Rare $B$ decays are at the forefront of Belle II physics program. As the Belle II experiment concludes four years of data taking, collecting a sample of $424 \text{ fb}^{-1}$, first results on rare $B$ decays have already been shown since 2021. The latest is the inclusive $\mathcal{B}(B \to X_s \gamma)$ measurement, described in this document. We then summarize the results related to $b \to s \ell \bar{\ell}$ ($\ell = e, \mu, \tau, \nu$) decays and provide the projections at higher statistics. Finally, we present two recent Belle searches of lepton flavor violating $b \to s \tau \ell$ transitions.
1. Introduction

Rare $B$ decays commonly denote the processes involving the transition of a $b$ quark into a non-charmed one ($u, d$ or $s$). In these proceedings, we focus on the $b \to s$ transitions, mediated in the Standard Model (SM) by loop-diagrams like the so-called electroweak ‘penguin’ (Fig. 1a). The study of these decays at $B$ factories aims at finding alterations and enhancements in the rates due to contributions of New Physics (NP) mediators, which can manifest either as new interactions at tree level (Fig. 1b) or as weaker GIM cancellations in loop corrections (Fig 1c).

![Figure 1: Diagrams representing $b \to s$ transitions: one of the SM contributions (a) and two possible NP scenarios involving respectively a leptoquark (b) and a charged Higgs (c).](image)

For the last two decades, Belle and BaBar first and now Belle II and LHCb experiments have enabled an outstanding advancement in the knowledge of $B$ physics. The Belle experiment operated for ten years and collected almost $800 \times 10^6$ $B\bar{B}$ pairs thanks to the excellent KEKB operation, which led to a world record luminosity in June 2009 [1]. The Belle II experiment results from the major upgrade of all the detector sub-parts [2] and the accelerator. By employing a special beam configuration called nano-beam scheme, SuperKEKB has already doubled the KEKB record for similar beam currents and produced more than $400 \text{fb}^{-1}$-equivalent of collision data, mostly collected at the center of mass energy of the $\Upsilon(4S)$ resonance. The design luminosity of the accelerator is $6 \times 10^{35} \text{cm}^{-2}\text{s}^{-1}$ and the Belle II experiment aims to ultimately record 50 $\text{ab}^{-1}$. Another significant change at Belle II is the software called basf2 [3], reflecting the detector changes and constantly improving to provide equal or better performance in simulation and reconstruction in the harsher SuperKEKB environment.

Searching for rare $B$ decays demands high performance at all sub-detector levels: high reconstruction efficiency and momentum resolution for charged particles and photons and excellent particle identification to effectively select charged kaons and leptons with low pion fake rates.

2. Inclusive $b \to s \gamma$ branching fraction measurement at Belle II

Looking for NP in flavor physics requires comparing the observable’s SM predictions to the measured values. A way to probe the $b \to s \gamma$ coupling is measuring the rates of $\mathcal{B}(B \to K^{(*)}\gamma)$ decays. However, the theoretical values for these branching fractions suffer from large theoretical uncertainties due to the estimation of the hadronic form factors. The inclusive measurement of $\mathcal{B}(B \to X_s \gamma) - X_s$ being the hadronic system containing an $s$ quark – overcomes this issue to a large extent but at the cost of being more experimentally challenging. The $B$ factories are the ideal
environment for this measurement, given the relatively clean environment compared to hadronic colliders. Below we list three possible approaches for the $\mathcal{B}(B \to X_s \gamma)$ measurement.

**Sum of exclusive.** The $X_s$ is treated as the sum of exclusive contributions $K, K^*, K\pi, \ldots$. This method heavily relies on the simulation of the $X_s$ system.

**Inclusive.** Adopted by Belle (see Ref. [4]), the inclusive approach consists in reconstructing the most energetic photon in the event and measuring the photon spectrum, from which all the expected background contributions are subtracted. The method clearly has high efficiency but large backgrounds as well as strong dependence on simulation.

**Hadronically tagged events.** The $B$ meson accompanying the $B \to X_s \gamma$, called $B_{\text{tag}}$, is fully reconstructed in a list of hadronic decays – at Belle II, through a multivariate algorithm called FEI [5]. This allows not only to compute the photon energy in the $B$-rest frame, but also to isolate the $X_s$ system and achieve higher signal purities. The main disadvantage comes from the lower hadronic $B_{\text{tag}}$ reconstruction efficiency – around 1%.

This last approach has been adopted at Belle II for a measurement on a $190 \text{ fb}^{-1}$ dataset [6].

The selection of $B_{\text{tag}} \gamma$ events produces large combinatorial background from $q\bar{q}$ ($q = u, d, s, c$) events with high-energy photons from $\pi^0$ and $\eta$ mesons, which is reduced using multivariate analysis. As a second step, the distribution of the beam-constrained $B_{\text{tag}}$ mass, defined as\footnote{$E_{\text{beam}}^2 = \sqrt{s}/2$, $\sqrt{s}$ being the available energy in the center-of-mass frame.} $M_{bc} = \left(\frac{E_{\text{beam}}^2 - |\vec{p}_{B_{\text{tag}}}^\gamma|^2}{c^2}\right)$, is fitted in bins of the gamma photon energy $E_{\gamma}^B$ to disentangle the correctly tagged events from the rest to remove the surviving $q\bar{q}$ and the non-peaking $B\bar{B}$ contributions (Fig. 2a). The remaining, correctly tagged $B\bar{B}$ events without $B \to X_s \gamma$ decays are subtracted based on simulation. Finally, the $B \to X_d \gamma$ events are also removed with the assumption of similar efficiency and photon spectrum shape for $b \to s\gamma$ and $b \to d\gamma$ events. Furthermore, the two branching fractions are expected to scale as the CKM matrix elements: $(|V_{td}|/|V_{ts}|)^2 \sim 4.3\%$. The final spectrum, unfolded to account for $E_{\gamma}^B$ bin migration due to resolution effects, is shown in Fig. 2b.

The result $\mathcal{B}(B \to X_s \gamma) \times 10^4 = 3.54 \pm 0.78\text{(stat)} \pm 0.83\text{(syst)}$ for $E_{\gamma}^B > 1.8 \text{ GeV}$ is comparable with BaBar’s, where similar tagging approach and dataset size were used [7], and consistent with the theoretical expectations [8]. The prospects for this measurement at higher statistics are provided at Ref. [9]. For a threshold $E_{\gamma}^B > 2.0 \text{ GeV}$, it is possible to exploit the statistical power up to $50 \text{ ab}^{-1}$. Theory predicts the inclusive rate with higher precision at lower thresholds, although the experimental challenge increases as better control of backgrounds is required. For example, at $E_{\gamma}^B > 1.4 \text{ GeV}$ the 5.2% systematic uncertainty becomes comparable to the statistical one at $10 \text{ ab}^{-1}$. In both cases, we consider an improved scenario, where the systematic effects are reduced, for example, regarding the $\pi^0 \to \gamma\gamma$ veto modeling.

3. **Status and prospects of $b \to s\ell\ell$ decays**

The $b \to s\ell^+\ell^-$ ($\ell = e, \mu$) transitions provide important observables to test the SM and constrain NP models. At $B$ factories, the measurement of $B \to K\ell\ell$ decays is challenged by the small available
samples due to the low SM branching fractions and the relatively low B meson production compared to hadronic machines. So far Belle II has measured with 190 fb$^{-1}$ the $\mathcal{B}(B \to K^*(892)\ell\ell)$ [10] and the properties of the tree-level $B \to J/\psi(\to \ell^+\ell^-)K$ decay [11], commonly used as control sample. These two measurements confirm the high performance in reconstructing and identifying electrons, comparably to muons. Concerning the $R_K(\ast)$ observables, the most precise measurement at $B$ factories is from Belle [12] while the projections for Belle II predict a 3% precision at 50 ab$^{-1}$, to be compared to the current 5% precision at LHCb [13] and the expected one in the coming years, around 1 - 2% [14]. Despite not being competitive, the Belle II measurement would provide a crucial clarification in a different experimental environment.

Uniqueness of Belle II is instead the inclusive measurement of $\mathcal{B}(B \to X_s\ell\ell)$, which can attain the 10% precision with 5 ab$^{-1}$ [15] and provide valuable information in the context of global $b \to s\mu\mu$ fits.

Searches of the $b \to s\tau^+\tau^-$ transitions have been performed at LHCb and $B$ factories [16–18] but with no observation, due to the challenging experimental reconstruction of $\tau$ decays and very low SM branching fractions, of $\mathcal{O}(10^{-7})$. However, significant enhancements to the rates are predicted for the same scenarios leading to an excess in $R(D^{(*)})$ [19].

For the $\mathcal{B}(B^0 \to K^{*0}\tau^+\tau^-)$, Belle has set an upper limit at 90% CL of $3.1 \times 10^{-3}$ [18]. Belle II aims to improve this result by exploiting the larger statistics, using the FEI algorithm for the hadronic $B$-tagging and a multivariate approach, and extending to additional $\tau$ modes for increased efficiency. The expected sensitivity at 50 ab$^{-1}$ is $5.3 \times 10^{-4}$ [9] assuming the above improvements while conservatively using Belle’s systematic uncertainty. Similarly to the $K^{*0}$ case, unprecedented sensitivities are expected for the $K^+$ mode.

$B \to K\nu\bar{\nu}$ decays are considered golden modes for Belle II, given the large amount of missing energy and not many distinct signatures. Such difficulties are overcome thanks to the relatively low track multiplicity, the known initial kinematics combined with the detector’s good hermeticity, and
the $B$-tagging algorithms allowing to infer on the invisible part of the decay. 

At $B$ factories, both hadronic and semileptonic $B$-tagging approaches have been tried, and overall the second proved to perform better [20]. At Belle II, an inclusive approach was exploited for the 2021 measurement with 63 fb$^{-1}$, showing a significant improvement in relative precision – 20% compared to the semileptonic $B$-tagging and 350% with respect to the hadronic. The method consists in selecting a high-momentum charged kaon and exploiting the properties of every other particle in the event, which should be compatible with a $B$ meson [21]. Despite increasing sensitivity, such a strategy requires a profound understanding of the detector and systematic effects. The projections at higher luminosities, based on the 2021 result, show that Belle II can establish the SM trying to simultaneously explain the discrepancies observed in $R$ [13] and $B(D^{(*)})$ would point to enhancements in the rates of lepton flavor violating (LFV) $b\to s\ell\ell'$ decays up to $O(10^{-6})$ – while being practically zero in the SM.

4. Recent lepton flavor violation searches at Belle

The Belle $Y(4S)$ and $Y(5S)$ datasets are still the largest ever collected at $B$ factories experiments and are currently the best suited samples for searches of very rare or forbidden $B$ decays. The high-level Belle detector information can be converted into a basf2-friendly format to perform these searches with the improved Belle II analysis tools [23].

The $b\to s\ell\ell'$ searches have been stimulated by the persistent anomalies in $B$ decays. Extensions of the SM trying to simultaneously explain the discrepancies observed in $R_{K^{(*)}}$ and $R(D^{(*)})$ would point to enhancements in the rates of lepton flavor violating (LFV) $b\to s\ell\ell'$ decays up to $O(10^{-6})$ – while being practically zero in the SM.

The Belle search for $B^+\to K^+\tau\ell(\ell=e,\mu)$ decays [24] consists of four distinct modes, representing each not only a specific set of couplings for theoretical interpretation but also a different background nature. The knowledge of the $e^+e^-$ collision kinematics and the hadronic reconstruction of the $B_{tag}$ via FEI allows us to infer the 4-momentum of the signal $B$ meson and to derive the recoil mass $M_{reco}$ (see Fig. 3a), peaking at the $\tau$ mass for signal events. A multi-variate analysis is performed to reduce the background coming from $q\bar{q}$ events and $B\bar{B}$ events – mainly containing $B^+\to \bar{D}^0(\to K^+X)X\ell^+\nu_\ell$ or $B^+\to \bar{D}^0(\to K^+\ell^+\bar{\nu}_\ell)X$ decays. As no evidence of signal is found, the upper limits on the branching fractions are set at 90% CL:

$$B(B^+\to K^+\tau^+\mu^-) < 0.59 \times 10^{-5}$$

$$B(B^+\to K^+\tau^+e^-) < 1.51 \times 10^{-5}$$

$$B(B^+\to K^+\tau^-\mu^+) < 2.45 \times 10^{-5}$$

$$B(B^+\to K^+\tau^-e^+) < 1.53 \times 10^{-5}.$$ 

These are the world’s best limits on $B^+\to K^+\tau\ell$ decays.

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$^2$Respectively 711 fb$^{-1}$, corresponding to $770 \times 10^6$ $B\bar{B}$ pairs and 121 fb$^{-1}$, equivalent to $17 \times 10^6$ $B^0\bar{B}^0$ mesons.

$^3$Although the LFU tension in the $b\to s\ell\ell$ sector has vanished with the LHCb updated measurement [13].
Figure 3: Belle LFV searches probing the $b \to s\tau\ell$ couplings. (a): $M_{\text{recoil}}$ fit to $B^+ \to K^+\tau^+\mu^-$ events. (b): $p_1^*$ fit to $B_s \to \tau\ell$ events, where $B_s$ refers to either $B_s^0$ or $\bar{B}_s^0$. The signal distribution shown in red corresponds to a branching fraction equal to $10^{-3}$.

Another recent result from Belle is the LFV $B_s \to \tau\ell$ search [25]. This measurement is not as constraining given the smaller $\Upsilon(5S)$ data sample, combined to the low $B_s^0$ production cross-section. For this reason, the more efficient semileptonic $B_s^0$-tagging is used, sacrificing the possibility of using the recoil mass as the extraction variable. Nonetheless, because of the two-body $B_s^0 \to \tau\ell$ kinematics, the light lepton energy is almost monochromatic in the center-of-mass frame and its momentum $p_1^*$ is almost beyond the endpoint of the lepton spectrum in semileptonic $B$ decays, which makes it a good extraction variable (see Fig. 3b). The upper limits at 90% CL are derived:

$$ B(B_s^0 \to \tau^+\mu^-) < 7.3 \times 10^{-4} $$
$$ B(B_s^0 \to \tau^+e^-) < 14.1 \times 10^{-4} $$

The achieved sensitivity is not competitive with LHCb in the $\mu^\pm$ channel; nonetheless, Belle has set the first upper limit in the $e^\pm$ channel. A better sensitivity is expected at Belle II with a larger $\Upsilon(5S)$ data sample and boosted analysis techniques, such as the hadronic $B_s^0$-tagging. Similarly, a significant improvement is expected in the $B_d^0 \to \tau\ell(\mu)$ channels, for which Belle has already set the upper limits on the branching fractions at 90% CL of $1.6(1.5) \times 10^{-5}$ [26].

5. Conclusion

The results and prospects on rare $B$ decays show that Belle II can be, depending on the process, healthily redundant, competitive, or world-leading.

The first Belle II measurement of the $B \to X_s\gamma$ branching fraction using hadronic $B$-tagging and 190 fb$^{-1}$ data shows consistency with SM and comparable precision with respect to BaBar’s. Clarifications on $B \to K^{(*)}\ell\ell (\ell = e, \mu)$ and $R_{K^{(*)}}$ require more data and will be provided with competitive precision with roughly 5 ab$^{-1}$. Coming to $\ell = \tau$, branching fraction measurements are more challenging, but world-leading sensitivity is expected with the full Belle II dataset for $B^{(\pm,0)} \to K^{(\pm,0)}\tau\tau$ modes.

The first Belle II $B^+ \to K^+\nu\bar{\nu}$ results have demonstrated the potential of the inclusive $B$-tagging
approach. Consistency with SM is predicted to be established at Belle II, impacting model-dependent and independent new physics interpretations.

On the Belle side, two LFV searches are performed, providing the first branching fraction upper limit on the $B^0_s \to \tau e$ decay and world’s best for $B^+ \to K^+ \tau \ell$ decays.

The first long shutdown of Belle II operations, which started in June 2022, is almost completed and has seen numerous upgrades to the collider and the detector. Data-taking should resume in early 2024, and all the presented measurements will be updated with larger data samples and improved performance.

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