physical correlation of the GH coefficients with energy and \(X_{\text{max}}\). For Figure 4, CONEX simulated showers were used. Physical correlations should be considered when
performing constrained shower profile fits. The correlations of \(X_0\) and \(\lambda\) with energy or \(X_{\text{max}}\) are complicated to quantify, given the strong correlation between \(X_0\) and \(\lambda\). On the other hand, the correlations of \(L\) are easy to consider and \(R\) variations represent negligible changes in the shape of the shower profile.

3. Definition of the constraints in the fit of the shower profile

Constraints on the shape parameters were first implemented in the \(\chi^2\) minimization of the Auger longitudinal profile [3] and are currently part of the profile likelihood fit as

\[
L = L_{\text{GH}} G(p_1; \langle p_1 \rangle, \sigma_{p_1}) G(p_2; \langle p_2 \rangle, \sigma_{p_2}),
\]

where \(L_{\text{GH}}\) is the likelihood for the fit of the energy deposit (Poissonian distribution in the number of photoelectrons) and \(G(p_i; \langle p_i \rangle, \sigma_{p_i})\) are Gaussian distributions for the shape parameters with variance \(\sigma_{p_i}^2\) and centred at the mean values \(\langle p_i \rangle\). The shape parameters \((p_1, p_2)\) can be \((X_0, \lambda)\) or \((L, R)\) depending on whether we use \(f_{\text{GH}}^{L,X_0}\) or \(f_{\text{GH}}^{R,L}\) to fit the shower profile.

The methods to derive \(\langle p_i \rangle\) and \(\sigma_{p_i}\) are of crucial importance to avoid biases in the shape of the profile. These can introduce biases in the calorimetric energy \(E_{\text{cal}}\) and \(X_{\text{max}}\). For that reason, the analysis attempts to derive \(\langle p_i \rangle\) from a study of real data. The values of \(\sigma_{p_i}\) are chosen so that they are large enough to avoid biases, but not too large in order to have an efficient constraint for showers with short track lengths.

3.1 Constraints to the \(f_{\text{GH}}^{L,X_0}\) function in the shower profile fit

Initially the \(f_{\text{GH}}^{L,X_0}\) function was used to fit the shower profiles. For the reconstruction of the data up to ICRC 2015 only \(X_0\) and \(\lambda\) were constrained (this includes data for the energy scale update presented at ICRC 2013 [4] and for the \(X_{\text{max}}\) publication [5]), but for the data presented at the ICRC 2017, a constraint to the width of the shower profile was also included in the fit [6].

3.1.1 Constraints to \(X_0\) and \(\lambda\)

The constraints for \(X_0\) and \(\lambda\) were estimated from data. There, \(X_0\) and \(\lambda\) were determined iteratively, where either \(X_0\) or \(\lambda\) were fixed to the value obtained in the previous iteration. The problem with this procedure is that the values obtained depend very much on the starting parameter. Once e.g. an initial \(X_0\) is picked, one recovers immediately the corresponding \(\lambda\) parameter and vice versa (as suggested by the left panel in Figure 2). Therefore, one of the central values is arbitrary (either \(X_0\) or \(\lambda\)), which depends on the arbitrary choice of the starting parameter. The average values and variances for the mean and standard deviation of the \(X_0\) and \(\lambda\) distributions are:

\[
\langle X_0 \rangle = -120.5 \text{ g/cm}^2 \quad \sigma_{X_0} = 171.7 \text{ g/cm}^2
\]

\[
\langle \lambda \rangle = 60.93 \text{ g/cm}^2 \quad \sigma_{\lambda} = 12.93 \text{ g/cm}^2
\]

They are used in the likelihood of the profile fit defined according to Eq. 4, using two independent Gaussian distributions,

\[
L = L_{\text{GH}} G(X_0; \langle X_0 \rangle, \sigma_{X_0}) G(\lambda; \langle \lambda \rangle, \sigma_{\lambda})
\]
3.1.2 Adding a constraint to the shower profile width

After the update of the energy scale presented at ICRC 2013, it was realized that the energy estimation for low energy events was affected by a rather large negative bias. In 2015 evidence was found in the data for “non-physical” values of the ratio of the calorimetric energy (the integral of the shower profile) and \((dE/dX)_{\text{max}}\) (the amplitude of the shower profile) for lower energy showers (Fig. 5, left). This ratio was called \(k\), and is approximately universal among different primaries and hadronic interaction models as shown in Figure 5 (right). The value of \(k\) is a measure of the width of the shower profile in units of g/cm\(^2\),

\[
k = \frac{E_{\text{cal}}}{(dE/dX)_{\text{max}}}. \tag{8}
\]

From simulations, the mean value of the expected \(k\) distribution changes from around 560 to 635 g/cm\(^2\) over the energy range from \(10^{17}\) to \(10^{20}\) eV (black solid line in Figure 5). The plot on the left in Figure 5 corresponds to the reconstructed average \(k\) values using real events (HeCo and FD events). The small \(k\) values reconstructed at lower energies are biased. The most straightforward solution to remove such a bias was to add a further Gaussian constraint on \(k\) in the fit of the shower profile,

\[
L = L_{\text{GH}} \ G(X_0; \langle X_0 \rangle, \sigma_{X_0}) \ G(\lambda; \langle \lambda \rangle, \sigma_{\lambda}) \ G(k; \langle k(E_{\text{cal}}) \rangle, \sigma_k) \tag{9}
\]

where the mean value of \(k\) is a function of the calorimetric energy.

Figure 5: (left) Reconstructed mean \(k\) values in real data. The solid line correspond to the average \(k\) over all compositions and models (see plot on the right). (right) Predicted mean of the \(k\) distribution as a function of energy, for different compositions and hadronic models. The overall average \(\langle k_{\text{had}} \rangle\) and the corresponding range for \(\sigma(k_{\text{had}})\) are indicated with black solid and dashed lines respectively.

The optimal values of \(\langle k \rangle\) and \(\sigma_k\) had to be derived from simulations. The parameterization of \(\langle k \rangle\) was obtained from the average of the QGSJetII–04, EPOS–LHC, and Sibyll2.3 predictions with a mixed proton and iron composition shown in Figure 5 (right). The value of \(\sigma_k\) was set in order to account for the different models, mass compositions and the shower-to-shower fluctuations. The parameterizations are [6]:

\[
\langle k(E_{\text{cal}}) \rangle = (332.6 + 13.67 \ \log_{10} E_{\text{cal}}) \ \text{g/cm}^2 \quad \sigma_k = 29 \ \text{g/cm}^2 \tag{10}
\]
3.2 Constraints to the $f_{GH}^{R,L}$ function in the shower profile fit

The $f_{GH}^{R,L}$ function was used in the shower profile fit for the ICRC 2019 data production and for this conference it was improved by refining the probability density function for the $L$ parameter.

3.2.1 Constraints that assume a Gaussian distribution for the $L$ and $R$ parameters

The shower profile fit using the $f_{GH}^{R,L}$ function was introduced after the publication, in 2019, of the paper on the measurement of the average shape of longitudinal profiles [7]. It was consequently used for the ICRC 2019 and ICRC 2021 data production and for the papers on the energy spectrum obtained with the 1500 m [8, 9] and 750 m [10] arrays.

The fit with the $f_{GH}^{R,L}$ function has several advantages. The parameter $L$ is to a very good approximation equivalent to $k$ since $L \approx k/\sqrt{2\pi}$, and therefore a fit very similar to that defined by Eq. 9 can be performed using only two constraints:

$$
\mathcal{L} = \mathcal{L}_{GH} G(L; \langle L \rangle, \sigma_L) G(R; \langle R \rangle, \sigma_R)
$$

(11)

Moreover, the average values of the shape parameters can be fixed using the measurements presented in [7], therefore improving the old approach where $\langle k \rangle$ was fixed using MC simulations.

The $\sigma$ of the constraints were defined following the same logic used for $\sigma_k$. The one for $L$ has been derived from that used for $k$ ($\sigma_L = \sigma_k/\sqrt{2\pi}$), while for $R$ we have done a dedicated study using MC simulations. The final parameterizations are:

$$
\langle L \rangle = \left[ 227.3 + 7.44 \left( \log_{10} E_{\text{cal}} - 18 \right) \right] \text{g/cm}^2 \quad \sigma_L = 11.5 \text{g/cm}^2
$$

(12)

$$
\langle R \rangle = 0.257 \quad \sigma_R = 0.055
$$

(13)

The systematic uncertainties in the measurements of $\langle L \rangle$ and $\langle R \rangle$ are 7.3 g/cm² and 0.040 respectively, well below the $\sigma$ of the constraints.

3.2.2 Constraints that assume an exponentially modified Gaussian distribution for the $L$ parameter

From shower simulations we noticed that the $R$ distributions are rather symmetric and the width of the constraint looks large enough to encompass the shower-to-shower fluctuations. However, the situation for the $L$ parameter is more complicated: while the distribution for iron showers is rather narrow and symmetric, the one for protons has a long tail toward large values of $L$ and the constraint is clearly not large enough to encompass all the values of $L$. We noticed that a small bias in $X_{\text{max}}$ was introduced for deep showers when the asymmetric distribution of $L$ was not taken into account.

The $L$ distributions for proton showers for different ranges of $X_{\text{max}}$ are well described by an exponentially modified Gaussian, i.e. the convolution of the normal and exponential probability density functions,

$$
G_{\text{exp}} = e^{-x/\tau_L} \otimes \frac{1}{\sigma_L \sqrt{2\pi}} e^{-((x-\mu)^2)/2\sigma_L^2} = \frac{1}{2\tau_L} \exp \left( \frac{1}{\tau_L} \left( -x + \mu + \frac{\sigma_L^2}{2\tau_L} \right) \right) \text{erfc} \left( \frac{-x + \mu + \sigma_L^2/\tau_L}{\sqrt{2}\sigma_L} \right)
$$

(14)

where $\tau_L$ characterizes the exponential decay and erfc is the complementary error function.
For proton showers the value for $\tau_L/\sigma_L$ increases linearly with $X_{\text{max}}$, regardless of the shower energy. For shallow $X_{\text{max}}$ ($X_{\text{max}}=700\text{ g/cm}^2$) $\tau_L/\sigma_L = 1$, and for deep showers ($X_{\text{max}}=1000\text{ g/cm}^2$) $\tau_L/\sigma_L = 3$. For iron showers all the $L$ distributions are well described by a normal p.d.f. with $\tau_L/\sigma_L \approx 0.6$.

The study of the $L$ distributions has led to a new definition of the likelihood for the fit with the $f_{\text{GH}}^{R,L}$ function. In this new version, the $R$ constraint is the same as the one defined in Section 3.2.1 and the one for $L$ is given by an exponentially modified Gaussian p.d.f.:

$$L = L_{\text{GH}} G_{\text{exp}}(L; \langle L_M \rangle, \sigma_L, \tau_L/\sigma_L) \ G(R; \langle R \rangle, \sigma_R)$$ (15)

$G_{\text{exp}}$ is characterized by three parameters: $L_M$ is the Mode (value of $L$ for which the p.d.f. has its maximum) and its mean value is parametrized as a function of $E_{\text{cal}}$ with Equation 12 using the measurements of the average shape of the longitudinal profiles [7] (justified by the fact that in the bulk of the data there are not so many deep showers), $\sigma_L = 11.5\text{ g/cm}^2$ (the same of the Gaussian constraint, see Equation 12) and $\tau_L/\sigma_L$ is conservatively fixed to the maximum value observed in simulated events, $\tau_L/\sigma_L = 3$ (corresponding to $\tau_L = 34.5\text{ g/cm}^2$). The resulting p.d.f. for the $L$ constraint, in comparison to the Gaussian constraint, is now large enough to encompass the shower-to-shower fluctuations even for the very deep showers.

4. Performance of the constrained fit of the shower profile

Figure 6 show the biases in reconstructed $X_{\text{max}}$ and energy as a function of $X_{\text{max}}$ for three different types of fit constraints. Panels on the left show the biases calculated using simulated events at energies between $10^{17.8}\text{ eV}$ and $10^{18.5}\text{ eV}$. Panels on the right show relative reconstruction differences in real events for energies between $10^{18.5}\text{ eV}$ and $10^{19.0}\text{ eV}$. The biases for $f_{\text{GH}}^{R,L}G$ and $f_{\text{GH}}^{R,L}G_{\text{exp}}$ fits are rather small (left plots), with a slightly better performance for the $f_{\text{GH}}^{R,L}G_{\text{exp}}$ fit.

In contrast, the $f_{\text{GH}}^{L,X_0}$ fit shows a rather large positive bias in $E_{\text{cal}}$ for deep showers. This bias looks correlated with the positive bias also observed for $X_{\text{max}}$. This is not surprising because, $f_{\text{GH}}^{L,X_0}$ fit tends to overestimate the profile width $L$ when the showers are deep. Larger profile widths clearly correspond to larger values of $E_{\text{cal}}$, and when the profile size is increased the fit tends to introduce a positive bias in $X_{\text{max}}$. The relative differences observed using real events take as reference the $f_{\text{GH}}^{R,L}G$ fit, and they are for intermediate energies, between $10^{18.5}\text{ eV}$ and $10^{19}\text{ eV}$.

The relative difference between $f_{\text{GH}}^{R,L}G_{\text{exp}}$ and $f_{\text{GH}}^{R,L}G$ fits show a moderate $X_{\text{max}}$ dependence. In the most extreme case for $X_{\text{max}} \approx 1000\text{ g/cm}^2$, the difference maximize at about 6 g/cm$^2$ for $X_{\text{max}}$ and 3% for energy. The $f_{\text{GH}}^{L,X_0}$ fit gives shifts consistent with those of the $f_{\text{GH}}^{R,L}G_{\text{exp}}$ fit, within 2 g/cm$^2$ and 1%, with the exception of the few events with $X_{\text{max}} \approx 1000\text{ g/cm}^2$.

For higher energies, above $10^{19}\text{ eV}$, the shifts are very small which means that the performance of the three fits are very similar. This is to some extent expected, as at the highest energies the statistical fluctuations of the measured $dE/dX$ are small and the profiles are in general well contained in the field of view of the telescopes.

5. Conclusion

In some cases the estimation of the calorimetric energy requires a significant extrapolation of the fit function beyond the range of the measurements. The constraints in the parameters that
Figure 6: (left) $X_{\text{max}}$ and $E_{\text{cal}}$ biases of the profile fit for the HECO events simulated with Sibyll 2.3d (simulations include a realistic energy spectrum and real atmosphere characteristics). The events were selected using appropriate FoV cuts. (right) $X_{\text{max}}$ and energy relative difference observed with real data for the $f_{\text{GH}}^R$/$f_{\text{GH}}$ fits taking as a reference the $f_{\text{GH}}$ fit. Notice that plots on the left and on the right correspond to different energy ranges.

characterize the shape of the shower profile allow us to take control of the extrapolation. This is particularly important at low energies where the showers are characterized by relatively short track lengths. We have carefully examined the impact on the energy reconstruction of various types of constrained fit. The improvement in the energy resolution and the better control of systematic uncertainties is remarkable when the shower width “L” is constrained in the fit of $f_{\text{GH}}^R$ to the measured profiles.

References


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The Fitting Procedure for Longitudinal Shower Profiles

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The Fitting Procedure for Longitudinal Shower Profiles
José Bellido

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The Fitting Procedure for Longitudinal Shower Profiles

José Bellido

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