

## Light dark matter and flavour

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Flavour physics and dark matter physics have a long and fruitful history of shared interests. We briefly review in this work these links for the case of light dark matter models and present some of the recent developments. A particular emphasis is put on the case of flavour non-universal or flavour-violating couplings between a new dark sector and the Standard Model fields.

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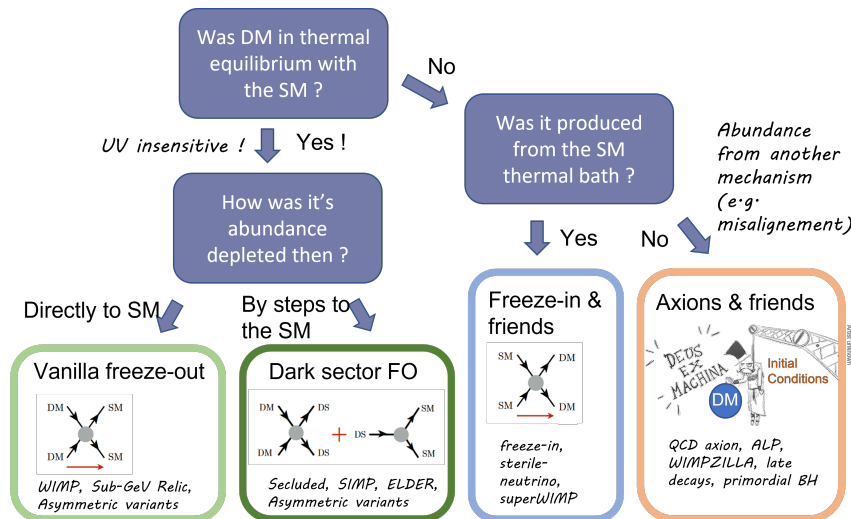
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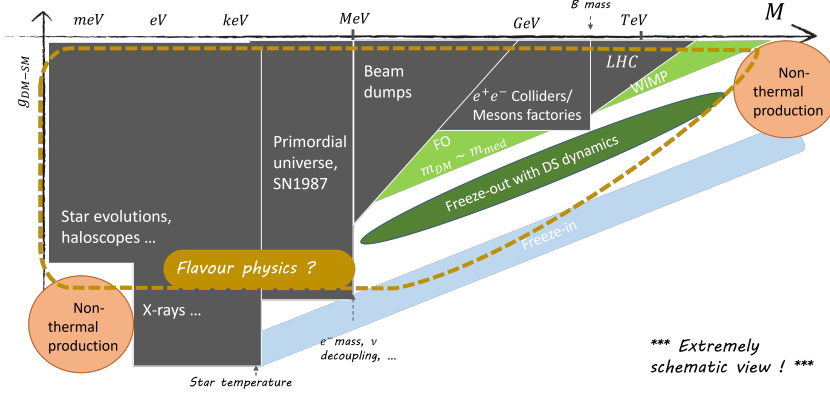
## 1. Introduction

Identifying the nature of dark matter (DM) is one of the key challenge of both the experimental and theoretical particle physics communities. While the former strives to find new ways of probing the existence and properties of dark matter, the later can also help by studying and classifying the various possible candidates. Two particular properties of dark matter stand out which can help in this classification: its stability (it cannot decays at a high rate), and its relic density  $\Omega h^2$  which has been precisely measured by the Planck collaboration [1]. It is therefore particularly relevant to classify DM models following how they predict the required relic density.

Such a classification, adapted slightly from [2, 3] is shown in Fig. 1. The first step is to divide the DM candidates between those that reach thermal equilibrium with the Standard Model (SM) and those which do not. For the models where DM reaches thermal equilibrium with the SM, its equilibrium abundance is typically much larger than the one required to obtain the measured  $\Omega h^2$  in the current universe, and it must therefore be depleted prior to its decoupling. If this depletion occurs directly by producing SM particles, we recover the standard “vanilla” freeze-out mechanism, which is used in a large range of models such as weekly-interacting massive particles (WIMP [4]) or many sub-GeV DM constructions (for a recent review see e.g. [5]). On the other hand, if this depletion proceeds by steps, a “dark sector” freeze-out first occurs (as for models such as secluded DM [6] or SIMP [7]), followed by entropy injection in the SM thermal bath from dark sector particle decays or annihilations. Instead, if the DM does not reach thermal equilibrium with the SM, one must find a mechanism which creates DM in the first place. Freeze-in DM and related models (for instance sterile neutrino DM [8, 9] or super-WIMP DM [10]) rely for instance on a tiny DM production rates from the SM thermal bath to reach the required relic density. Finally, the DM production can be completely unrelated to the SM thermal bath, with the most well-known model being arguably the misalignment mechanism for axion DM models (see [11] for a recent review).



**Figure 1:** A simple flowchart combining various DM candidates based on the mechanism by which they reach the observed relic density, adapted from [2].



**Figure 2:** A schematic view of the explored parameter space (grey area) for FIPs as function of their mass and dominant coupling to the SM. The typical scales and coupling ranges of the types of DM candidate presented in Fig. 1 is overlaid in colored area. The typical domain of flavour physics experiment is shown as a dotted orange line.

This classification can in turn enlightens the search strategies that must be pursued. In particular, the larger couplings between DM and the SM particles required by the freeze-out mechanism typically imply that these particles could be directly produced at particle accelerators or from meson decays in the case of light DM. For the well-studied case of a WIMPs, the interaction with the SM arises from EW interactions, making DM closely resembling SM fermions. Since the later come in three generations, a rich model-building and phenomenological program went on in considering that DM itself could have a flavour structure (as its relic density can be obtained from EW interactions anyway), thus leading to inherently flavourful signatures in collider experiments. This Flavour Dark Matter paradigm have been extensively studied in the last decades and presented in previous FPCP conferences (for recent works still, see e.g. [12–14]). In this short review, we will focus instead on the cases where the DM interaction with the SM particles does not arise from the electroweak sector. This implies exploring more broadly the model space of DM presented above, and in particular considering the cases where the DM particles are lighter than the electroweak scale, and thus constrained by Lee-Weinberg bound [15] to have an interaction with the SM particles beyond the electroweak ones. This interaction then typically proceeds via so-called “Feebly Interacting Particles” (FIPs), neutral particles which interact with the SM via suppressed new interactions, and which are at the center of the interplay studied here between DM and flavour physics.

## 2. Portals, flavour and dark matter

There is an extremely rich literature on new “mechanisms” to obtain the relic density for models of DM where the DM mass is below the electroweak scale (see a recent review in [5]). However, as most FIP models can be embedded in a light DM setup (with of course various level of complexity), the classification of FIP interactions with the SM is a good first step toward exploring and experimentally testing these models. We show in Fig. 2 a schematic representation of the

existing constraints on FIPs along with the corresponding regions of the parameter space for the various types of DM classified above. In general, an interaction between a FIP and SM particles must be written as a Lagrangian contribution of the form

$$\mathcal{L} \supset \frac{1}{\Lambda^{4-n-m}} O_{\text{SM}}^n O_{\text{NP}}^m, \quad (1)$$

where the so-called "portal" operator  $O_{\text{SM}}^n$  of dimension  $n$  is composed exclusively of SM fields, while  $O_{\text{NP}}^m$  includes the FIPs and potentially the DM particles and  $\Lambda$  is a new physics scale. The dimension of the  $O_{\text{SM}}^n$  operator represents a convenient way of classifying such interactions and its structure will determine the flavourful or flavour-independent nature of the interaction. Let us consider in turns the renormalisable possibilities.

- The scalar portal [16, 17]. Based on the SM Higgs boson, this portal corresponds to the operator

$$O_{\text{SM}}^{\text{Higgs}} = HH^\dagger, \quad (2)$$

and typically leads to a mixing between a scalar FIP  $S$  and the SM Higgs. This implies that the light new scalar inherits the SM Higgs flavourful couplings, making this class of interactions inherently flavourful. While DM can annihilate via this portal directly into SM particles, flavour constraints from mesons decays (such as  $B \rightarrow KS$  or  $K \rightarrow \pi S$ ) are all but covering the relevant parameter space [18, 19]. Secluded annihilation, corresponding to the case where DM annihilates first into the new light scalars which later decay into SM particles must then be used, with a large parameter space still uncovered [18].

- The vector portal [20] relies instead on the hypercharge gauge boson and corresponds to the operator

$$O_{\text{SM}}^{\text{vector}} = F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (3)$$

This portal is fully flavour-independent since it leads to kinetic mixing and thus to an interaction proportional to the  $\gamma$  and  $Z$  currents for new vector FIP (for instance a dark gauge boson). It is nonetheless one of the most looked upon mediator for light DM as it allows for vanilla freeze-out scenarios that are still not fully covered by the various experimental searches [3]. As a particular example, one can cite the inelastic DM models first proposed in the context of the DAMA anomaly [21, 22], and then advocated in [23, 24] in the context of light DM. Despite the fact that this interaction is flavour independent, some of the best limits arise from flavour-motivated and neutrino-motivated experiments as they offer an excellent trade-off between precision and energy [3, 5].

- The neutrino portal [25, 26], relies instead on the combination of a lepton and a Higgs doublet:

$$O_{\text{SM}}^{\text{neutrino}} = L \cdot H. \quad (4)$$

This portal is inherently flavourful as it typically leads to new neutral fermions (called Heavy Neutral Leptons – HNL– or right-handed neutrinos depending on the context) to mix with

the SM neutrinos and thus interact with the  $W$  current. While quite constrained, HNLs can also be DM candidates (see e.g. [8, 9]), being thus one of the most economical light DM candidates since they do not require another mediator to be added to the theory.

- Finally, the fermion portal uses the contraction of two SM fermions

$$O_{\text{SM}}^{\text{fermion}} = \bar{\Psi}_{L,R} \Gamma \Psi_{L,R} , \quad (5)$$

where  $\Psi_L$  and  $\Psi_R$  are any of the SM fermions and  $\Gamma$  is a combination of Dirac matrices [3, 27]. This portal is the last class of operators which can lead to a renormalisable interaction when coupled to a bosonic FIP (for instance a new gauge boson). It is also the only one which can have direct flavour-violating operators and we will discuss in more details its flavour aspects in the next section.

It is worth noting that at larger dimensions an infinity of portals can be further constructed in an EFT spirit, including in particular most axion-like particle (ALP) couplings, which can also be used when the ALP acts as a mediator particle for DM annihilation (see e.g. the discussions in [28–32]). The above cases are however the only ones that allow for the renormalisable (and thus unsuppressed even at small scales) operators to be constructed as in Eq. (1). Higher dimensional portals, such as the ALP ones, are effective operators and thus require additional UV physics, which may play an important role in the FIP production and detection strategies (e.g.  $Z, h \rightarrow \text{ALP } \gamma$  at LHC).

### 3. Fermion portal to dark matter and flavour

In this last section, we will study in more details the last type of operators listed above: the “fermion” portal. While this class of operators allow for direct flavour-violation, it is clear that flavour-violating operators typically cannot be used as an effective path for the annihilation of light DM in the early universe. Indeed tree-level processes mediating flavour-violation from mesons or lepton decays are extremely constrained, making freeze-out of light DM from a flavour-violating fermion portal typically impossible. As an illustration, let us consider a  $\bar{b}\gamma^\mu(\gamma^5)s$  portal operator. The vector operator can be associated to a new dark gauge boson (for instance from an horizontal flavour gauge) leading to an interaction term of the form  $c_{V,bs} V_\mu \bar{b}\gamma^\mu s$ , while the axial case could be seen as an ALP coupling of the form  $c_{A,bs} \partial_\mu a \bar{b}\gamma^\mu \gamma^5 s$ . Constraints from  $B \rightarrow K$  inv. processes imply that the coupling  $c_{V,bs}$  must be smaller than around  $10^{-8}$  for a FIP mass around the GeV [33], while for the ALP case it leads to  $c_{A,bs} \lesssim 10^{-6} \text{ GeV}^{-1}$  [11]. In order to get the proper relic density for DM with such small couplings, one must use either the freeze-in approach or rely on non-thermal production mechanisms. In fact this represent one of the few cases where experimental constraints in laboratory experiments could be strong enough to test this type of scenario (see e.g. [34] for a recent study for the case of ALPs).

On the other hand, operators with non-universal interactions are significantly less constrained, and in particular flavour non-universal leptonic interactions can be used to construct viable freeze-out models of light DM. The main limitation – or interest, depending on the point of view – of such models is that the coupling to DM typically arises from the same gauge interaction as the one leading to the flavour non-universal interaction to SM particles. The two couplings are thus expected to

be of the same order up to charges whose values are furthermore limited by the requirements of anomaly cancellations [43, 44]. One the most famous example of such construction relies on a  $L_\mu - L_\tau$  gauge boson mediator (see also e.g. [35–39] for recent other examples), which corresponds to a Lagrangian of the form

$$\mathcal{L} \supset g_{\mu\tau} V_\rho (\bar{\ell}_2 \gamma^\rho \ell_2 - \bar{\ell}_3 \gamma^\rho \ell_3 + \bar{\mu}_R \gamma^\rho \mu_R - \bar{\tau}_R \gamma^\rho \tau_R), \quad (6)$$

where  $\ell_{2,3}$  are the second and third generation leptons  $SU(2)_L$  doublets. This type of gauge interactions can be obtained, e.g., as the  $\tau_3$ -generator of a broken  $SU(2)_{fL}$  flavour gauge symmetries [40], or just as an abelian subgroup of a bigger flavour gauge groups. It is additionally anomaly-free with just the SM fermionic content and is one of the simplest  $U(1)_X$  model still standing for the  $(g-2)_\mu$  anomaly [41]. The annihilation of DM into neutrinos makes this boson a viable mediator for a light DM scenario (see e.g. [42]).

Finally, it is important to point out that in these models the renormalisation group evolution of the FIP interactions can play an important role and must be estimated quantitatively. This is particularly critical for models which try to prevent the appearance of flavour-violation couplings or couplings to first-generation fermions for phenomenological reasons. For instance, for a vector FIP, kinetic mixing typically arises back with a loop factor from SM fermions loop, and plays a critical role in experimental searches [41, 45–49]. Similarly for an axion-like particle, most couplings re-appear radiatively in the RGE, including possible flavour-violating interactions [50–57]. Altogether, one must therefore be particularly careful when dealing with large FIP couplings to second and third generations as they will probably re-introduce couplings to first-generation fermions or to photons, both of which can be easily constrained in precision frontier experiments.

## 4. Conclusion

The last decade has seen strong links built between the communities studying light DM models and the flavour physics world. Indeed, a vast literature now show that flavour physics, and more generically flavour physics experiments, are extremely effective at testing the mediator particle typically required in these light DM models for two main reason. First, the precision of the measurements in flavour experiments can be leveraged to search for very rare, light DM-inspired, events which are not necessarily flavourful, such as rare mono-photon searches in, e.g. Belle-II. Second, many light DM models have inherently flavourful interactions, allowing for clean rare flavourful processes with invisible energy in the final states which can be searched for by intensity frontier collaborations. In this review, we presented an overview of the interplay between both fields from the point of view of the feebly-interacting particle (FIP) mediator between the DM (or dark sector) and the SM. The classification in portals has been reviewed with an eye on the flavourfulness of each portal operator, showing that in many cases flavourful interactions are expected and lead to a rich phenomenology in precision-frontier experiments. We finally analysed in more details the case of a “fermion portal”, showing that flavour physics and flavour experiments are key players in probing these scenarios.

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