

## Modelling the Transport of Particles in the Interstellar Medium

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To reveal the nature of high-energy gamma-ray sources and to understand the associated emission and acceleration mechanisms, we need detailed models capable of reproducing the observed energy spectra and morphologies. Gamma rays can be produced in non-thermal radiation processes involving protons and electrons interacting with the interstellar medium, magnetic fields and soft photon fields. These protons and electrons originate from cosmic-ray accelerators, such as supernova remnants or pulsar wind nebulae. However, the gamma-ray emission is not necessarily produced at the centre of the accelerators, as they can expand and cosmic rays can diffuse in ambient magnetic fields, ultimately transporting them through the interstellar medium.

In this contribution, we will present a software framework to model 3D distributions of cosmic-ray and gamma-ray emission around accelerators of varying complexity and to optimise the model parameters. We will also demonstrate the importance of modelling in 3D and discuss the challenges in determining the 3D location of the interstellar gas.

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## 1. multiverse: A Software Framework for Modelling Galactic Cosmic-Ray, Gamma-Ray and Neutrino Distributions in 3D

Recent high-impact discoveries, such as ultra-high-energy photons from Galactic gamma-ray sources [1] and high-energy neutrinos from the Galactic plane [2], and the century-long question of the origin of Galactic cosmic rays, spark interest in tools to support finding the responsible accelerators and deciphering the underlying mechanisms. Due to the vast amount of astrophysical objects qualifying as accelerator candidates, combined with a multitude of target fields and radiative processes to produce the observed emission, and many unknown properties of the objects and mechanisms, we need a flexible tool capable of running large-scale model variations. At the same time, for example once the most promising accelerators and model parameters have been found, or for dedicated investigations of a specific source, we need a tool for deriving detailed emission distributions, to reliably understand the object, its environment and mechanisms.

The software framework `multiverse` allows modelling Galactic multi-messenger and multi-wavelength distributions in 3D, for different scenarios and with varying complexity. This enables not only large-scale models and variations, but also constraining the parameter space with low-complexity models, before fine-tuning complex but time-consuming models.

## 2. Cosmic-Ray Particle Transport

Galactic astrophysical objects, such as supernova remnants (SNRs) or pulsar wind nebulae (PWNe), can accelerate cosmic-ray (CR) particles, such as electrons or protons, which can escape their accelerator and are released into the interstellar medium (ISM). Their distribution  $f \equiv f(\gamma, \mathbf{r}, t)$ , here expressed in terms of their Lorentz factor  $\gamma$ , at time  $t$  and location  $\mathbf{r}$ , depends on characteristics of their source and the environment. It is commonly described by the so-called transport equation (e.g. [3–5]):

$$\frac{\partial f}{\partial t} = S + \nabla \cdot (D_{rr} \nabla f) - \frac{\partial}{\partial \gamma} (\dot{\gamma} f) - \nabla \cdot (\mathbf{v} f) + \frac{1}{3} \frac{\partial}{\partial \gamma} (\gamma (\nabla \mathbf{v}) f) + \frac{\partial}{\partial \gamma} \left( p^2 D_{\gamma\gamma} \frac{\partial}{\partial \gamma} \left( \frac{1}{\gamma^2 f} \right) \right) - \frac{1}{\tau_f} f - \frac{1}{\tau_r} f. \quad (1)$$

This partial differential equation includes effects of spatial diffusion with coefficient  $D_{rr} \equiv D_{ijk}(\gamma, \mathbf{r})$ , advection with velocity  $\mathbf{v} \equiv \mathbf{v}(\gamma, \mathbf{r}, t)$ , energy losses from radiative processes and adiabatic expansion, diffusion from re-acceleration with acceleration rate  $D_{\gamma\gamma}$ , losses by radioactive decay and fragmentation ( $\tau_f, \tau_r$ ) and the injection  $S \equiv S(\gamma, \mathbf{r}, t)$  of particles.

This kind of equations requires numerical solutions. However, under specific assumptions, analytical solutions can be derived. Neglecting advection, re-acceleration and losses by radioactive decay and fragmentation, and assuming isotropic, homogeneous diffusion, we can derive, for instance, a solution for an impulsive injection  $S(\gamma, \mathbf{r}, t) = S_0(\gamma) \delta(\mathbf{r}) \delta(t)$ . As an example, we present the radially symmetric solution for an exponential powerlaw injection spectrum for cosmic-ray protons, released dependent on energy from an expanding shell with time, as in the case of SNRs:

$$f(\gamma, R, t) = N_0 \gamma^{-\alpha} \exp\left(-\frac{\gamma \exp(t/\tau_{pp})}{\gamma_c}\right) \exp\left(-\frac{(\alpha-1)t}{\tau_{pp}}\right) \cdot \frac{f_0}{\pi^{3/2} l_d^3} \exp\left(-\frac{(R - R_{\text{esc}})^2}{l_d^2}\right), \quad (2)$$

with the normalisation

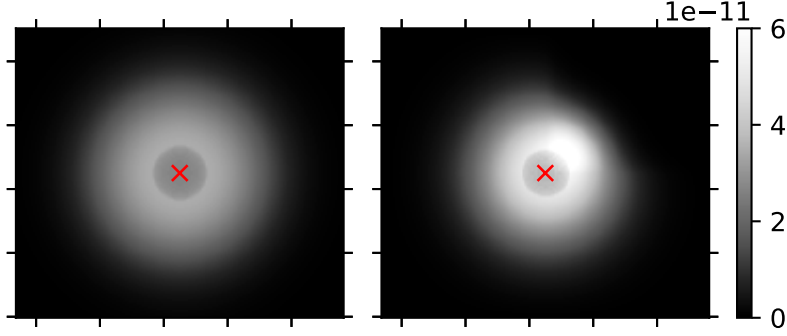
$$f_0 = \frac{\sqrt{\pi} l_d^3}{(\sqrt{\pi} l_d^2 + 2 \sqrt{\pi} R_{\text{esc}}^2) l_d + 4 R_{\text{esc}} l_d^2} \quad (3)$$

and diffusion length

$$l_d = \sqrt{4 D(\gamma) t \frac{\exp(\delta t / \tau_{\text{pp}} - 1)}{\delta t / \tau_{\text{pp}}}}. \quad (4)$$

A derivation of the diffusion coefficient  $D(\gamma)$  is provided in [6]. When modelling the Sedov phase of the SNR evolution, particles escape the shock front with radius  $R_{\text{esc}}(t_{\text{esc}}(\gamma))$  [7] at a time  $t_{\text{esc}}(\gamma)$  [8] after the supernova explosion. The above provided analytical solution was used to derive the results presented in [9–11].

Numerical solutions are specifically important for modelling electrons, as they lose energy through radiative cooling rapidly, or for regions with large inhomogeneities that have strong effects on the particle distribution (cf. Figure 1).



**Figure 1:** Comparison of proton cosmic-ray distributions between analytical (left) and numerical solution (right). The shell of confined cosmic rays around the accelerator (red cross) is visible. In the numerical model, the magnetic field is inhomogeneous and is ten times stronger in the upper right quarter. Cosmic rays in this region diffuse shorter distances.

### 3. Gamma-Ray and Neutrino Production

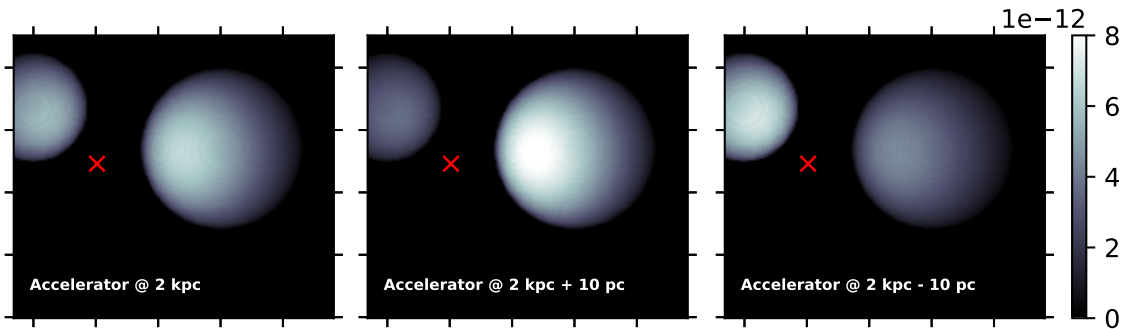
In hadronic interactions, heavy particles are generated in proton-proton inelastic collisions, producing many pions, which decay to both gamma rays and neutrinos. To calculate their emission, we use parameterisations of the differential cross sections by [14, 15]. Neutrinos oscillate between their three flavours. For neutrinos travelling Galactic distances, we approximate that they oscillate from a  $(\nu_e:\nu_\mu:\nu_\tau)$  flavour ratio of (1:2:0) at the source to (1:1:1) at Earth [13].

In leptonic scenarios, gamma rays can be produced via Bremsstrahlung, where electrons scatter of the ambient interstellar medium [16], via Synchrotron radiation, which is produced by charged particles in the presence of magnetic fields [17] or Inverse Compton scattering, where soft photons from a seed photon field scatter off relativistic electrons. We use descriptions of the emission provided by [16–19].

Gamma rays are subject to pair absorption with interstellar radiation fields (ISRFs) as they travel to Earth. We take this into account by following a description in [20], using the ISRF model developed in [21]. The pair absorption optical depth values required to perform these calculations are obtained from GALPROP<sup>1</sup>.

#### 4. Importance and Challenges of Modelling in 3D

Many terms of the equations to derive distributions of CRs, gamma rays or neutrinos are not homogeneous and include a spatial dependency. Even under the assumption of a homogeneous diffusion, we have to combine a spherically symmetric CR density with an arbitrary distribution of an interstellar gas distribution to produce gamma rays via pion decay. Translating only one of the 3D distributions in any direction, will lead to very different densities to be considered at a given coordinate, resulting in large changes of the produced gamma-ray emission. Using the CR density model described in section 2 (Equation 2 to 4), we place two hydrogen clouds at a distance of (2 kpc + 15 pc) and (2 kpc - 15 pc) from Earth. Then we place the accelerator at different distances of (2 kpc + 10 pc), (2 kpc - 10 pc) and 2 kpc and obtain the gamma-ray emission. Figure 2 visualises the huge impact such small changes can introduce.



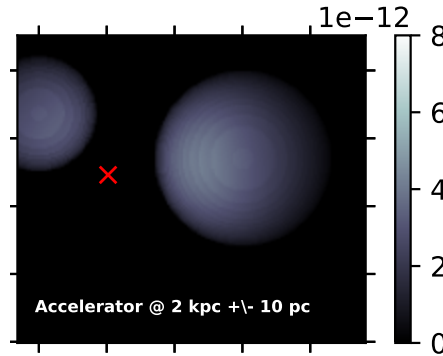
**Figure 2:** Gamma-ray distribution for different distances of the accelerator (red cross) in relation to the clouds in a hadronic scenario in 3D. The clouds are positioned at (2 kpc - 15 pc) and (2 kpc + 15 pc). It is demonstrated that the morphology of the gamma-ray emission is very sensitive to small changes. It is also obvious that these effects can only be observed in 3D.

Figure 3 demonstrates the importance of modelling in 3D by using cosmic-ray and gas distributions in 2D instead of 3D. It is shown that using 2D distributions, different distances between the accelerator and individual clouds cannot be considered and the resulting gamma-ray morphology can be very different.

However, modelling in 3D comes with a range of challenges. One of the major challenges is the derivation of the ISM gas distribution in position-position-position (PPP) space (sky coordinates and distances to Earth). Observations tracing the ISM are obtained in position-position-velocity (PPV) space (sky coordinates and local-standard-of-rest velocities) and need to be transformed. Models of the Milky Way's rotation provide a method to translate radial velocities to kinematic distances; however, for most cases ambiguous solutions of a 'near' and a 'far' distance exist. Another challenge

<sup>1</sup><https://galprop.stanford.edu>

is that the measured velocities are a combination of radial motions and local motions of the gas. As we cannot measure the local motions of the gas to subtract, a large uncertainty on the distance arises (in the order of hundreds of parsecs).



**Figure 3:** Gamma-ray distribution of the same scenario as in Figure 2, derived with cosmic-ray and gas distributions in 2D instead of 3D. The distances between the accelerator (red cross) and the individual clouds cannot be considered in this scenario. Consequently, the gamma-ray morphology is quite different compared to the one derived with 3D distributions and the importance of modelling in 3D is evident.

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