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A parametric analytical model of the atmospheric muon flux at sea-level and its application in the field of the muon transmission radiography

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Atmospheric muons represent the primary component of cosmic radiation detected at sea-level. Their specific characteristics - natural abundance, energy-based ability to penetrate objects, harmlessness - make them an essential tool for performing non-destructive imaging of the internal structure of objects by means of muon transmission radiography (MTR), a 2D technique optimised for studying large objects such as mountains and volcanoes, and multiple scattering muon tomography (MSMT), a 3D technique exploited for relatively small objects. On the other hand, the considerable presence of atmospheric muons and secondary particles produced by them influences the measurements of low background experiments, designed to reveal dark matter and neutrinos signals. In all cases, the need to have a reliable model that accurately describes the energy spectrum and the angular distribution of the flux of atmospheric muons at sea-level is evident. This contribution describes the development of a parametric analytical model built by fitting a selected set of empirical models developed in the last decades to the data measured by ADAMO (Florence, Italy, 2004) and other experiments. The developed model was subsequently used as a generator tool for Monte Carlo simulations in some applications of the MTR technique and the results will be presented and discussed.

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1. Introduction

Muons are a significant component of cosmic radiation. When a high energy particle, mostly an atomic nucleus roaming across the universe (a so-called "primary cosmic ray") collides with a nucleus in the upper atmosphere, mostly Nitrogen and Oxygen, it produces a shower of particles (the so-called "secondary cosmic rays"). Among these secondary particles are short-lived charged pions and kaons that in their decay chain produce muons, at a height of around 15 km above sea-level. Muons are unstable particles, decaying into electrons and neutrinos with a lifetime $\tau \sim 2.2\ \mu$s at rest, but a well-known relativistic effect dilates their observed lifetime allowing many of them to reach the sea-level from every direction with a large distribution of momenta. At sea-level, muons constitute the vast majority of the charged particles, arriving at a rate of roughly 100 Hz/m^2 [1].

Several muons properties make them an essential tool for performing non-destructive imaging of the internal structure of a scanned object: natural abundance, absence of strong nuclear interactions, negligible probability of producing electromagnetic cascades (up to very large momenta, $\sim$ 500 GeV/c), relatively small energy losses by ionization, harmlessness to the structure of object. In particular, depending on the features of data collection and analysis, it is possible to investigate the internal structure of targets by means of Muon Transmission Radiography (MTR), a 2D technique optimised for studying large objects such as mountains and volcanoes, and Multiple Scattering Muon Tomography (MSMT), a 3D technique exploited for relatively small objects.

The features of atmospheric muons are also capable of influencing neutrino physics experiments. On the one hand, they represent the most abundant signal in a neutrino telescope and they can be used to calibrate the detector and to check its expected response to the passage of charged particles. On the other hand, however, they can represent a background source because downward-going muons can incorrectly be reconstructed as upward-going particles and mimic high energy neutrino interactions [2]. Likewise, atmospheric muons can be a nuisance in the measurements of low background experiments, designed for example to reveal dark matter signals [3].

In all cases, the need to have a reliable model that accurately describes the energy spectrum and the angular distribution of the flux of atmospheric muons at sea-level is then essential.

This paper illustrates the development of a parametric analytical model describing the atmospheric muons flux at sea-level built by fitting a selected set of empirical models developed in the last decades to the data measured by the Altazimuth Detector for Atmospheric Muons Observation (ADAMO) spectrometer and by other experiments present in literature. The developed model was subsequently used as a generator tool for Monte Carlo simulations in some applications of the MTR technique performed with the Muon Imaging for Mining and Archaeology (MIMA) tracker. In detail: in section 2 the selected parametric analytical model is described, in section 3 the features of the data collected by the ADAMO spectrometer and by other selected experiments present in literature are described and the result of the performed fit is presented, in section 4 the features of the MIMA tracker are explained and the results of a data collection campaign in MTR conducted with it by using the parametric analytical model previously built are presented and discussed.

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2. The spectrum of incident cosmic muons and some empirical models describing it

Two different approaches have been developed in order to obtain the distribution of the muon flux at sea-level [4]:

- use Monte Carlo methods to simulate the interaction process of primary cosmic ray incident into the Earth's atmosphere, the subsequent atmospheric cascade shower and finally obtain the distribution of the muon flux at sea-level;
- derive a parametric analytical model by fitting an empirical model to a set of measured data describing the muon flux at sea-level.

Since the 1950s there has been a great effort by the international community working in the field of cosmic rays to build some empirical models capable of interpreting the available experimental data. The practice was to build the experimental apparatus capable of measuring the flux of cosmic rays on the ground and subsequently analyse the data and build models on the basis of the knowledge that the community was gradually acquiring on their behaviour. Each parametric analytical model can then derive from an empirical model based on the physical process of production and transport of muons or can be a simple parametric model devoid of physical meaning.

Measurements have been performed all over the world in different time periods, with different duration, at different latitudes and altitudes and with different technologies that allowed to measure the spectrum of muons at sea-level as a function of their momentum and of the zenith and azimuth angles of arrival. A huge amount of data is then available to determine the flux of muons at sea-level for energies E_μ up to 1 TeV and for different zenith angles $\theta \in [0°, 90°]$ [2, 5]. In many cases the characteristics of the data collections have made it possible to combine the results of the different experiments: it was so possible to extend over the years the data on which to fit the models that have gradually been designed.

A series of interesting models are those that, starting from the model popularised by Gaisser [6] that describes the vertical flux of muons at sea-level, extend its use to a general zenith angle [4].

The formula proposed by Gaisser can be reported as:

$$\frac{dI_\mu}{dE_\mu} = 0.14 \left(\frac{E_\mu}{\text{GeV}}\right)^{-2.7} \left[\frac{1}{1 + \frac{1.1\, E_\mu \cos\theta}{115\ \text{GeV}}} + \frac{0.054}{1 + \frac{1.1\, E_\mu \cos\theta}{850\ \text{GeV}}}\right]$$

where I_μ is the muon differential flux at sea-level, E_μ is the muon energy expressed in GeV and θ is the zenith angle.

This formula has been used for calculating muon-induced background for various underground experiments but is valid under the hypotesis that the curvature of the Earth can be neglected and that muon decay is negligible. The model proposed by Guan [7] suggests a modified parametrization that can overcome these shortcomings by proposing the formula:

$$\frac{dI_\mu}{dE_\mu} = 0.14 \left[\frac{E_\mu}{\text{GeV}}\left(1 + \frac{a\ \text{GeV}}{E_\mu (\cos\theta^*)^b}\right)\right]^{-2.7} \left[\frac{1}{1 + \frac{1.1\, E_\mu \cos\theta^*}{115\ \text{GeV}}} + \frac{0.054}{1 + \frac{1.1\, E_\mu \cos\theta^*}{850\ \text{GeV}}}\right]$$

where

$$\cos\theta^* = \sqrt{\frac{(\cos\theta)^2 + P_1^2 + P_2\,(\cos\theta)^{P_3} + P_4\,(\cos\theta)^{P_5}}{1 + P_1^2 + P_2 + P_4}}$$

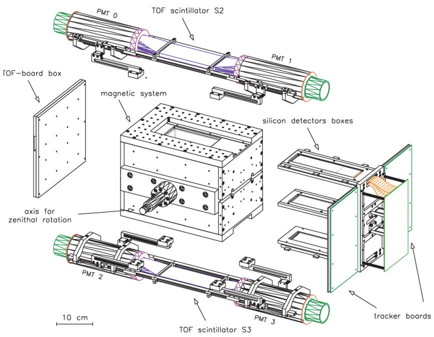
is a convenient parametrization of the effect of the curvature of the Earth and parameters P_1, P_2, P_3, P_4 and P_5 are reported in [7].

The values of the parameters $a = 3.64$ and $b = 1.29$ are obtained from a fit on a dataset available in literature and well described in [7].

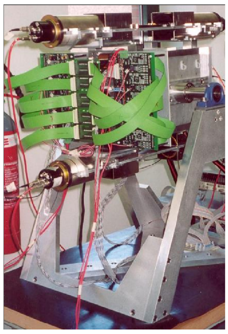
3. A parametric analytical model built by using data collected by the ADAMO spectrometer

The Altazimuth Detector for Atmospheric Muons Observation (ADAMO) [8] is a magnetic spectrometer that has been developed to allow a precise measurement of the spectra of the main cosmic-ray charged components at ground level. The ADAMO detector, a representation of which is given in figure 1, is composed of three main subsystems:

- a permanent magnet with a cavity of $(60 \times 140 \times 210)$ mm^3 with a magnetic field of ~ 0.4 T;
- a tracking system made of five double sided silicon micro-strip detecting units with spatial resolutions of 3 μm and 11 μm along the two sides of the silicon sensors;
- a trigger system working as a Time Of Flight (TOF) system in order to allow particle discrimination at low momentum.



(a) schematic configuration of the detector.

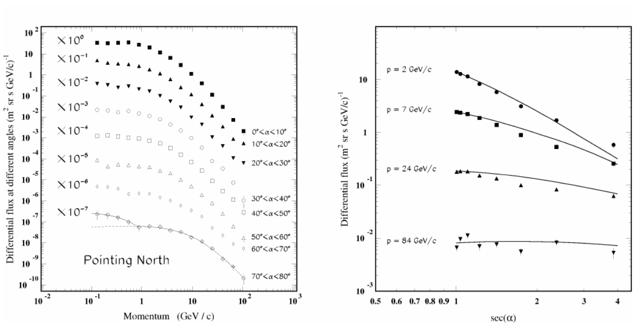


(b) a picture of the detector.

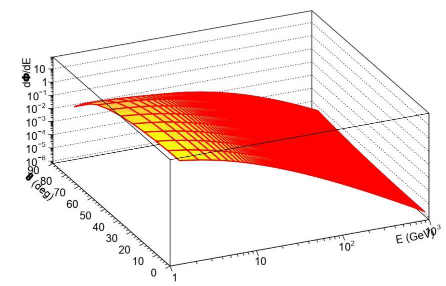
Figure 1: the ADAMO magnetic spectrometer.

The whole spectrometer, which dimensions are $(25 \times 35 \times 25)$ cm^3, allows the measurement of charged particles in the wide range of momentum $[0.1, 150]$ GeV$/c$.

A set of data was collected in 2004 in Florence (Italy) by the ADAMO magnetic spectrometer, without the TOF system being functional, with zenith angles in the range $0° - 80°$ and reconstructed particle's momentum in the range $[0.1, 130]$ GeV$/c$. The *all-particle* cosmic-ray spectra at sea-level are reported in figure 2a: a high electron contamination is evident at low energy, especially at high zenith angles. A 2-dimensional fit in (E_μ, θ), whose graphical representation is reported in figure 2b, was performed on the set of data collected in Florence (Italy) by the ADAMO [8] detector (latitude: 44° 16' N) and a set of data collected in 1977-1978 in Tel Aviv (Israel) by the DEIS [9] detector (latitude: 32° 4' N). The addition of the second dataset allowed to extend the fit at energies E_μ up to 1 TeV and zenith angles $\theta \in [80°, 90°]$. The fit was performed with $E_\mu \in [1, 1000]$ GeV in order to avoid the electron contamination in the ADAMO dataset. The parameters obtained in the fit, $a = 3.629 \pm 0.021$ and $b = 1.368 \pm 0.003$, are close to those obtained by Guan *et al.* [7].



(a) spectra measured by the ADAMO [8] detector. Left: differential flux for eight zenith angle intervals. Right: differential flux for fixed momentum values as a function of the zenith angle.



(b) 2-dimensional fit obtained with the model proposed by Guan *et al.* [7] on a set of data collected by ADAMO [8] and DEIS [9] detectors.

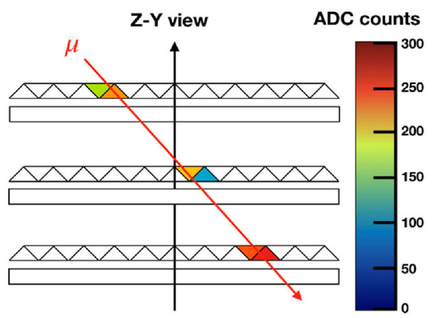
Figure 2: set of data used and results of the fit performed on them by using a selected empirical model.

4. Applications of the muon transmission radiography technique with the developed parametric analytical model by using the MIMA tracker

The 2-dimensional function obtained by fitting the ADAMO [8] and DEIS [9] datasets with the parametric analytical model proposed by Guan *et al.* [7] was used as a generator tool for Monte Carlo simulations in some applications of the muon transmission radiography technique.

The detector used for the data collection is the Muon Imaging for Mining and Archaeology (MIMA) tracker [10], a light, rugged, low power muon detector conceived as a portable apparatus to allow on-field tests of muon transmission radiography in different fields of application, following a multidisciplinary approach. The MIMA detection system, shown in figure 3a, is composed of six tracking planes with a surface of (40×40) cm^2, positioned as three ortogonal pairs, each of which assembled as an array of 21 scintillator strips with triangular section readout at both ends by Silicon PhotoMultiplier (SiPM) sensors. The particular shape of the strips is derived from a FERMILAB

(a) a picture of the detector.



(b) an example of a reconstructed muon track.

Figure 3: the MIMA tracker.

design which, using a clustering algorithm taking into account the signals produced by particles crossing adjacent strips, partially overlapped, allows achieving a spatial resolution around 1.6 mm in the muon hit point reconstruction, rather better than using the so-called "digital algorithm" with a 20 mm readout pitch. Considering muons trajectories traversing all the three tracking modules, as reported in figure 3b, the resulting geometrical factor is approximately 1000 cm^2 sr. The total power consumption is 30 W approximately, thus allowing the apparatus to be operated using a small photovoltaic system, if no mains electricity is available. The tracking modules are housed in a protective aluminum box and the resulting assembly, a $(50\times50\times50)$ cm^3 cube, is operated on-board an altazimuth mechanical support which allows fixing the detector's pointing direction by setting the associated zenith and azimuth angles. MIMA is designed as a robust and portable instrument to be easily installed inside mines, in tiny spaces inside archaeological sites and inhospitable sites. The masses of the systems are about 50 kg for the detector and 20 kg for the optional mount.

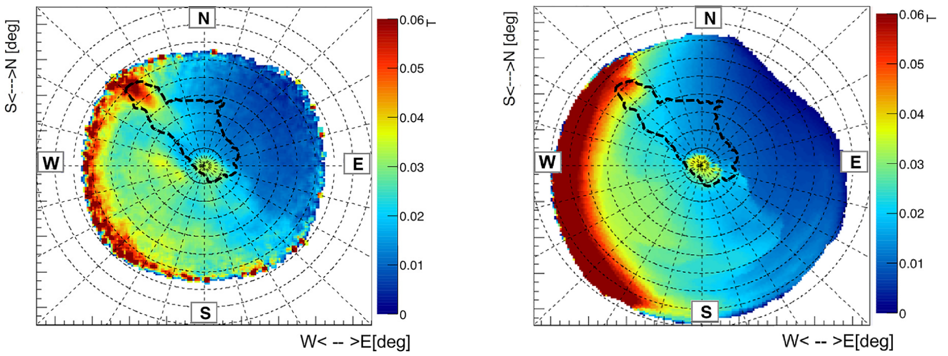
A set of muon transmission radiography measurements have been carried out, in the context of archaeological and geological studies, at the Temperino mine (Livorno, Italy) for the search and three-dimensional visualisation of cavities [11] or ore body prospecting [12]. This mine has been exploited since Etruscan times until recently (1973), and is now an active tourist attraction with public access to the tunnels. Apart from the archaeological interest, the importance of mapping the cavities within this mine lies in identifying the areas where the extraction ores were found and also in the safety issues arising from the tourist presence inside the mine.

Regarding the data analysis, what is directly measured by a muon detector in a "target" configuration (i.e. with the scanned object and the sky in its field of view) is the muon flux coming from the various directions within its acceptance. Comparing this flux to the one obtained in the "free-sky" configuration (i.e. without the scanned object in its field of view, manteining the previous pointing configurations), this yields the probability for a muon to be absorbed by a given target along a certain line of sight (also known as *measured muon transmission* $T_M(\theta, \varphi)$ where θ is the angle between the line of sight and the vertical direction, called the zenith angle, φ is the pointing angle from the north direction, known as the azimuth angle) [10].

The simulation tool requires the knowledge of the digital model of the terrain (obtained on-field or from the literature), the minimum energy that a muon must have to reach the detector crossing a given target (obtained from the literature) and the differential muon flux at sea-level (this developed parametric analytical model). Having these informations it is possible to evaluate the *simulated muon transmission* $T_S(\theta, \varphi, \bar{\rho})$, where $\bar{\rho}$ is the average density in the (θ, φ) direction [10].

In figure 4 the measured and simulated muon transmissions are reported in the polar reference frame, showing a good agreement between measured data and the simulation obtained with the new generator. The profile of a well known big cavity, named Gran Cava, is shown as a dashed line.

The presence of a cavity is identified by an excess in the measured transmission relative to the simulated one. The relative transmission, defined as $R(\theta, \varphi, \bar{\rho}) = \frac{T_M(\theta,\varphi)}{T_S(\theta,\varphi,\bar{\rho})}$, will be roughly equal to unity in case the correct average rock density is chosen, whereas it will be greater than unity when the average density for a certain line of sight is lower than the one used for the simulation (which could be related to the presence of a cavity) and conversely. By changing the density value for each direction in order to normalise the relative transmission to unity, we can obtain the average density distribution $\bar{\rho}$ as a function of the direction (θ, φ). Figure 5 shows the average density distribution $\bar{\rho}(\theta, \varphi)$ obtained from three different measurement positions.



(a) measured muon transmission $T_M(\theta, \varphi)$.

(b) simulated transmission $T_S(\theta, \varphi, \bar{\rho})$ for a target average density $\bar{\rho} = 2.65$ g/cm^3.

Figure 4: muon radiography performed at the Temperino mine. The profile of the Gran Cava known cavity is shown as a dashed line. The figures are reported in a polar reference frame with a grid representing the zenith angles θ (circles) and azimuth angles φ (lines). The simulation was performed in a larger acceptance.

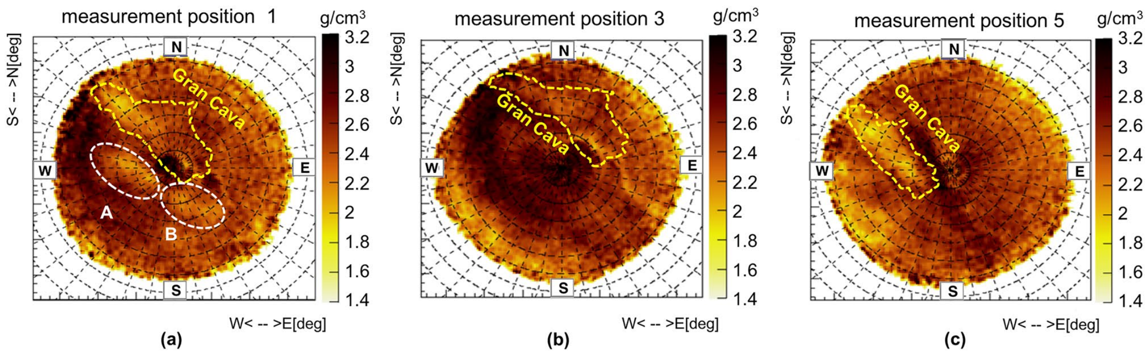


Figure 5: two-dimensional average density $\bar{\rho}(\theta, \varphi)$ obtained from three different measurement positions.

The Gran Cava profile corresponds to zones whose density is less than $2.0 - 2.2$ g/cm^3 of the typical rocks that can be found in the mine and therefore can be associated with the presence of a cavity. Other areas with lower average densities, such as those named A and B, could correspond to cavities that have never been mapped before [11]. The high density values present in the Gran Cava area indicate the presence of skarn material below the cavity [12].

A three-dimensional imaging can be achieved using two algorithms: the first one involves a triangulation of two or more measurements performed at different locations, the other is based on the back-projections [13] of reconstructed muon tracks. The latter requires only a single muographic data taking and is to be preferred in applications where more than one site location can be difficult to access. The quality of the three-dimensional imaging was evaluated by comparing the results of the cavity reconstruction [11] and the ore body prospecting [12] with laser scan profiles obtained for a field survey carried out inside the accessible tunnels and with data present in literature.

The results of this muon transmission radiography survey demonstrate the reliability of the parametric analytical model of the atmospheric muon flux at sea-level obtained by fitting the ADAMO [8] and DEIS [9] datasets with the empirical model proposed by Guan *et al.* [7]. This study pave also the way for future data taking with the ADAMO spectrometer, in order to improve the used dataset and to fit on it other promising empirical models developed by the community working in the field of cosmic rays.

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