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The saturation of the Bell instability and its implications for cosmic ray acceleration and transport

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The non-resonant (Bell) streaming instability driven by energetic particles is crucial for producing amplified magnetic fields that are key to the acceleration of cosmic rays (CRs) in supernova remnants, around Galactic and extra-galactic CR sources, and for the CR transport. We present a covariant theory for the saturation of the Bell instability, substantiated by self-consistent kinetic simulations, that can be applied to arbitrary CR distributions and discuss its implications in several heliospheric and astrophysical contexts.

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1. Introduction

Collisionless shocks associated with supernova remnants (SNRs) are believed to be the primary source of Galactic cosmic rays (CRs) up to the "knee", at rigidities of a few PV. Particles are energized via diffusive shock acceleration (DSA) [1, 2], which requires CRs to be confined close to the shock, and hence strong, turbulent magnetic fields [3]. Magnetic turbulence and acceleration are closely related and the study of the growth and saturation of CR-driven instabilities is crucial to explain the origin of high-energy CRs.

While the left-handed modes driven by the resonant CR streaming instability [4–6] were initially thought to be the only ones important for the self-confinement of CRs, Bell [7] showed that the right-handed modes may grow faster and saturate at much larger amplitudes. Such a *nonresonant streaming instability*, (or "Bell instability"), is crucial for the dynamics of non-relativistic shocks [e.g., 8, 9], as well as for the determining the highest energy achievable in SNRs [e.g., 10–14].

The linear theory shows that the Bell instability grows faster than the resonant instability when the maximally unstable wavelength is much smaller that the CR gyroradius, i.e., $k_{\max} r_L \gg 1$. This happens for large CR currents/energy densities (as discussed, e.g., in §4.3 of [7] and §3 of [15]), provided that the growth rate does not exceed Ω_{ci}, the gyrofrequency of thermal ions. In this regime right-handed, circularly-polarized modes are driven unstable and the wavenumber and the growth rate of the fastest-growing mode read

$$k_{\max} = \frac{4\pi}{c}\frac{J_{\rm cr}}{B_0} = \frac{1}{2}\left(\frac{n_{\rm cr}}{n_{\rm g}}\right)\left(\frac{v_{\rm d}}{v_{\rm A,0}}\right) d_i^{-1}; \quad \gamma_{\max} = k_{\max} v_{\rm A,0} = \frac{1}{2}\left(\frac{n_{\rm cr}}{n_{\rm g}}\right)\left(\frac{v_{\rm d}}{v_{\rm A,0}}\right)\Omega_{ci}\,, \tag{1}$$

where e and m are the proton charge and mass, $J_{\rm cr} = e n_{\rm cr} v_{\rm d}$ is the CR current, $n_{\rm cr}$ and $v_{\rm d}$ are the CR number density and drift velocity relative to the background plasma, B_0 and $n_{\rm g}$ are the background magnetic field and plasma number density, $v_{\rm A,0} \equiv B_0/\sqrt{4\pi m n_{\rm g}}$ is the initial Alfvén speed, $\Omega_{ci} \equiv eB_0/(mc)$, and $d_i \equiv v_{\rm A,0}/\Omega_{ci}$ is the ion inertial length.

Note that, depending on the CR distribution, many differnt modes, both parallel and transverse to the magnetic field may grow, as discussed in the thorough review by Bret [16], where the growth rate of Weibel, two-stream, Buneman, filamentation, Bell, and cyclotron instabilities are compared. Nevertheless, for most astrophysical applications, and surely for SNR shocks, the Bell instability is the most prominent channel for generating magnetic turbulence [e.g., 11, 12, 17–19].

1.1 Simulating the Bell Instability

MHD simulations have shown that, for a fixed CR current, large amplification factors can be achieved [e.g., 7, 20, 21]. Nevertheless, these simulations cannot self-consistently capture the backreaction of the growing modes on the CRs, hence they cannot be used to assess the saturation of the Bell instability. Particle-in-cell (PIC) simulations [22–25] confirmed that for typical CR currents the Bell instability grows as expected and saturates to levels of $\delta B/B_0 \gg 1$, unless the current is too strong [26], in which case a transverse filamentary mode grows faster than Bell. Such a saturation was ascribed to the background plasma being accelerated in the direction of the CR drift velocity, which reduces the effective CR current $\mathbf{J}_{\rm cr}$ [22, 24].

Reville et al. [27] used a MHD+Vlasov code to run *driven simulations* in which the CR current is kept constant, a situation more akin to a shock precursor, where the upstream plasma is constantly

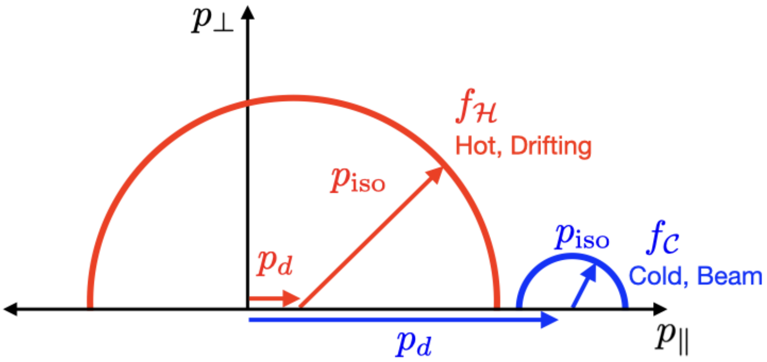

Figure 1: Diagram of the initial CR distributions, distinguishing the hot and cold cases.

exposed to "fresh" CRs; in this work the growth of the field was time-limited, and no saturation of the magnetic field was achieved. Kobzar et al. [28] performed PIC 2D simulations in a non-periodic box to follow the spatio-temporal evolution of the instability, though the very large current they used led to the formation of a shock, which should not happen for arbitrary CR distributions.

In our work [29], we study the Bell instability via hybrid simulations using the massively parallel code `dHybridR` [18, 30], where ions are treated kinetically and their relativistic dynamics is retained. We perform *driven simulations* in which CRs are injected in the simulation box at a constant rate on the left side and are free to leave from the right, while the thermal background plasma and the electro-magnetic fields are subject to periodic boundary conditions. This setup allows for a self-consistent coupling between CRs and thermal plasma, which eventually leads to the saturation of the instability. We explore a large range of parameters that characterize the CR current, always in the regime in which Bell is the fastest growing instability, and use a suite of 1D, 2D, and 3D simulations to investigate how the amplified magnetic field at saturation scales with the CR parameters. We remand to the full paper for the details of the runs and the discussion of the linear and non-linear stages of our benchmark runs, as well as for convergence studies in number of spatial dimensions, box size, particles per cells, and space/time resolution.

In these proceedings we summarize our main finding, namely the formula that gives the amplified magnetic field at saturation as a function of the initial CR net momentum flux and compare such an expression with the original ansatz put forward by Bell based on a heuristic argument. We briefly discuss the implications of this new saturation value, which is typically smaller that Bell's, for astrophysical applications, viz., the maximum energy achievable in SNRs.

2. A General Formula for the Saturation of the Bell Instability

We consider CRs with number density $n_{\rm cr}$ and isotropic monochromatic momentum $p_{\rm iso} \equiv \gamma_{\rm iso} m v_{\rm iso}$ in their rest frame, which drift with velocity $\mathbf{v}_{\rm d} = v_{\rm d}\hat{\mathbf{e}}_x$ relative to the thermal plasma; this corresponds to a current $\mathbf{J}_{\rm cr} = e n_{\rm cr}\mathbf{v}_{\rm d}$; also, be $\mathbf{B} = B_0\hat{\mathbf{e}}_x$ the initial magnetic field.

While the CR current, and hence the growth rate, only depend on $n_{\rm cr}$ and $v_{\rm d}$, the CR net momentum and energy fluxes depend on $p_{\rm iso}$, too. Therefore, we may have two limiting cases: one in which the CRs are a cold beam with $p_{\rm d} \gg p_{\rm iso}$, or a hot drifting distribution with $p_{\rm d} \ll p_{\rm iso}$ (Fig. 1). The goal of this work is to provide a prescription for the amplified magnetic field related to the free energy/momentum flux in the initial CR distribution.

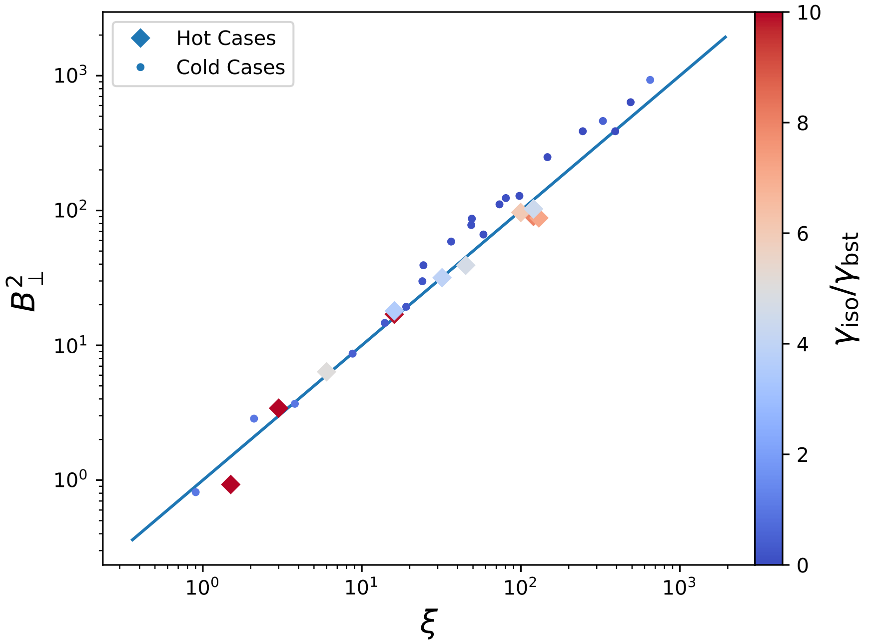

Figure 2: Scaling of the self-generated magnetic field with the parameter introduced in Eq. 5, $\delta B^2/B_0^2 \simeq \xi/4$, for many different hot/cold CR currents (see [29] for more details about the runs).

If we look at the saturation prescription suggested by Bell [7] [see also 31], in the limit in which CRs are relativistic and the drift is not, the final amplification reads:

$$\xi_{\rm Bell} \equiv \left(\frac{\delta B}{B_0}\right)^2 = \frac{U_{\rm cr}}{U_{\rm B}}\frac{v_{\rm d}}{2c} = \eta\gamma_{\rm iso}\frac{v_{\rm d}c}{v_A^2}; \tag{2}$$

where $U_{\rm cr} \equiv \gamma_{\rm iso} m n_{\rm cr} c^2$ and $U_{\rm B} \equiv m n_{\rm g} v_A^2/2$ are the CR and magnetic energy densities and we introduced $\eta \equiv n_{\rm cr}/n_{\rm g}$. Also note that, if one poses $p_\perp \approx \gamma_{\rm iso} mc$ (hot, relativistic CRs), then

$$k_{\rm max} r_{\rm L} = \eta\frac{v_{\rm d}}{v_A}\frac{p_\perp}{m v_A} \simeq \xi_{\rm Bell}. \tag{3}$$

The saturated magnetic field is thus related to the value of $k_{\rm max} r_{\rm L} \gg 1$, in the sense that Bell's *ansatz* is equivalent to asking that at saturation $k_{\rm max}(\delta B) r_{\rm L}(\delta B) \sim 1$.

Eq. 2 is deceitfully similar to the ratio of CR and magnetic *energy fluxes*, but the denominator is *not* the magnetic energy flux (waves do not move at c); this raises the questions of what are the physical quantities that balance out at saturation, and what is the covariant expression that encompasses and generalizes Eq. 2 for arbitrary CR distributions.

In the CR rest frame, the CR mass density is $\tilde{\rho}_{\rm cr}$ and their total (including the rest mass) energy density and isotropic pressure are $e_{\rm cr} = \gamma_{\rm iso}\tilde{\rho}_{\rm cr}c^2$ and $p_{\rm cr} = \frac{1}{3}\gamma_{\rm iso}\tilde{\rho}_{\rm cr}c^2\beta_{\rm iso}^2$. In an arbitrary frame that moves with velocity $\mathbf{v}_{\rm bst}$ and has a corresponding Lorentz factor $\gamma_{\rm bst}$, the CR density is $\rho_{\rm cr} = \gamma_{\rm bst}\tilde{\rho}_{\rm cr}$ and the components of the CR stress tensor read [see, e.g., §133 of 32]:

$$T^{\mu\nu} = (e_{\rm cr} + p_{\rm cr})u_{\rm bst}^{\mu}u_{\rm bst}^{\nu} + p_{\rm cr}\eta^{\mu\nu}, \tag{4}$$

where $u_{\rm bst}^{\mu}$ is the four-velocity constructed with $\mathbf{v}_{\rm bst}$ and $\eta^{\mu\nu}$ is the Minkovski metric. Note that, if CRs are relativistic, then $v_{\rm bst}$ is generally *larger* than the CR drift velocity in the final frame; the

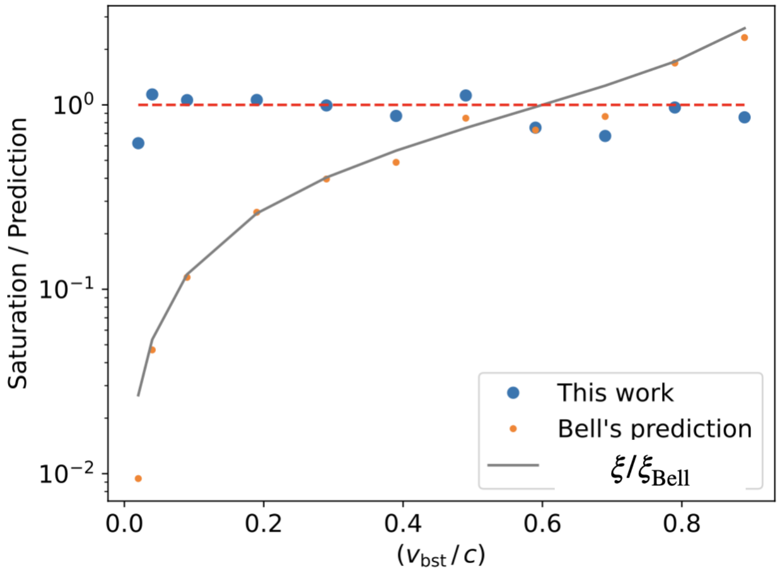

Figure 3: Comparison of Bell's original prescription for saturation with the one in Eq. 5. Our simulations suggest that the system saturates when it runs out of CR momentum flux, rather than energy flux.

two are connected (see, e.g., [25]) and since simulations are setup with an effective boost, we will give provide the saturation as a function of $v_{\rm bst}$ rather than $v_{\rm d}$.

We remand to the full paper [29] for the explanation of how we figured out the components of $T^{\mu\nu}$ that matter and report here just the final result. Fig. 2 shows that the transverse, self-generated, component of the magnetic field at saturation nicely correlates with the following quantity:

$$\xi = \frac{T^{11} - p_{\rm cr}}{P_{B,0}} = \frac{T^{01}}{P_{B,0}}\frac{v_{\rm bst}}{c} = \frac{T^{00} - \tilde{\rho}_{\rm cr}c^2}{P_{B,0}} = 2\gamma_{\rm iso}\frac{n_{\rm cr}}{n_g}\frac{c^2}{v_A^2}\left(\gamma_{\rm bst} - 1 + \frac{\gamma_{\rm bst}}{3}\frac{v_{\rm bst}^2}{c^2}\frac{v_{\rm iso}^2}{c^2}\right), \tag{5}$$

with the final amplified field being $\delta B^2/B_0^2 \simeq \xi/4$. Here ξ has the meaning of the net (i.e., anisotropic) momentum flux in the CRs (see the terms with T^{11} and T^{01}), or equivalently of the density in kinetic energy in the CR drift (term with T^{00}), normalized to the initial magnetic pressure. The prescription is validated by 1D, 2D, and 3D simulations, listed in [29], with many different CR distributions, both in the hot and cold regimes (color code and legend in Fig. 2).

In the Bell limit (relativistically hot, non-rel drift), Eq. 5 reduces to:

$$\xi \simeq \gamma_{\rm iso}\eta\frac{v_{\rm bst}^2}{v_A^2} = \xi_{\rm Bell}\frac{2v_{\rm bst}}{c}, \tag{6}$$

which (except for a factor of ~ 2) corresponds to a saturation smaller than Bell's ansatz by a factor $v_{\rm bst}/c$. The physical meaning of this result is that it is more likely for the instability to quench because of the system running out of net momentum (pressure), rather than of energy flux. Fig. 3 confirms that this is the case: the new prescription for ξ (Eq. 5) provides a better agreement with the saturated $B_\perp$ than Eq. 2, the discrepancy being largest for smaller values of $v_{\rm bst}/c$.

At first sight, this result may exacerbate the issue of accelerating CRs up to the knee in SNRs, which already with Bell's prescriptions comes short of about one order of magnitude for typical remnants [e.g., 33–35]. However, one has to remember that magnetic field amplification in SNRs occurs both in the precursor, due to diffusing CRs (a hot distribution with $v_{\rm d} \sim v_{\rm sh} \ll c$), and far upstream, due to escaping CRs (a rather cold beam with $v_{\rm d} \sim c$) [e.g., 12, 36]. Escaping CRs are fewer in number, by a factor of $\sim v_{\rm sh}/c$ for a p^{-4} distribution, but maximize the drift velocity, having $v_{\rm d} \sim c$; since $\xi \propto v_{\rm d}^2$, the net result is that escaping CRs are predicted to amplify the field more than diffusing ones, at a level comparable with Bell's ansatz when considering diffusing CRs.

3. Conclusions

We have used controlled hybrid simulations to investigate the saturation of the Bell instability [7] for a wide range of CR distributions, spanning from cold beams to hot-drifting cases (Fig. 1). We used a suite of 1D, 2D, and 3D simulations to assess the final amplitude of the self-generated magnetic field and our main finding is that what controls the saturation is the net momentum flux in CRs, not their energy flux (Eq. 5). This suggests that in shocks most of the amplification must be driven by escaping particles, rather than by CRs diffusing in the precursor, but it does not change the expectation for the highest energy CRs may achieve in SNRs.

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References

[1] A. R. Bell. The acceleration of cosmic rays in shock fronts. I. *MNRAS*, 182:147–156, January 1978.

[2] R. D. Blandford and J. P. Ostriker. Particle acceleration by astrophysical shocks. *ApJL*, 221:L29–L32, April 1978.

[3] Pasquale Blasi, Elena Amato, and Damiano Caprioli. The maximum momentum of particles accelerated at cosmic ray modified shocks. *MNRAS*, 375(4):1471–1478, 2007.

[4] R. Kulsrud and W. P. Pearce. The effect of wave-particle interactions on the propagation of cosmic rays. *ApJ*, 156:445, May 1969.

[5] E. G. Zweibel. Energetic particle trapping by Alfven wave instabilities. In J. Arons, C. McKee, and C. Max, editors, *Particle Acceleration Mechanisms in Astrophysics*, volume 56 of *American Institute of Physics Conference Series*, pages 319–328, November 1979.

[6] A. Achterberg. Modification of scattering waves and its importance for shock acceleration. *A&A*, 119:274–278, March 1983.

[7] A. R. Bell. Turbulent amplification of magnetic field and diffusive shock acceleration of cosmic rays. *MNRAS*, 353:550–558, September 2004.

[8] Colby C. Haggerty and Damiano Caprioli. Kinetic Simulations of Cosmic-Ray-modified Shocks. I. Hydrodynamics. *ApJ*, 905(1):1, December 2020.

[9] Damiano Caprioli, Colby C. Haggerty, and Pasquale Blasi. Kinetic Simulations of Cosmic-Ray-modified Shocks. II. Particle Spectra. *ApJ*, 905(1):2, December 2020.

[10] D. Caprioli and A. Spitkovsky. Cosmic-Ray-induced Filamentation Instability in Collisionless Shocks. *ApJL*, 765:L20, March 2013.

[11] D. Caprioli and A. Spitkovsky. Simulations of Ion Acceleration at Non-relativistic Shocks: I. Acceleration Efficiency. *ApJ*, 783:91, March 2014.

[12] D. Caprioli and A. Spitkovsky. Simulations of Ion Acceleration at Non-relativistic Shocks: II. Magnetic Field Amplification. *ApJ*, 794:46, October 2014.

[13] D. Caprioli and A. Spitkovsky. Simulations of Ion Acceleration at Non-relativistic Shocks. III. Particle Diffusion. *ApJ*, 794:47, October 2014.

[14] P. Crumley, D. Caprioli, S. Markoff, and A. Spitkovsky. Kinetic simulations of mildly relativistic shocks: Particle acceleration in high mach number shocks. *MNRAS*, 485:5105–5119, June 2019.

[15] E. Amato and P. Blasi. A kinetic approach to cosmic-ray-induced streaming instability at supernova shocks. *MNRAS*, 392:1591–1600, February 2009.

[16] A. Bret. Weibel, Two-Stream, Filamentation, Oblique, Bell, Buneman...Which One Grows Faster? *ApJ*, 699(2):990–1003, July 2009.

[17] D. Caprioli, H. Zhang, and A. Spitkovsky. Diffusive shock re-acceleration. *JPP*, January 2018.

[18] Colby C. Haggerty and Damiano Caprioli. dHybridR: A Hybrid Particle-in-cell Code Including Relativistic Ion Dynamics. *ApJ*, 887(2):165, 12 2019.

[19] Luca Orusa and Damiano Caprioli. Fast particle acceleration in 3D hybrid simulations of quasi-perpendicular shocks. *PRL, in press*, page arXiv:2305.10511, May 2023.

[20] A. R. Bell. The interaction of cosmic rays and magnetized plasma. *MNRAS*, 358:181–187, March 2005.

[21] J. H. Matthews, A. R. Bell, K. M. Blundell, and A. T. Araudo. Amplification of perpendicular and parallel magnetic fields by cosmic ray currents. *MNRAS*, 469:1849–1860, August 2017.

[22] Mario A. Riquelme and Anatoly Spitkovsky. Nonlinear Study of Bell's Cosmic Ray Current-Driven Instability. *ApJ*, 694(1):626–642, March 2009.

[23] Yutaka Ohira, Toshio Terasawa, and Fumio Takahara. Plasma Instabilities as a Result of Charge Exchange in the Downstream Region of Supernova Remnant Shocks. *The Astrophysical Journal Letters*, 703(1):L59, 2009.

[24] L. Gargaté, R. A. Fonseca, J. Niemiec, M. Pohl, R. Bingham, and L. O. Silva. The Nonlinear Saturation of the Non-resonant Kinetically Driven Streaming Instability. *ApJL*, 711:L127–L132, March 2010.

[25] Siddhartha Gupta, Damiano Caprioli, and Colby C. Haggerty. Lepton-driven Nonresonant Streaming Instability. *ApJ*, 923(2):208, December 2021.

[26] J. Niemiec, M. Pohl, T. Stroman, and K.-I. Nishikawa. Production of Magnetic Turbulence by Cosmic Rays Drifting Upstream of Supernova Remnant Shocks. *ApJ*, 684:1174–1189, September 2008.

[27] B. Reville and A. R. Bell. Universal behaviour of shock precursors in the presence of efficient cosmic ray acceleration. *MNRAS*, 430:2873–2884, April 2013.

[28] Oleh Kobzar, Jacek Niemiec, Martin Pohl, and Artem Bohdan. Spatio-temporal evolution of the non-resonant instability in shock precursors of young supernova remnants. *MNRAS*, 469(4):4985–4998, August 2017.

[29] Georgios Zacharegkas, Damiano Caprioli, and Colby Haggerty. Modeling the Saturation of the Bell Instability using Hybrid Simulations. *arXiv e-prints*, page arXiv:2210.08072, October 2022.

[30] L. Gargaté, R. Bingham, R. A. Fonseca, and L. O. Silva. dHybrid: A massively parallel code for hybrid simulations of space plasmas. *Computer Physics Communications*, 176(6):419–425, March 2007.

[31] P. Blasi, E. Amato, and M. D'Angelo. High-energy cosmic ray self-confinement close to extra-galactic sources. *Physical Review Letters*, 115(12):121101, September 2015.

[32] L. D. Landau and E. M. Lifshitz. *Fluid Mechanics, Second Edition: Volume 6 (Course of Theoretical Physics)*. Course of theoretical physics / by L. D. Landau and E. M. Lifshitz, Vol. 6. Butterworth-Heinemann, 2 edition, January 1987.

[33] A. R. Bell, K. M. Schure, B. Reville, and G. Giacinti. Cosmic-ray acceleration and escape from supernova remnants. *MNRAS*, 431:415–429, May 2013.

[34] Martina Cardillo, Elena Amato, and Pasquale Blasi. On the cosmic ray spectrum from type ii supernovae expanding in their red giant presupernova wind. *Astroparticle Physics*, 69:1–10, 2015.

[35] Pierre Cristofari, Pasquale Blasi, and Damiano Caprioli. Microphysics of Diffusive Shock Acceleration: Impact on the Spectrum of Accelerated Particles. *ApJ*, 930(1):28, May 2022.

[36] D. Caprioli, P. Blasi, E. Amato, and M. Vietri. Dynamical feedback of self-generated magnetic fields in cosmic ray modified shocks. *MNRAS*, 395:895–906, May 2009.