New constraints on Lorentz invariance violation using the extraordinary flare of Mrk 421 in 2014

Giacomo D’Amico, Jelena Strišković and Tomislav Terzić on behalf of the MAGIC collaboration

An extraordinary flare from blazar Mrk 421 was recorded on the night of April 24, 2014, by the Major Atmospheric Gamma Imaging Cherenkov (MAGIC) telescopes at very high energy (VHE) from 100 GeV up to 10 TeV. Several quantum-gravity models indicate that the photon group velocity in vacuum may be energy dependent, known as in-vacuum dispersion. Violation of Lorentz Invariance (LIV) of this kind can lead to an observable effect on the time of flight of VHE photons from cosmic origin. Observations of fast and distant phenomena, such as the flare from Mrk 421, are excellent testing grounds for this hypothesis. Using an innovative binned likelihood analysis, for the first time from this source we were able to search for arrival-time delays scaling linearly or quadratically with the energy of the photon. The non-detection of energy-dependent time delays led us to set constraints on the parameter space of quantum-gravity predictions.
1. Introduction

Quantum gravity (QG) exploration, dealing with quantum behavior of gravity, arises from incompatibility between general relativity and quantum field theory [13, 17, 26–28]. Many QG theories allow for Lorentz Invariance Violation (LIV), captured via the Standard Model Extension approach or alternative methods like double special relativity [5, 6, 8, 9, 16, 20, 21].

We use a modified photon dispersion relation under the assumption of an ultra-relativistic particle where the observed gamma-ray energy $E$ is less than QG energy scale ($E \ll E_{QG}$ with $E_{QG}$ the QG energy scale expected to be on the level of the Planck energy $1.22 \cdot 10^{19}$ GeV), which can be written as:

$$E^2 = p^2 c^2 \times \left[ 1 + \sum_{n=1}^{\infty} S_n \left( \frac{E}{E_{QG,n}} \right)^n \right]. \quad (1)$$

Astrophysical testing via Time of Flight (ToF) measurements from distant sources can offer significant constraints on $E_{QG,n}$ [7, 11, 12]. Given an energy-dependent group velocity, photons of varying energies, emitted at the same time, arrive at different times. The delay $\Delta t$ from a source at redshift $z_s$ can be expressed as [19]:

$$\Delta t_n \equiv -S_n \frac{n+1}{2} \frac{E^n}{H_0 E_{QG,n}} \cdot \kappa_n(z_s) \equiv \eta_n \times E^n, \quad \text{with} \quad \kappa_n(z_s) = \int_0^{z_s} \frac{(1+z)^n}{\sqrt{\Omega_\Lambda + \Omega_m (1+z)^3}} \, dz, \quad (2)$$

where the parameters $S_n$ have value $-1$ in the scenario where higher-energy photons arrive later (subliminal) or $+1$ in the scenario where they arrive earlier (superluminal). We also introduce the “spectral lag” parameter

$$\eta_n = -S_n \frac{n+1}{2} \frac{\kappa_n(z_s)}{H_0 E_{QG,n}} \quad (3)$$

having the dimension of time over the $n$-th power of energy.

Gamma-ray astronomy sources suitable for LIV investigations need to have high energy emissions, be distant, and exhibit rapid variability. Previous studies have used GRBs, Pulsars, and Active Galactic Nuclei flares [1, 18, 23, 24, 29].

In this work constraints on $E_{QG}$ for both linear and quadratic scenarios have been set. Our study focuses on ToF measurements from the 2014 flare of Mrk 421, as observed by the MAGIC telescopes.

2. MAGIC observations and data analysis

The MAGIC telescopes comprise two 17-meter diameter Imaging Atmospheric Cherenkov Telescopes (IACTs). Situated at the Roque de los Muchachos Observatory on La Palma, Canary Islands, Spain, at an altitude of 2200 meters above sea level [4]. Mrk 421, an active galactic nucleus, is one of the most intensely studied sources in the VHE energy range. Located in the northern hemisphere, Mrk 421 resides at a redshift of 0.031 (approximately 400 million light-years) and is hosted by the galaxy UGC 6132 (R.A. = 116.11 h, DEC. = 38.21°). During a regular monitoring observation on the night of April 25th to 26th, 2014, Mrk 421 exhibited an exceptional gamma-ray
flare. All observations were conducted in the “wobble” mode [4, 14] under dark sky conditions [3], with atmospheric transmission exceeding 90% throughout the observation period, as measured by the MAGIC LIDAR system [15].

The data underwent reduction and analysis using the MAGIC Analysis and Reconstruction Software (MARS) [4, 31] and the open-source gammaPy [2, 10] software. After data quality cuts, the flare counted approximately $9 \cdot 10^3$ events above 100 GeV in the signal region with the last event being detected 8136 seconds after $T_0$ (corresponding to April 25, 2014, at 22:26:34 Universal Time) and with only about 7% of them expected to be background contamination. The event list containing arrival times and reconstructed energies of individual events was extracted to perform the ToF analysis. The intrinsic energy distribution at the source is derived by unfolding the energy distribution of the events’ reconstructed energy, i.e., factoring out instrumental effects and the impact of extragalactic background light (EBL) absorption. The resultant spectrum aligns best with a log-parabolic model given by

$$\phi(E) = A \left( \frac{E}{E_0} \right)^{-\gamma - \beta \log \left( \frac{E}{E_0} \right)}.$$

### 3. Probing Lorentz invariance violation through a binned likelihood analysis

In this work, instead of employing an unbinned likelihood analysis, introduced for LIV studies in 2009 [25] and used in many works thereafter (see Refs. [22, 24] for instance), we opted for this LIV study to implement for the first time a binned likelihood analysis. This new likelihood solves certain drawbacks of the unbinned likelihood analysis, namely:

- In an unbinned likelihood analysis the selection of the interpolation algorithm or the function intended for fitting to the low-energy LC points is somewhat arbitrary and challenging to justify.

- After obtaining the LC template, it becomes difficult to incorporate the Poissonian uncertainties inherent in the low-energy LC points into the likelihood. This may often result in biases in the analysis which are difficult to estimate.

The binned likelihood is defined as

$$L = \prod_{i=1}^{N_i} \prod_{j=1}^{N_j} \mathcal{P}(s_{i,j}, b_{i,j} | N_{on,i,j}, N_{off,i,j}),$$

where the quantities $s_{i,j}, N_{on,i,j}, b_{i,j},$ and $N_{off,i,j}$ respectively signify the expected signal counts, the observed counts in the ON region, the expected background counts, and the observed counts in the OFF region, each in the $i$-th time and $j$-th energy bin. Furthermore, $\alpha$ is the ratio of exposure in the ON and OFF regions, specifically set to 1/3 for this analysis. $\mathcal{P}$ is the ON/OFF Poissonian term expressed as

$$\mathcal{P}(s, b) = \frac{(s + \alpha b)^{N_{on}}}{N_{on}!} e^{-(s+\alpha b)} \frac{b^{N_{off}}}{N_{off}!} e^{-b}.$$
Here, \( N_1 \) is the count of time bins excluding wobble bins and border bins, and \( N_2 \) is the total count of energy bins.

The expected signal counts \( s_{i,j} \) for the \( i \)-th time bin and \( j \)-th energy bin can be obtained from

\[
s_{i,j} = \int_{\Delta E_j} dE \sum_{k=1}^{N_1} \int dE' \, \text{EBL}(E') \, \phi_k(E', \mu) \, \text{IRF}_i(E, E') \, \Delta t_{i,k}(\eta_n \cdot E'),
\]

where \( \text{IRF}_i(E, E') \) is the instrument response function of the MAGIC telescopes, \( \Delta E_j \) is the \( j \)-th energy bin, \( \text{EBL}(E') \) represents the EBL absorption as a function of true energy \( E' \), and \( \phi_k(E', \mu) \) is the intrinsic flux per energy and time in the \( k \)-th time bin (\( \mu \) denotes the nuisance parameters such as the spectral index).

The time-width migration, denoted as \( \Delta t_{i,k}(\eta_n \cdot E') \), is a matrix that considers the intrinsic flux contribution from the \( k \)-th bin to the \( i \)-th one, factoring in the LIV-induced delays.

The likelihood ratio, used as the statistic in this analysis, is given by:

\[
-2\Delta \log \mathcal{L}(\eta_n) = -2 \log \left( \frac{L(\eta_n; \hat{A}, \hat{b}, \hat{\mu})}{\tilde{L}} \right),
\]

where \( \tilde{L} \) signifies the maximum value of the likelihood, while for all nuisance parameters in the analysis the “tilde” on top of the variable represent the values of that variable that maximize the likelihood for a given \( \eta_n \).

According to the Wilks’ theorem [30] the statistic computed using equation 8 follows a \( \chi^2 \)-distribution with 1 degree of freedom that allows a straightforward way of obtaining the 95\% upper limits \( \eta_{95} \) on the spectral lag parameter \( \eta_n \):

\[
-2\Delta \log \mathcal{L}(\eta_{95}) = 3.84
\]

4. Results and discussion

We selected a binning arrangement for our dataset comprising 70 bins in time (on a linear scale) and 10 bins in energy (on a logarithmic scale). Adopting a log-parabola for the energy flux as discussed in section 2 where the parameters \( \gamma \) and \( \beta \) are treated as free parameters we got that the null hypothesis of no spectral lag (\( \eta_n = 0 \) s/TeV\( ^n \), with \( n = 1, 2 \) for the linear and quadratic case, respectively) is compatible with the observation as the value \( \eta_n = 0 \) s/TeV\( ^n \) lies in the 68\% CI defined by the interval in which \(-2\Delta \log \mathcal{L} \leq 1 \) (see equation 8). From equation 9, we derive the following limits on the spectral lag \( \eta_1 \) for the linear case:

\[
\eta_{95} = -37.6 \text{ s/TeV}, \quad +30.9 \text{ s/TeV}.
\]

In a similar way, the quadratic case provides us with:

\[
\eta_{95} = -15.8 \text{ s/TeV}^2, \quad +16.3 \text{ s/TeV}^2.
\]

From these values of \( \eta_{95} \) (using equation 3), the derived lower limits on the QG energy scale across all investigated scenarios - subluminal and superluminal for both linear and quadratic cases - are reported in table 1, where also the impact of systematic uncertainties on these estimates is
Table 1: 95% lower limits on the GQ energy scale without and with the systematic uncertainties.

<table>
<thead>
<tr>
<th>Case</th>
<th>No systematics</th>
<th>Including systematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear scenario: $E_{QG,1}/$GeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>superluminal</td>
<td>$3.55 \cdot 10^{17}$</td>
<td>$2.66 \cdot 10^{17}$</td>
</tr>
<tr>
<td>subluminal</td>
<td>$4.82 \cdot 10^{17}$</td>
<td>$2.36 \cdot 10^{17}$</td>
</tr>
<tr>
<td>Quadratic scenario: $E_{QG,2}/$GeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>superluminal</td>
<td>$3.58 \cdot 10^{10}$</td>
<td>$2.51 \cdot 10^{10}$</td>
</tr>
<tr>
<td>subluminal</td>
<td>$3.52 \cdot 10^{10}$</td>
<td>$2.25 \cdot 10^{10}$</td>
</tr>
</tbody>
</table>

Considered. We examined the potential systematic uncertainties stemming from various factors. These include the choice of binning in both time and energy domains, the calibration of the energy scale, and potential temporal variations in the SED parameters.

Our results (see table 1) align with previous constraints obtained using AGN data. However, our limits are more robust as they do not depend on defining a LC template, thereby fully integrating Poissonian statistics into the analysis. While our estimates are more conservative, they reflect a more complete statistical treatment of the data.

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References

Constraints on LIV with MAGIC

Giacomo D’Amico


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Giacomo D’Amico

30 Croatian MAGIC Group: University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture (FESB), 21000 Split, Croatia
31 Croatian MAGIC Group: Josip Juraj Strossmayer University of Osijek, Department of Physics, 31000 Osijek, Croatia
32 Finnish MAGIC Group: Finnish Centre for Astronomy with ESO, University of Turku, FI-20014 Turku, Finland
33 Japanese MAGIC Group: Department of Physics, Tokai University, Hiratsuka, 259-1292 Kanagawa, Japan
34 Università di Siena and INFN Pisa, I-53100 Siena, Italy
35 Saha Institute of Nuclear Physics, A CI of Homi Bhabha National Institute, Kolkata 700064, West Bengal, India
36 Inst. for Nucl. Research and Nucl. Energy, Bulgarian Academy of Sciences, BG-1784 Sofia, Bulgaria
37 Japanese MAGIC Group: Department of Physics, Yamagata University, Yamagata 990-8560, Japan
38 Finnish MAGIC Group: Space Physics and Astronomy Research Unit, University of Oulu, FI-90014 Oulu, Finland
39 Japanese MAGIC Group: Chiba University, ICEHAP, 263-8522 Chiba, Japan
40 Japanese MAGIC Group: Department of Physics, Kyoto University, 606-8502 Kyoto, Japan
41 Japanese MAGIC Group: Institute for Space-Earth Environmental Research and Kobayashi-Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, 464-8001 Nagoya, Japan
42 Croatian MAGIC Group: Ruđer Bošković Institute, 10000 Zagreb, Croatia
43 INFN MAGIC Group: INFN Sezione di Perugia, I-06123 Perugia, Italy
44 INFN MAGIC Group: INFN Roma Tor Vergata, I-00133 Roma, Italy
45 Japanese MAGIC Group: Department of Physics, Konan University, Kobe, Hyogo 658-8501, Japan
46 also at International Center for Relativistic Astrophysics (ICRA), Rome, Italy
47 now at Institute for Astro- and Particle Physics, University of Innsbruck, A-6020 Innsbruck, Austria
48 also at Port d’Informació Científica (PIC), E-08193 Bellaterra (Barcelona), Spain
49 also at Institute for Astro- and Particle Physics, University of Innsbruck, A-6020 Innsbruck, Austria
50 also at Department of Physics, University of Oslo, Norway
51 also at Dipartimento di Fisica, Università di Trieste, I-34127 Trieste, Italy
52 Max-Planck-Institut für Physik, D-80805 München, Germany
53 also at INAF Padova
54 Japanese MAGIC Group: Institute for Cosmic Ray Research (ICRR), The University of Tokyo, Kashiwa, 277-8582 Chiba, Japan