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LCSR predictions for $b \to s$ hadronic form factors

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We address the calculation of mesonic $B \to P$ form factors where P is a light pseudoscalar meson using Light Cone Sum Rules with B-meson Distribution Amplitudes, and present preliminary results for $B \to K$ form factors at zero momentum transfer.

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1. Introduction

Deviations from the Standard Model have long been observed in semileptonic B-meson decays, notably $b \to s\ell\ell$ transitions, triggering speculations on potential New Physics effects in this sector. After the recent update of $R_{K^{(*)}}$ [1] and $\mathrm{BR}(B_{(s)} \to \mu\mu)$ [2] by the LHCb collaboration, the remaining significant deviations from the SM in flavour-changing neutral currents B decays are found in the branching ratios of mesonic decays involving $b \to s\mu\mu$ [3] and in the angular observable P_5' [4–7].
Unlike $R_{K^{(*)}}$ and $\mathrm{BR}(B_{(s)} \to \mu\mu)$, the observables $\mathrm{BR}(B_{(s)} \to M\mu\mu)$ ($M = K^{(*)}, \phi, \dots$) are theoretically challenging to predict accurately because of their high sensitivity to non-perturbative QCD contributions, both local and non-local. These contributions yield a theoretical error of the order of 30%, which can be as large as (sometimes larger than) the experimental uncertainty, and clearly hamper the potential of these observables for discovery [8, 9].

At low hadronic recoil these form factors can be computed using lattice QCD methods [10, 11], while in the large recoil region the analytical method of Light Cone Sum Rules (LCSR) has mostly been used [9, 12, 13] -although the HPQCD collaboration [11] recently announced results in the whole momentum transfer range using lattice QCD.

In this study, we utilize the LCSR method with B-meson Distribution Amplitudes (DAs) to present preliminary findings on form factors at zero momentum transfer in $B \to K$ transitions.

2. Form Factors

The relevant form factors for the $B \to P$ transitions are $f_0^{B\to P}$, $f_+^{B\to P}$ and $f_T^{B\to P}$. They can be defined as follows:

$$\langle P(k)|\,\bar{q}\gamma^\mu b\,|B(P_B = k+q)\rangle = \left((2k+q)^\mu - \frac{m_B^2 - m_P^2}{q^2}q^\mu\right) f_+^{B\to P}(q^2) + \frac{m_B^2 - m_P^2}{q^2}q^\mu f_0^{B\to P}(q^2), \tag{1}$$

$$\langle P(k)|\,\bar{q}\sigma^{\mu\nu}q_\nu b\,|B(P_B = k+q)\rangle = \frac{i f_T^{B\to P}(q^2)}{m_B + m_P}\left(q^2(2k+q)^\mu - (m_B^2 - m_P^2)q^\mu\right). \tag{2}$$

P_B denotes the momentum of the B-meson, k that of the light meson and q^2 is the momentum transfer squared. The relation $f_+^{P\to B}(q^2 = 0) = f_0^{P\to B}(q^2 = 0)$ ensures that (1) is well-defined at $q^2 = 0$.

3. Correlation function and analyticity

LCSR with B-meson DAs are constructed from a B-meson to vacuum correlation function [14]:

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x}\,\langle 0|\,T J^\nu_{\text{int}}(x) J^\mu_{\text{weak}}(0)\,|\bar{B}(P_B = q+k)\rangle\,. \tag{3}$$

The correlation function in (3) is of interest because on the one hand it can be expressed as a function of the relevant form factors at positive k^2, and on the other hand it can be computed pertubatively in QCD at large negative k^2. Matching both expressions results in the so-called LCSR [15]. In the following we take an on-shell B-meson: $P_B = m_B v$, $v^2 = 1$ and an off-shell light meson: $k^2 \neq m_P^2$.

We work in the heavy quark effective theory (HQET), hence the heavy b-quark field is replaced by the HQET field h_v, where v denotes the velocity of the B-meson.

We define $J^{\nu}_{\text{int}} \equiv \bar{d}(x)\Gamma_2^{\nu} q_1(x)$ and $J^{\mu}_{\text{weak}} \equiv \bar{q}_1(0)\Gamma_1^{\mu} h_v(0)$ as in [9]. $\Gamma_{1,2}$ expressions can be read from Table 1. q_1 is the exchanged internal quark, of mass m_1.

Process	J^{ν}_{int}	J^{μ}_{weak}	Form Factors
$\bar{B}^0 \to \bar{K}^0$	$\bar{d}\gamma^{\nu}\gamma_5 s$	$\bar{s}\gamma^{\mu} h_v$ $\bar{s}\sigma^{\mu\nu} q_{\nu} h_v$	$f_{+}^{B\to K}(q^2)$, $f_{+/-}^{B\to K}(q^2)$ $f_T^{B\to K}(q^2)$

Table 1: Weak and interpolating currents in the correlation function.

For the non-perturbative expression, following [15, 16], the correlation function is analytic in k^2. For a negative k^2, using the Cauchy formula on the contour C in Fig. 1 and Schwartz reflection principle, the B-meson to vacuum correlation function is

$$\Pi^{\mu\nu}(q,k) = \frac{1}{\pi}\int_{t_{min}}^{\infty} ds \frac{\mathrm{Im}\Pi^{\mu\nu}(s)}{s - k^2}, \tag{4}$$

where t_{min} is below every hadronic treshold.

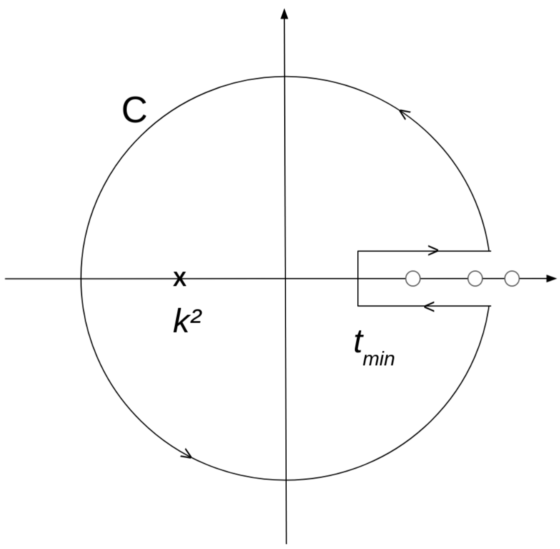

Figure 1: Contour in the complex plane. The circles at $k^2 > 0$ denote hadronic thresholds.

By inserting the complete set of intermediate hadronic states between the two currents, one obtains a hadronic dispersion relation for the correlation function

$$\Pi^{\mu\nu}(q,k) = \frac{\langle 0| J^{\nu}_{\text{int}} |M(k)\rangle \langle M(k)| J^{\mu}_{\text{weak}} |\bar{B}(q+k)\rangle}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_{cont}}^{+\infty} ds \frac{\rho^{\mu\nu}(s)}{s-k^2}, \tag{5}$$

where s_{cont} is the threshold of the lowest continuum state. M is the lightest meson of the aforementioned set of hadronic states, whose contribution has been singled out. ρ is the density of the excited and continuum states. The M to vacuum matrix element on the r.h.s. can be expressed with the light meson decay constant

$$\langle 0| \bar{q}_2 \gamma^{\nu}\gamma_5 q_1 |M(k)\rangle = ik^{\nu} f_P \tag{6}$$

for $M = K$ in our case. The $B \to M$ matrix element is proportional to the transition form factors of interest. Depending on the inserted current J_{weak}, this matrix element is related to the form factors by the definitions in (1) or (2).

4. Perturbative expansion

Let us define $\tilde{q}$ by $q = m_b v + \tilde{q}$. Thus $k + \tilde{q} = (m_B - m_b)v$. In order for the correlation function (3) to be computable perturbatively in QCD [17]:

$$|k^2|, |\tilde{q}^2| \gg \Lambda_{QCD}^2. \tag{7}$$

The LCSR method relies on an expansion of the products of currents near the light-cone. The kinematical regime has to be such that the dominant contributions to (3) arise from light-like distance $x^2 \sim 0$, while respecting the condition of perturbativity (7). The integral (3) is dominated by the region where $k \cdot x \lesssim O(1)$. We choose the frame of reference such that

$$k \cdot x = k_0 x_0 - k_3 x_3. \tag{8}$$

For $k^2 < 0$ we can write:

$$k_0 = \frac{m_B^2 + k^2 - q^2}{2m_B}, \qquad k_3 = \sqrt{k_0^2 - k^2}, \qquad x_3 = \frac{k_0 x_0 - k \cdot x}{k_3}, \tag{9}$$

which yields

$$x^2 = -\frac{4m_B \left(x_0^2 k^2 m_B - x_0 (k\cdot x)(k^2 + m_B^2 - q^2) + m_B (k\cdot x)^2\right)}{k^4 + (m_B^2 - q^2)^2 - 2k^2(m_B^2 + q^2)}. \tag{10}$$

The light-cone expansion of the product of currents is valid if $x^2 \ll 1/\Lambda_{QCD}^2$. The B to vacuum distribution amplitudes are suppressed at large x_0, typically a few $1/\Lambda_{QCD}$, hence the correlation function is suppressed at $x_0 \to +\infty$.
Thus, keeping in mind that $x_0 \leqslant O(1/\Lambda_{QCD})$, the dominant contribution to (3) which comes from the region $k \cdot x \lesssim O(1)$ does arise from the light-cone $x^2 \to 0$ when $k^2 \to -\infty$ for any finite q^2. This also satisfies condition (7), since

$$\tilde{q}^2 = q^2 (1 - \frac{m_b}{m_B}) + k^2 \frac{m_b}{m_B} + m_b^2 - m_B m_b. \tag{11}$$

Therefore at large negative k^2, (3) can be computed in perturbative QCD and be expanded on the light-cone. We find no strong restriction on the transfer momentum in this limit.

For the perturbative expression we only retain the 2-parton (quark-antiquark) and 3-parton (quark-gluon-antiquark) contributions of the Fock expansion for the B-meson, leading to two-particle contributions and 3-particles contributions. The internal quark propagator is expanded on the light-cone [18]. We adopt the same notation as in [9]:

$$\Pi^{\mu\nu}|_{2p} = \int d^4x \int \frac{d^4p'}{(2\pi)^4} e^{i(k-p').x} \left[\Gamma_2^\nu \frac{\not p' + m_1}{m_1^2 - p'^2} \Gamma_1^\mu \right]_{\alpha\beta} \langle 0| \bar{q}_2{}^\alpha(x) h_v^\beta(0) |\bar{B}(v)\rangle, \tag{12}$$

$$\Pi^{\mu\nu}|_{3p} = \int d^4x \int \frac{d^4p'}{(2\pi)^4} \int_0^1 du\, e^{i(k-p').x} \left[\Gamma_2^\nu \frac{(1-u)(\not p' + m_1)\sigma^{\lambda\rho} + u\sigma^{\lambda\rho}(\not p' + m_1)}{2(m_1^2 - p'^2)^2} \Gamma_1^\mu \right]_{\alpha\beta} \times \langle 0| \bar{q}_2{}^\alpha(x) G_{\lambda\rho}(ux) h_v^\beta(0) |B\bar(v)\rangle, \tag{13}$$

where $G_{\lambda\rho}(ux) = g_s(\lambda^a/2)G^a_{\lambda\rho}(ux)$ is the gluon field strength tensor evaluated at ux, a fraction of the distance x. The B to vacuum matrix elements in the above equations are given directly as integrals over the B-meson DAs [19]. Schematically:

$$\langle 0| \bar{q}_2{}^\alpha(x)[x,0] h_v^\beta(0) |\bar{B}(v)\rangle = -\frac{i f_B m_B}{4} \int_0^{+\infty} d\omega e^{-i\omega v.x} \Phi_{2p}(\omega)^{\beta\alpha}, \tag{14}$$

$$\langle 0| \bar{q}_2{}^\alpha(x)[x,ux] G_{\lambda\rho}(ux)[ux,0] h_v^\beta(0) |\bar{B}(v)\rangle = \frac{f_B m_B}{4} \int_0^{+\infty} d\omega_1 \int_0^{+\infty} d\omega_2 e^{-i(\omega_1+u\omega_2)v.x} \Phi_{3p}(\omega_1,\omega_2)^{\beta\alpha}, \tag{15}$$

where $\Phi_{2p}(\omega)$ and $\Phi_{3p}(\omega_1,\omega_2)$ refer to combinations of B-meson DAs and spin structures. The brackets $[x,0]$ and such denote Wilson lines that render the DAs gauge invariant. We work in the Fock-Schwinger gauge $x^\mu A^a\mu(x)\lambda^a/2 = 0$ where the Wilson lines are **1**.

The correlation function can be written in the form

$$\Pi^{\mu\nu} = \int_0^{+\infty} d\sigma \sum_{n=1}^{+\infty} \frac{I_n^{\mu\nu}(\sigma)}{(s(\sigma)-k^2)^n}, \tag{16}$$

where the function s is defined as $s(\sigma) = \sigma m_B^2 + \frac{m_1^2 - \sigma q^2}{1-\sigma}$. $I_n^{\mu\nu}(\sigma)$ are functions of the B-DAs and have 2-particle and 3-particle contributions:

$$I_n^{\mu\nu(2p)}(\sigma,q^2) = \frac{f_B m_B}{\bar{\sigma}^n} \sum_{\psi_{2p}} C_n^{\mu\nu(2p)}(\sigma,q^2)\, \psi_{2p}(\sigma m_B), \tag{17}$$

where ψ_{2p} = 2-particle B-DAs and

$$I_n^{\mu\nu(3p)}(\sigma,q^2) = \frac{f_B m_B}{\bar{\sigma}^n} \int\limits_0^{\sigma m_B} d\omega_1 \int\limits_{\sigma m_B - \omega_1}^{\infty} \frac{d\omega_2}{\omega_2} \sum_{\psi_{3p}} C_n^{\mu\nu(3p)}(\sigma,u,q^2)\, \psi_{3p}(\omega_1,\omega_2) \Big|_{u=(\sigma m_B - \omega_1)/\omega_2}, \tag{18}$$

where $\psi_{3\mathrm{p}}$ = 3-particle B-DAs. In (17) the variable change $\sigma = \omega/m_B$ was performed while in (18) we took $\sigma = (\omega_1 + u\omega_2)/m_B$.

5. Quark-hadron duality

At this point one can match both non-perturbative and perturbative expressions:

$$\Pi^{\mu\nu}(q,k) = \frac{\langle O|\, J^{\nu}_{\mathrm{int}}\, |M(k)\rangle\langle M(k)|\, J^{\mu}_{\mathrm{weak}}\, |\bar{B}(q+k)\rangle}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_{cont}}^{+\infty} ds \frac{\rho^{\mu\nu}(s)}{s-k^2}, \tag{19}$$

$$= \int_0^{+\infty} d\sigma \sum_{n=1}^{+\infty} \frac{I_n^{\mu\nu}(\sigma)}{(s(\sigma)-k^2)^n}. \tag{20}$$

To obtain an expression for the relevant transition form factors, one needs to estimate the integral over the density of the excited and continuum states using quark-hadron duality [15, 20]. At sufficiently large negative k^2, the semi-global quark-hadron duality is

$$\int_{s_{cont}}^{+\infty} ds \frac{\rho(s)}{s-k^2} \approx \frac{1}{\pi}\int_{s_0}^{+\infty} ds \frac{\mathrm{Im}\Pi^{\mathrm{LCOPE}}(s)}{s-k^2}, \tag{21}$$

where s_0, the quark-hadron duality threshold, has to be determined.

One can note that the expression obtained pertubatively in (16) has different powers of the denominator. As such it cannot be directly identified with the r.h.s of (21). After integration by parts we can rewrite the correlation function (calculated perturbatively) as:

$$\Pi^{\mu\nu} = \int_0^{+\infty} d\sigma \sum_{n=1}^{+\infty} \frac{1}{(n-1)!}\frac{1}{s-k^2}\left(\frac{d}{d\sigma}\frac{1}{s'}\right)^{n-1} I_n^{\mu\nu}(\sigma) \tag{22}$$

$$+ \sum_{n=2}^{+\infty} \frac{1}{(n-1)!} \sum_{j=1}^{n-1} \frac{(n-j-1)!}{(s-k^2)^{n-j}}\frac{1}{s'}\left(\frac{d}{d\sigma}\frac{1}{s'}\right)^{j-1} (I_n^{\mu\nu}(\sigma))|_{\sigma=0},$$

where in the above equation $s' = \frac{ds}{d\sigma}$ and we adopt the notation

$$\left(\frac{d}{d\sigma}\frac{1}{s'}\right)^{m} F(\sigma) = \frac{d}{d\sigma}\frac{1}{s'}\left(\frac{d}{d\sigma}\frac{1}{s'}...(F(\sigma))\right). \tag{23}$$

The first term on the r.h.s in (22) has the proper structure, but there is also a "boundary" term evaluated at $\sigma = 0$. In [21] it is argued that since $s(\sigma = 0) = m_1^2 \ll s_0$, the singularity in that boundary term is not contained in the OPE integral above s_0, thus the quark-hadron duality subtraction only concerns the first term. Let us define σ_0 such that $s(\sigma_0) = s_0$. After the quark-hadron duality subtraction, we obtain:

$$\frac{\langle O|\, J^{\nu}_{\mathrm{int}}\, |M(k)\rangle\langle M(k)|\, J^{\mu}_{\mathrm{weak}}\, |\bar{B}(q+k)\rangle}{m_M^2 - k^2} = \int_0^{+\infty} d\sigma \sum_{n=1}^{+\infty} \frac{I_n^{\mu\nu}(\sigma)}{(s(\sigma)-k^2)^n} \tag{24}$$

$$- \int_{\sigma_0}^{+\infty} d\sigma \sum_{n=1}^{+\infty} \frac{1}{(n-1)!}\frac{1}{s(\sigma)-k^2}\left(\frac{d}{d\sigma}\frac{1}{s'}\right)^{n-1} I_n^{\mu\nu}(\sigma).$$

One should note that in the perturbative expression the integral is over σ and not s. For the quark-hadron duality to work we argue that $s(\sigma)$ should vary monotonously from m_1^2 to $+\infty$ as it is the case when the integral is over s varying from m_1^2 to $+\infty$. Since $s'(\sigma) = m_B^2 + \frac{m_1^2 - q^2}{(1-\sigma)^2}$, this is only true if $q^2 < m_1^2$. Thus the LCSRs we derive are only valid at $q^2 < m_1^2$.

6. Borel transformation

The semi-global quark-hadron duality is an approximation which comes with a poorly understood systematic error. In order to suppress the impact of the quark-hadron duality approximation, it is customary to perform the so-called Borel transformation [9, 15, 22]. The Borel transformation is defined as

$$F(M^2) \equiv \mathcal{B}_{M^2} F(k^2) = \lim_{-k^2, n \to \infty \text{ and } \frac{-k^2}{n} = M^2} \frac{(-k^2)^{n+1}}{n!} \left(\frac{d}{dk^2} \right)^n F(k^2), \tag{25}$$

with M^2 the Borel parameter. A typical Borel transformation is

$$\mathcal{B}_{M^2} \left(\frac{1}{(m^2 - k^2)^n} \right) = \frac{1}{(n-1)!} \frac{exp(-m^2/M^2)}{M^{2(n-1)}}. \tag{26}$$

This allows to exponentially suppress the unknown contributions from the excited states and the continuum.

From (24), selecting the proper Lorentz structure and applying a Borel transformation, one can extract the form factors in the following form (F denoting one of the form factors):

$$\begin{aligned} F = & \frac{1}{K^{(F)}} \sum_{n=1}^{\infty} \left\{ \int_0^{\sigma_0} d\sigma \; e^{(-s(\sigma,q^2)+m_M^2)/M^2} \frac{1}{(n-1)!(M^2)^{n-1}} I_n^{(F)} \right. \\ & \left. + \left[\frac{1}{(n-1)!} e^{(-s(\sigma,q^2)+m_M^2)/M^2} \sum_{j=1}^{n-1} \frac{1}{(M^2)^{n-j-1}} \frac{1}{s'} \left(\frac{\mathrm{d}}{\mathrm{d}\sigma} \frac{1}{s'} \right)^{j-1} I_n^{(F)} \right]_{\sigma=\sigma_0} \right\}, \end{aligned} \tag{27}$$

with $K^{(F)}$ normalisation factors involving the light-meson decay constant.

7. Numerical results

The last step before numerically evaluating the sum rule is to set the two following parameters: the Borel parameter M^2 and the duality threshold s_0.

The Borel parameter has to account for a compromise. It has to be large enough so that the expansion in growing twists -supressed by powers of $1/M^2$- converges sufficiently fast, but not too large, so that the contribution of the density over excited and continuum states is still exponentially suppressed.

For $B \to K$ we find that the window $0.5 \leqslant M^2 \leqslant 1.5$ GeV^2 is suitable. We take M^2 to be uniform over that window.

The duality threshold is determined at fixed M^2 by imposing that the form factor of interest is independent of the Borel parameter. Taking the derivative of our perturbative expression for the form factor with respect to the Borel parameter as null leads to a second sum rule, called the daughter sum rule, which predicts the mass of the light meson. One can recquire the daughter sum rule to accurately predict the light-meson's mass, which sets the value of the duality threshold.

For the other input parameters, we assume they follow independent Gaussian distributions. Their values are summarised in (Table 2).

Parameter	Value
m_B	5.27966 ± 0.00012 GeV
m_K	0.497611 ± 0.000013 GeV
f_B	0.1894 ± 0.0014 GeV
$1/\lambda_B^+$	2.2 ± 0.6 GeV^{-1}
λ_E^2	0.03 ± 0.02 GeV^2
λ_H^2	0.06 ± 0.03 GeV^2
f_K	0.1557 ± 0.0007 GeV

Table 2: List of input parameters. Masses are taken from [23], f_B from [24], f_K from [25] and $1/\lambda_B^+$, λ_E^2 and λ_H^2 from [9].

We obtain the following preliminary results (Table 3) which are in agreement with the literature.

Form Factor ($q^2 = 0$)	Our result	Other Results
$f_+^{B\to K}$	0.19 ± 0.07	0.27 ± 0.08 [9]
		0.331 ± 0.041 [12]
		0.31 ± 0.04 [14]
		0.395 ± 0.033 [13]
		0.364 ± 0.05 [26]
$f_T^{B\to K}$	0.20 ± 0.08	0.25 ± 0.07 [9]
		0.358 ± 0.037 [12]
		0.27 ± 0.04 [14]
		0.381 ± 0.027 [13]
		0.363 ± 0.08 [26]

Table 3: Our preliminary results with a comparison to literature.

8. Conclusion

In light of the evolving B-anomalies landscape and the latest experimental insights, the need for accurate computations of transition form factors has become more critical than ever before. In this endeavor, we have employed the Light-Cone Sum Rules method in conjunction with B-meson distribution amplitudes to compute the $B \to K$ form factors at zero momentum transfer. Our preliminary findings are in agreement with the existing body of literature.

Our final results for the $B \to K$ form factors are to be implemented in *SuperIso*, a public program for the calculation of flavour physics observables [27–30]. The next step will be undertaking the computation of the non-local contribution, which constitute the main source of theoretical uncertainty in the predictions of $B \to Ml^+l^-$ observables.

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