

Galactic diffuse gamma-ray emission at VHE from discrete CR sources

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We propose a new model of the TeV to PeV Galactic diffuse gamma-ray emission, that takes into account the discreteness of cosmic-ray injection sites, by implementing numerically the analytical solutions of the diffusion equation obtained by means of the Green's functions formalism. As an application of our model, we compute the VHE diffuse gamma-ray emission for two different cosmic-ray diffusion processes and demonstrate that our approach leads to a more realistic description of the gamma-ray sky. In particular, our simulations show that for some relevant regions of the parameters space, such as considering that only a small subset of the cosmic-ray sources are PeVatrons, the discreteness of the cosmic-ray sources can no longer be ignored and induces some clumps on the gamma-ray sky maps. In the light of these new results, we expect that near-future observations will provide relevant constraints on the cosmic-ray production and propagation mechanisms.

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1. Introduction

The recent observations of the gamma-ray sky by Tibet AS γ [1] and LHAASO [2] at sub-*PeV* energies have opened a new observational window on our Galaxy. Motivated by these observational advances, we present our new model, based on the Green's functions formalism, for the description of the cosmic-ray density and subsequently the very high energy (VHE) gamma-ray emission in our Galaxy. This formulation, by providing a consistent framework for a discontinuous injection of cosmic rays by sources located at discrete positions, allows for a more realistic description of the gamma-ray sky by accounting for the local effects induced by individual sources in their vicinity. Adopting this formalism, we investigate the imprint of the discrete character of the cosmic-ray injection and of the diffusion mechanisms, responsible for cosmic ray propagation on the VHE gamma ray sky maps and, in particular, the diffuse gamma-ray background. We find that this approach has the advantage to produce very complex sky maps, reflecting the latest gamma-ray observations and offering the perspective of a direct comparison of observations with our numerical simulations. Here we focus on the morphology of the gamma-ray sky maps and the constraints that can be deduced from it. In this optic, we organize this proceeding as follows: In Section II, we introduce our cosmic-ray model, and the main equations employed to evaluate the gamma-ray emission. In Section III, we briefly describe our code and discuss the numerical setup adopted in this work. In Section IV, we present our main results and discuss the clumpiness of the gamma ray sky maps and the abundance of hadronic PeVatrons. Finally, we summarize our work and present our conclusions in Section V.

2. Model

We consider Galactic hadronic cosmic rays injected by transient sources, such as supernova remnants (SNRs), and diffusing isotropically in the interstellar medium. In this context, the cosmic-ray density $n(E, \vec{r}, t)$ can be described by the diffusion equation:

$$\frac{\partial n(E, \vec{r}, t)}{\partial t} - \nabla \cdot (D(E, t) \nabla) n(E, \vec{r}, t) = N(E) \delta(\vec{r} - \vec{r}_s) \delta(t - t_s) \quad (1)$$

where \vec{r}_s and t_s represent respectively the location of the source and the injection time of cosmic rays, $D(E, t)$ the isotropic diffusion coefficient and $N(E) \propto E^{-\gamma}$ with $2 \leq \gamma \leq 2.4$ the cosmic-ray injection energy spectrum, assumed to be a power-law.

Concerning the diffusion mechanisms, we adopt two choices:

A time-independent diffusion coefficient, as a very conservative case, described by equation [4]:

$$D(E) = 10^{28} D_{28} \left(\frac{R}{3 \text{ GV}} \right)^\delta \text{ cm}^2/\text{s} \quad (2)$$

with $D_{28} = 1.33 \times \frac{H}{\text{kpc}}$ for $\delta = 1/3$, corresponding to a Kolmogorov turbulence of the magnetic field. This form, also used in GALPROP code, ensures that the B/C ratio is satisfied for rigidities $R > 3 \text{ GV}$.

As for the other case, we consider the situation where the diffusion coefficient is smaller for a short period of time after the cosmic-ray injection and then starts to increase and reaches the value

given by equation (2). In this context, the diffusion coefficient can be expressed through:

$$D(E, t) = D_{start} \left(\frac{\alpha + 2}{2} + \frac{\alpha}{2} \tanh(\beta(\tau - \tau_{change})) \right) \quad (3)$$

where D_{start} is the diffusion coefficient after the cosmic-ray injection, $\alpha = D_{end}/D_{start} - 1$, D_{end} is the final diffusion coefficient given by equation (2), τ_{change} is the age at which the change in the diffusion coefficient is observed and β the increase pace. We choose this form because it allows to better account for the early stages of the cosmic-ray propagation, when the higher cosmic-ray density in the vicinity of the source induces important current densities, which might in turn lead to a local and momentaneous suppression of the diffusion, and because of its flexibility, as it allows to easily control the increase of the diffusion coefficient in a way that satisfies the observational constraints, such as the B/C ratio¹.

In the absence of boundary conditions, for both of the time-independant and the time-dependant diffusion coefficients analytical solutions to the diffusion equation can be found. For the first case, a the solution can be obtained by means of Feynman's path integrals [3] and reads as:

$$n(E, \vec{r}, \tau) = \frac{N(E)}{(4\pi D(E)\tau)^{3/2}} \exp\left(-\frac{|\vec{r} - \vec{r}_s|^2}{4D(E)\tau}\right) \quad (4)$$

while for the second case, the Green's functions formalism gives:

$$n(E, \vec{r}, \tau) = \frac{N(E) \exp\left(-\frac{|\vec{r} - \vec{r}_s|^2}{4D_{start}\left(\frac{\alpha+2}{2}\tau + \frac{\alpha}{2\beta}(\ln(\cosh(\beta\tau - \beta\gamma)) + \ln(2) - \beta\gamma)\right)}\right)}{\left[4\pi D_{start}\left(\frac{\alpha+2}{2}\tau + \frac{\alpha}{2\beta}(\ln(\cosh(\beta\tau - \beta\gamma)) + \ln(2) - \beta\gamma)\right)\right]^{3/2}} \quad (5)$$

where, in both situations, $\tau = t - t_s$, and, for cosmic-ray sources assumed be SNRs injecting a fraction η of their total energy budget ε in cosmic rays whose energy ranges between E_0 and E_{max} , the normalization factor of the energy spectrum can be expressed by an analytical extension of the formula given in [4]:

$$N(E) = \begin{cases} \frac{(\gamma-2)\eta\varepsilon}{E_0^2} \frac{1}{\left(1 - \left(\frac{E_{max}}{E_0}\right)^{-\gamma+2}\right)} \left(\frac{E}{E_0}\right)^{-\gamma}, & \gamma > 2 \\ \frac{\eta\varepsilon}{\ln\left(\frac{E_{max}}{E_0}\right)} E^{-2}, & \gamma = 2 \end{cases} \quad (6)$$

We assume that hadronic cosmic rays create most of the photons contributing to the diffuse gamma-ray flux through their interaction with the interstellar gas. In particular, we limit ourselves to the computation of the gamma-ray flux produced by hadronic cosmic rays and neglect the leptonic contribution to the diffuse gamma-ray background, as, for low latitudes and high energies, it is expected to contribute to less than 10% of the diffuse flux [5]. With such assumptions, the gamma-ray flux of energy E_γ observed at Earth in the direction (l, b) can be obtained by a simple integration

¹It should be noted that, so far, all the measurments of the B/C ratio have been obtained at much lower energies than those considered in this work and that the observational constraints considered here arise from extrapolations of the GeV – TeV data.

over the line of sight [5]:

$$\phi_\gamma(E_\gamma, l, b) = \int_0^{t_{max}} Q(E_\gamma, l, b, t) e^{-\tau(E_\gamma, l, b, t)} dt \quad (7)$$

where $Q(E_\gamma, \vec{r})$ represents the production rate of gamma-rays, $e^{-\tau(E_\gamma, l, b, t)}$ an attenuation factor and $\tau(E_\gamma, l, b, t)$ the optical depth accounting for the absorption of gamma-rays due to their interactions with the cosmic microwave background (CMB) photons. The absorption due to the CMB photons can be considered as isotropic and treated to a very good approximation as a black body radiation in thermal equilibrium [6], inducing an isotropic attenuation factor $e^{-\tau(E_\gamma, t)}$, where $\tau(E_\gamma, t)$ is proportional to the distance traveled by gamma-rays.

Concerning the production rate $Q(E_\gamma, \vec{r})$, we first evaluate the gamma-ray production rate contributed by the decay of neutral pions π^0 and η mesons produced by collisions between cosmic-ray protons and the protons of the interstellar gas, described by the density presented in [5]. We compute the production rate by using expression [7]:

$$Q(E_\gamma, \vec{r}) \equiv \frac{dN_\gamma}{dE_\gamma} = cn_H(\vec{r}) \int_{E_\gamma}^{E_{max}} \sigma_{inel}(E_p) n(E_p, \vec{r}, t) F_\gamma\left(\frac{E_\gamma}{E_p}, E_p\right) \frac{dE_p}{E_p} \quad (8)$$

where $\sigma_{inel}(E_p)$ represents the inelastic cross-section of the $p-p$ interactions, and $F_\gamma\left(\frac{E_\gamma}{E_p}, E_p\right)$ represents the energy spectrum of the gamma rays emitted. We subsequently account for the collisions between heavier nuclei by introducing a nuclear enhancement factor $\epsilon \sim 1.8$ [8].

3. Numerical setup

We model the galaxy as a cylinder of infinite radius and height $H = 5$ kpc, corresponding to the height of the galactic halo, and assume that all the cosmic-ray sources and most of the interstellar gas are contained within the disc, whose height is fixed to 0.2 kpc. For this geometry, we borrow the solution to the diffusion equation with the boundary conditions $n(E, \vec{r}, \pm H, t) = 0$ and $D(E) \frac{\partial n(E, \vec{r}, t)}{\partial z}$ at $z = \pm H$ representing the escape flux, from [4] and supplement it with the approximation given in [9] to make it numerically convenient. We generate sources of random ages at a rate of 2 sources per century at random locations following the surface distribution [10]:

$$n(r) \propto \left(\frac{r}{R_\odot}\right)^{0.7} \exp\left(-3.5 \frac{r - R_\odot}{R_\odot}\right) \quad (9)$$

where the sun's distance to the galactocenter is set to $R_\odot \simeq 8.5$ kpc. However, we consider that only a fraction of all supernovae are PeVatrons, which will contribute to the VHE gamma ray emission, and ignore the remaining sources which will rather contribute at smaller energies. For each percentage of sources that are PeVatrons, we fix the value of the injection index in a way that fits the cosmic ray flux at Earth at PeV energy and consider that each source accelerates cosmic rays to energies ranging from $E_0 = 1$ GeV to $E_{max} = 4$ PeV and releases a total energy of $\varepsilon = 10^{51}$ erg of which a fraction $\eta = 10\%$ goes to particle acceleration.

Starting from these assumptions, we first randomly generate a list of sources including a given percentage of sources that are PeVatrons, with the corresponding spectral index, we evaluate the

flux at Earth and restart the process until generating a list that satisfies the flux of cosmic rays at Earth. We then allow their cosmic rays, injected as bursts, to propagate in the Galaxy for each of the two diffusion mechanisms described in the previous section and compute the resulting gamma-ray flux.

As stated above, the case of the time-independent diffusion coefficient is described by equation (2) and the case of a time-dependent diffusion coefficient is described by equation (3), where we fix the starting diffusion coefficient to $D_{start} = 10^{28} \text{ cm}^2/\text{s}$, the increase pace to $\beta = 100$ and the increase age to $\tau_{change} \simeq 10 \text{ kyr}$. The choice of these parameters has been made so that cosmic rays spend only a short period of time in the slow diffusion regime compared to their confinement time in the Galaxy, in order to prevent them from accumulating too much boron close to their sources and therefore, preserve the B/C ratio.

4. Results

In this section we present our main results for the two models considered in this work. We notably highlight the general conclusions, arising for the two diffusion mechanisms, and comment on their range of validity, and then, provide a case by case discussion of the impact of the propagation mechanism on the VHE gamma-ray sky maps.

4.1 Impact of the discrete injection on the diffuse gamma-ray flux

Since we have considered a discrete cosmic-ray injection by a stochastic population of sources, unlike the Lipari & Vernetto model [5] or GALPROP code, our calculations lead to a gamma-ray flux that includes the contribution of the diffuse gamma-ray background as well as that of point-like and extended sources. Thus, in order to evaluate the diffuse flux, the contribution of the other components should be removed according to a given criterion directly related to the detectability of a source by a given experiment, which depends on several parameters, such as its brightness compared to its background or the amount of statistics collected. Since the question of the detectability of cosmic-ray sources depends on the characteristics of any given experiment, which is beyond the scope of this paper, we limit ourselves to removing the contribution of the brightest source in the direction of each light of sight while we treat the remaining sources as unresolved sources and/or count their contribution as a part of the diffuse background.

Following this procedure, we have calculated the diffuse flux, shown in Fig. 1, for the two cases of the time-independent and time-dependent diffusion mechanisms. It should be noted that our calculations do not reproduce the large peak present at $l = 0$ in [5] which is contributed by the interaction of cosmic rays with the molecular hydrogen, dominant close to the galactic center, which we have excluded from our calculations, as it is not relevant for the main conclusions of this study. We observe that for low percentages of sources able to accelerate cosmic-rays to PeV energies, both the curves of the time-independent and time-dependent diffusion coefficients generally follow the Lipari & Vernetto factorized model (see Ref. [5]) but also present some local deviations from this model. These deviations are due to the important variations of the cosmic-ray density over short distances, rather than due to the variations of the interstellar gas density. These fast variations are, in turn, due to the important and local contribution of some of the remaining sources, compared to the background, which is less dominant than at lower energies for which the contribution of

individual sources would be negligible. We further notice that this behaviour tends to diminish with the increase of the percentage of sources that are PeVatrons, which becomes more similar to the situation observed at lower energy.

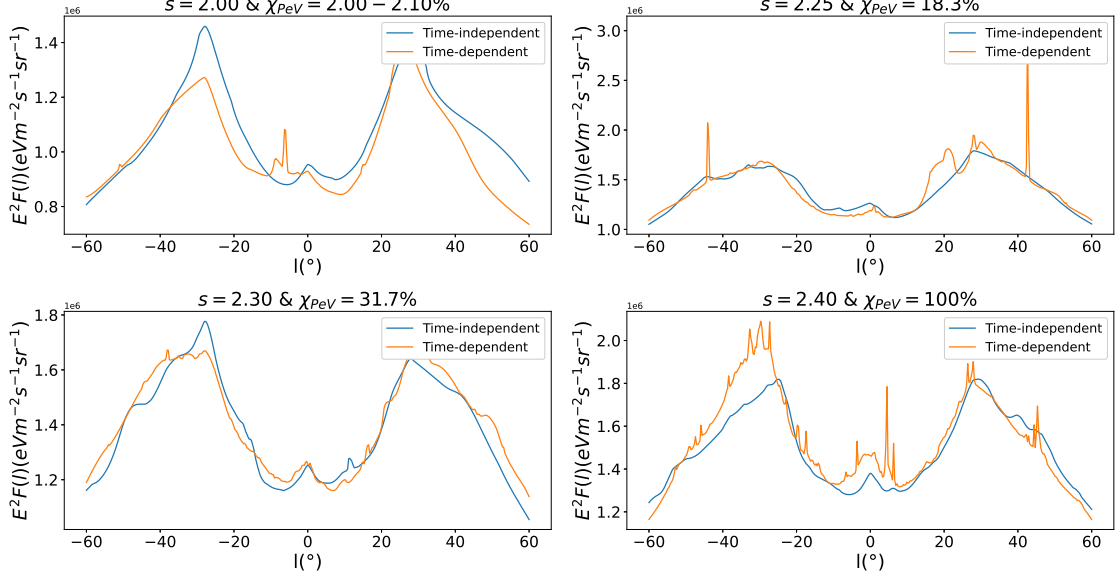


Figure 1: Gamma-ray flux as a function of l for different percentages of sources that are PeVatrons (χ_{PeV}) in the time-independent, and time-dependent diffusion mechanisms at $E_\gamma = 200$ TeV.

4.2 Impact of the propagation mechanism on the gamma-ray sky maps

In this part, we discuss the imprint of the cosmic-ray propagation mechanism on VHE gamma-ray maps. Thus, and to better account for the differences between the two diffusion mechanisms, we show the maps as they are without removing the observable sources.

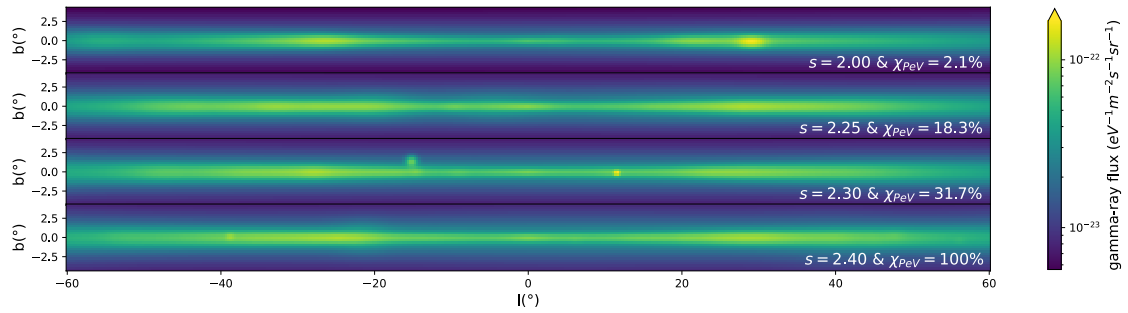


Figure 2: Gamma-ray flux observed at Earth in the time-independent diffusion mechanism case for $-60^\circ \leq l \leq +60^\circ$ and $-4.2^\circ \leq b \leq +4.2^\circ$ at $E_\gamma = 200$ TeV for different percentages of cosmic-ray sources that are PeVatrons (χ_{PeV}) and their corresponding spectral indexes (s).

We find that (see Fig. 2 and Fig. 3) the diffusion mechanism has a significant impact on the morphology of the gamma-ray sky maps and can lead to very different results. In particular, Fig. 2 shows that considering the scenario of a time-independent, isotropic and homogeneous

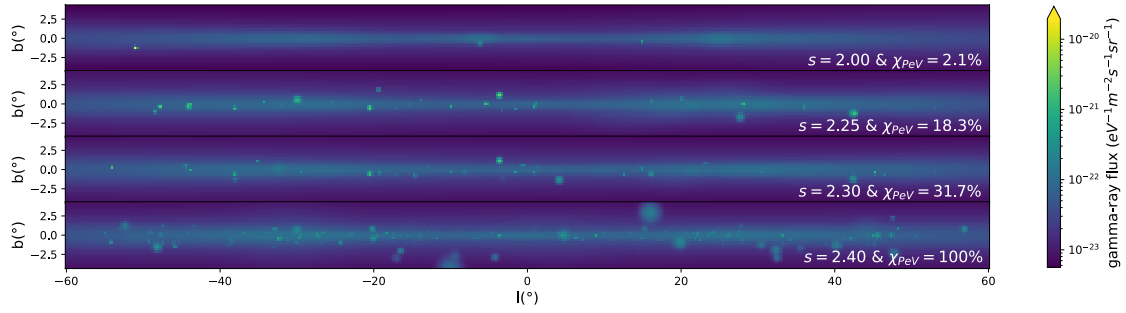


Figure 3: Gamma-ray flux observed at Earth in the time-dependent diffusion mechanism case for $-60^\circ \leq l \leq +60^\circ$ and $-4.2^\circ \leq b \leq +4.2^\circ$ at $E_\gamma = 200$ TeV for different percentages of cosmic-ray sources that are PeVatrons (χ_{PeV}) and their corresponding spectral indexes (s).

diffusion can only lead to a very small number of sources that are detectable at VHE, due to the fast propagation of their cosmic rays, which might explain the very small number of hadronic PeVatrons reported by the LHAASO collaboration in their catalog [11], even in the scenario of a VHE diffuse gamma-ray emission mainly contributed by hadronic cosmic rays. On the other hand, Fig. 3 shows that even a short period of suppressed diffusion can lead to the appearance of several cosmic-ray sources whose contributions overcome that of the diffuse background. As a consequence, in this case, a large number of hadronic sources would be detectable.

5. Conclusion

In this work, we have presented our model of the Galactic diffuse gamma-ray emission based on a discrete injection of cosmic-rays by point-like sources. We have notably demonstrated the impact of the diffusion mechanism responsible for the propagation of cosmic-rays on the morphology of the gamma-ray sky maps, we have found that considering the discrete character of the cosmic-ray injection becomes necessary at VHE, and that a small modification of the parameters leads to a rich variety of phenomena. Finally, we expect that comparing our findings with near-future observations, will help discriminate between different models and allow to better understand cosmic-ray acceleration and propagation at VHE.

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