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SM prediction for the CP asymmetries in two-body hadronic charm meson decays

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Charm Physics is highly topical in the current flavour landscape, especially after the announcement by LHCb of the measurement of direct CP asymmetries in the separate decays of $D^0 \to K^+K^-$ and consequently $D^0 \to \pi^+\pi^-$, preceded by the measurement of the difference of these two asymmetries. The experimental result is extremely difficult to interpret, as the fully hadronic decays of charm entail significant QCD uncertainties, precluding tests of the Kobayashi-Maskawa mechanism in the up-type sector. In this work we address the problem of the theoretical determination of the strong amplitudes involved by considering very general properties of amplitudes, namely unitarity and analyticity. We implement these properties in two-channel dispersion relations which describe the final state interactions between the pion and kaon pairs, using data-driven parameterizations of all the strong rescattering quantities. While reproducing the experimental branching fractions of the aforementioned decays as well as of the ones related through isospin, we predict CP asymmetries which are way below the experimental values. Moreover, we argue that even without considering the specifics of the uncertainty-plagued inelasticity between the pions and kaons, the two-channel hypothesis always yields a severe underestimation of the CP asymmetries.

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1. Introduction

Searches for CP violation have always been an integral part of the Flavour Physics programme. In the quark sector, the Standard Model (SM) contains only one parameter that can induce a violation of the CP symmetry, namely the phase of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. The elements of said matrix, derived from four independent parameters in total, have been extensively probed from various observables such as B- and K-meson related ones, exhibiting a remarkable level of agreement between the different measurements. Charm meson Physics provides a complementary domain giving access to CKM parameter combinations from up-type quarks.

Recently the experimental interest has been directed towards various types of charm meson processes, such as rare decays, $D-\overline{D}$ mixing and the subject of this work, which is the measurement of the CP asymmetries in D^0 decays to $\pi^+\pi^-$ and to K^+K^- [1, 2]. The latter measurements, which constitute the only observation of CP violation in the charm sector up to date, have motivated various theoretical estimations (see e.g. [3, 4]) as well as ignited the discussion about different New Physics scenarios [5]. However the measured values of the asymmetries are extremely difficult to interpret, the reason being that their theoretical calculation involves hadronic S-matrix elements which are dominated by non-perturbative QCD. Unfortunately, there is no definitive suitable framework for the calculation of such matrix elements, contrary to K decays where one can make use of chiral perturbation theory or lattice results, and B decays, where various well-developed tools help mitigate the uncertainties related to hadronic effects.

In this work [6] we carry out a data-driven calculation in the Standard Model, based on the assumption that the main effect driving the hadronic matrix elements is s-channel rescattering between the final states of two pions and two kaons, respecting the isospin symmetry. We exploit the property of analyticity of scattering amplitudes to connect the amplitudes involved in these decays through two-channel dispersion relations which we solve numerically. The non-rescattering part of the hadronic matrix elements is calculated in the large number of colours expansion.

The importance of a realistic determination of the strong amplitudes can be illustrated schematically with the following: The decay amplitude of a D^0 meson can be parameterized in terms of two amplitudes

$$\mathcal{A}(D^0 \to f) = A(f) + ir_{CKM}B(f) \tag{1}$$

$$\mathcal{A}(\overline{D^0} \to f) = A(f) - ir_{CKM}B(f) \tag{2}$$

where in our case $r_{CKM} = \mathrm{Im}\frac{V^*_{cb}V_{ub}}{V^*_{cd}V_{ud}} \approx 6.2 \cdot 10^{-4}$, hence the direct CP asymmetry is very well approximated by

$$a^{dir}_{CP} \approx 2r_{CKM}\frac{|B(f)|}{|A(f)|} \cdot \sin\arg\frac{A(f)}{B(f)} \tag{3}$$

Here the weak phases are factored out and the phases of the amplitudes A and B are only induced by the strong interactions. It is then clear that the determination of the magnitudes of the interfering amplitudes, as well as of their strong phase-shift difference, is crucial for the calculation of the CP asymmetry.

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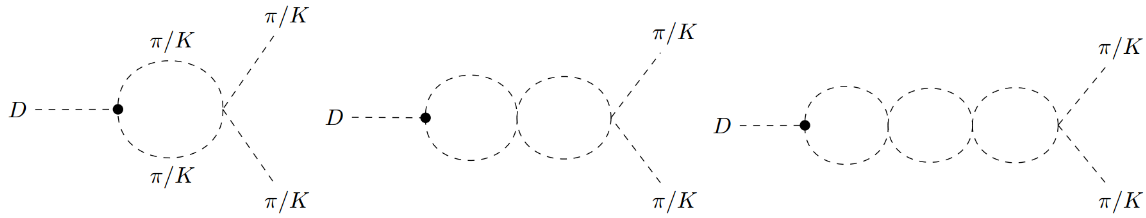

Figure 1: Diagrams of s-channel rescattering "bubbles".

2. Short-distance treatment

For the decays of interest, the weak Hamiltonian at a scale $\mu < m_b$ has the following form:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\Sigma_{i=1}^{2} C_i(\mu) \left(\lambda_d Q_i^d(\mu) + \lambda_s Q_i^s(\mu) \right) - \lambda_b (\Sigma_{i=3}^{6} C_i(\mu) Q_i(\mu) + C_{8g}(\mu) Q_{8g}(\mu)) \right] + h.c.$$

where the operators 1 and 2 are the current-current ones

$$Q_1^q = (\bar{q}c)_{V-A}(\bar{u}q)_{V-A}$$
$$Q_2^q = (\bar{q}_j c_i)_{V-A}(\bar{u}_i q_j)_{V-A}$$

$(q = d, s)$ and the operators 3 to 6 are the so-called penguin operators, which arise when matching the effective theory below the mass of the b quark to the one of $m_b < \mu < m_W$:

$$Q_3 = (\bar{u}c)_{V-A} \Sigma_q (\bar{q}q)_{V-A}$$
$$Q_4 = (\bar{u}_j c_i)_{V-A} \Sigma_q (\bar{q}_i q_j)_{V-A}$$
$$Q_5 = (\bar{u}c)_{V-A} \Sigma_q (\bar{q}q)_{V+A}$$
$$Q_6 = (\bar{u}_j c_i)_{V-A} \Sigma_q (\bar{q}_i q_j)_{V+A}$$

The Wilson coefficients of this basis have been calculated up to next to leading order (NLO) with some NNLO ingredients available [7]. The chromomagnetic operator does not give significant contributions (see comment in the next section) and is ignored. The penguin operators also have very small Wilson coefficients but need to be taken into account, for reasons that will be discussed later. The determination of the CP asymmetry requires the calculation of the hadronic matrix elements $\langle \pi^+\pi^- | Q_i^q | D^0 \rangle$ and $\langle K^+K^- | Q_i^q | D^0 \rangle$ for i=1,2,4,6 and $q = d, s$ (contributions from the operators 3 and 5 are suppressed according to the treatment explained in the following section).

3. Non-perturbative QCD treatment

We consider that the long-distance QCD effects on the decay amplitudes are encapsulated predominantly in the s-channel rescattering of the final states (Fig. 1). Given that isospin is a symmetry of the strong interactions (whose breaking by electromagnetic interactions and quark masses is irrelevant for the targeted level of precision), we view rescattering as taking place between isospin-invariant final states. This relates the matrix elements of $D^0 \to \pi^+\pi^-$, $D^0 \to \pi^0\pi^0$, $D^0 \to K^+K^-$ and $D^0 \to K^0\overline{K^0}$ with one another, through the reduced matrix elements of $D \to \pi\pi$ and

$D \to KK$ with isospin zero, $D \to \pi\pi$ with isospin 2 and $D \to KK$ with isospin 1. Inelastic rescattering between the pion and kaon pairs thus only happens in the isospin-zero channels; we also assume the isospin-1 and -2 states to rescatter only elastically. Unitarity of the S-matrix, as well as of the strong S-submatrix, both projected onto the S-wave isospin-zero final states, yields then a relation between the decay amplitudes and the $\pi\pi$, KK rescattering amplitudes:

$$\begin{pmatrix} A(D \to \pi\pi_{I=0}) \\ A(D \to KK_{I=0}) \end{pmatrix} = \underbrace{\begin{pmatrix} \eta e^{i2\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{i2\delta_2} \end{pmatrix}}_{(S_{strong})^0_0} \cdot \begin{pmatrix} A^*(D \to \pi\pi_{I=0}) \\ A^*(D \to KK_{I=0}) \end{pmatrix} \tag{4}$$

with an analogous expression holding for the CP-odd amplitudes B in place of A. In the elastic limit $\eta \longrightarrow 1$ one recovers Watson's theorem, which would identify the strong phase of the respective decay amplitude with the strong phase of the $\pi\pi$ or KK rescattering (this is the case for the isospin-1 and -2 channels). It is clear that the strong phases of the amplitudes are related to the ones of the rescattering amplitudes, which as we will see are experimentally known at low energies.

Simultaneously, we employ the property of analyticity. This is a model-independent, fundamental property of the amplitudes arising as a necessary condition for causality to be respected. By extending the Mandelstam variables to the complex plane we can apply Cauchy's theorem, taking a contour around the physical region $Re\{s\} > 4m_\pi^2 \equiv s_{thr}$ (left-hand cuts are absent because there is no crossing symmetry for the decay amplitudes). This results in a relation of the form

$$ReA(s) = \frac{1}{\pi} PV \int_{s_{thr}}^{\infty} ds' \frac{ImA(s')}{s'-s} \tag{5}$$

By also implementing the result from unitarity, in the case of elastic rescattering we have

$$ReA(s) = \frac{1}{\pi} PV \int_{s_{thr}}^{\infty} ds' \frac{\tan\delta(s')}{s'-s} ReA(s') \tag{6}$$

This integral equation (rather its once-subtracted form) has an explicit solution

$$|A_I(s)| = \underbrace{A_I(s_0)}_{\text{ampl. when } \Omega^{(I)}=1} \underbrace{\exp\{\frac{s-s_0}{\pi} PV \int_{4M_\pi^2}^{\infty} dz \frac{\delta_I(z)}{(z-s_0)(z-s)}\}}_{\text{Omnes factor } \Omega^{(I)}} \tag{7}$$

where the so-called Omnes factor arises from the strong rescattering. In the case of two-channel rescattering the relevant hadronic matrix elements for the operators Q_i, $i = 1, 2, 4, 6$ (with Q_i^q implied for Q_1 and Q_2) are written as

$$\begin{pmatrix} \langle\pi\pi_{I=0}| Q_i |D\rangle \, (s = m_D^2) \\ \langle KK_{I=0}| Q_i |D\rangle \, (s = m_D^2) \end{pmatrix} = \Omega^{(0)}(s = m_D^2) \cdot \begin{pmatrix} \langle\pi\pi_{I=0}| Q_i |D\rangle \, (s_0) \\ \langle KK_{I=0}| Q_i |D\rangle \, (s_0) \end{pmatrix} \tag{8}$$

where the Omnes factor is now a two-by-two matrix, which is found by solving numerically (following the method introduced by [11]) the system of dispersion relations

$$\mathrm{Re}[\Omega^{(0)}(s)] = \frac{s-s_0}{\pi} PV \int_{4M_\pi^2}^{\infty} ds' \frac{T_0^{0*}(s')\Sigma(s')\Omega^{(0)}(s')}{(s'-s_0)(s'-s)} \tag{9}$$

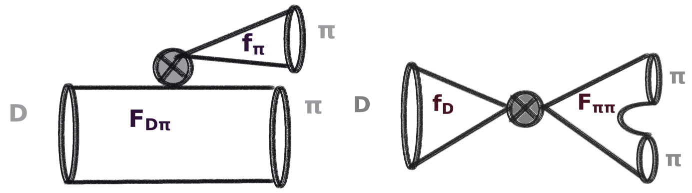

Figure 2: Examples of tree (left) and annihilation (right) topologies.

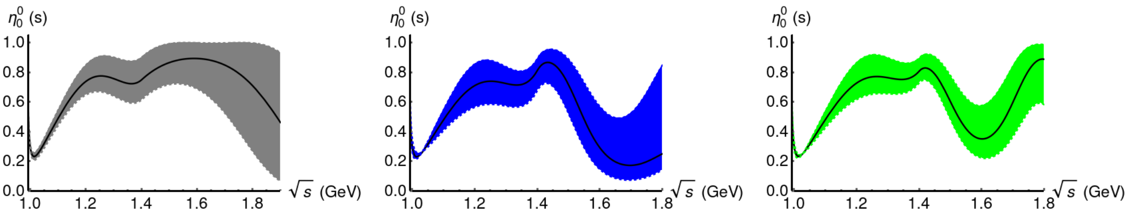

Figure 3: Three different possible solutions for the inelasticity of the S-wave isospin-zero $\pi\pi$ channel, taken from [9].

with $T_0^0(s)$ defined from $(S_{strong})_J^I = \mathbf{1} + 2i\, \Sigma^{1/2}(s)\, T_J^I(s)\, \Sigma^{1/2}(s)$, and the matrix of kinematical factors $\Sigma(s) = \mathrm{diag}\left[\Theta(s-4M_\pi^2)\,\sigma_\pi(s)\,,\, \Theta(s-4M_K^2)\,\sigma_K(s)\right]$, $\sigma_i(s) = (1-4M_i^2/s)^{1/2}$.

If the strong phase $\delta_I(z)$ was absent, as seen from Eq. (7) the amplitude would reduce to the amplitude at the subtraction point; therefore $A_I(s_0)$ needs to be taken as the no-rescattering limit. This is given by the leading term in the large number of colours (N_C) expansion, in which the hadronic matrix elements are found by insertions of the operators into tree and annihilation topologies, with no gluon exchange between the two currents; see Fig. 2. Hence the leading-N_C Q_3 and Q_5 contributions vanish in the isospin limit, because the only surviving insertions are into tree topologies involving the matrix element $\langle\pi^0|\overline{u}\gamma^\mu\gamma_5 u + \overline{d}\gamma^\mu\gamma_5 d|0\rangle$, where the operator is of $\Delta I = 0$. The contribution of the chromomagnetic operator is also zero for $N_C \to \infty$. The calculation of the relevant insertions of the other operators requires knowledge of a vector form factor $D \to \pi$ or $D \to K$, or the pion scalar form factor, and one decay constant. These quantities are calculated in lattice or chiral perturbation theory to good precision.

For the isospin-1 and -2 channels, we assume only KK and $\pi\pi$ states to be present and extract the magnitudes of the Omnes factors (at $s = m_D^2$) from the experimental branching ratios of the related $D^+ \to K^+\overline{K^0}$ and $D^+ \to \pi^+\pi^0$ decays. The phase-shifts of these isospin decay amplitudes are left free. The physical inputs for the description of final state interactions in the isospin-zero channels include two rescattering phase-shifts and one inelasticity (see Eq. (4)). These are known from various experiments and there exist data-driven parameterizations [8–10] as functions of the center-of-mass energies, which have been constructed to respect the expected properties of phase-shifts, encoding the effects of known intermediate resonances. Uncertainties from conflicting sets of data are taken into account and multiple solutions are considered, which is why the inelasticity appears to fall in more than one possible broad-spanning "bands" for energies above 1.6 GeV, see Fig. 3. We carefully extrapolate all the parameters for energies above the range provided, respecting requirements for their behaviour as $s \to \infty$, such as that $\eta \to 1$.

4. Results and conclusions

The large uncertainties involved especially in the inelasticity result in a plethora of possible solutions for the Omnes matrix, which in turn determine the decay amplitudes of all the $D \to \pi\pi$, KK channels. To remove some of the uncertainties we consider the appropriate Omnes matrices to be the ones that predict the branching fractions of all the decay channels close to their experimental values, for some value of the free parameters of the isospin-1 and 2 phase-shifts. This requirement restricts the viable inelasticity inputs to the ones of solution I (grey band in Fig. 3). The allowed Omnes matrices appear to have large off-diagonal elements, which as seen from Eq. (8) means that the final decay amplitudes receive contributions of the same order from initial $D \to \pi\pi$ and from initial $D \to KK$ weak decays. This is equivalent to claiming that the tree topologies are of the same order as the long-distance penguin topologies, in the isospin-zero channels. On the other hand, the Omnes matrix elements do not present any symmetry pattern (note that if e.g. $\Omega^{(0)}_{11} = \Omega^{(0)}_{21}$ and $\Omega^{(0)}_{12} = \Omega^{(0)}_{22}$ the isospin-zero amplitudes would be the same for the pion and kaon pair modes); this result constitutes a manifestation of U-spin breaking. Another implicit source of $SU(3)_f$ breaking effects comes from the fact that the isospin-1 and 2 channels, which exist uniquely for the kaon and pion pairs respectively, are taken from experiment, namely, $D^+ \to \pi^+\pi^0$ and $D^+ \to K^+K^0$ branching ratios, without $SU(3)_f$ being enforced in the process.

With the branching fractions correctly estimated, the same Omnes matrices can be used to determine the CP asymmetries. The asymmetry of each decay mode can be seen as coming from two sources: one is that of the interference of isospin-zero amplitudes, since inelastic rescattering creates amplitudes with two diffent strong and weak phases, besides those introduced also by penguin operators; the other is that of the interference between the isospin-1 or 2 and the isospin-zero invariant amplitudes. It is worth noting that the additional contributions from hadronic matrix elements of the penguin operators are sizable for Q_6, despite being suppressed from the smallness of their Wilson coefficients, due to enhancement factors that accompany the scalar-pseudoscalar annihilation topologies.

For all the viable final state interaction inputs, we find that the CP asymmetry of the $\pi^+\pi^-$ is predicted to be too small compared to the experimental values, while the difference of the asymmetries as defined in [1] is

$$\Delta a_{CP}^{dir,theo} \approx 5 \cdot 10^{-4} \tag{10}$$

also too small compared to experiment. The asymmetries predicted in $\pi^+\pi^-$ and K^+K^- have opposite signs, although a same-sign scenario is in principle not excluded by our predictions. We highlight that a similar level of CP asymmetries is found for the neutral meson decays to $\pi^0\pi^0$ and $K^0\overline{K^0}$.

We have also examined [12] the possibility of the existence of bounds on the asymmetries without implementing any specific $\pi\pi - KK$ inelasticity parameterization, but only assuming two-channel unitarity for the isospin-zero states and implementing the form of the $\pi\pi + KK$ strong phase-shift. We find that the CP asymmetries which result from the interference of isospin-zero amplitudes are bounded to a few times 10^{-4}, while the CP asymmetries from the interference of isospin-2 and -0 for the two-pion channel could be larger, but only if some underlying strong dynamics were present that would significantly enhance simultaneously all the isospin-zero Omnes matrix elements.

In summary, we have followed a data-driven approach that exploits the consequences of unitarity and analyticity through dispersion relations that describe the s-channel final state interactions, in order to predict the decay amplitudes of D^0 to a pion or a kaon pair. We have assumed elastic scattering in isospin 2 and 1 and only rescattering between the pion and kaon pairs in isospin 0. We exploit the experimentally available branching fractions to help discard some of the strong rescattering inputs, and consequently predict the CP asymmetries in the decays of all the channels. The difference of asymmetries as well as the asymmetry in the charged pion decay mode are predicted to be too small compared to the LHCb values. In the future, we plan to examine the effect of further channels which the final states could rescatter to, although this effect is not expected to be significant; we also plan to study other CP asymmetries in related two- and multi-body decays that serve as complementary tests of the SM.

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References

[1] R. Aaij et al. (LHCb collaboration), *Observation of CP Violation in Charm Decays*, Phys.Rev.Lett. 122 (2019) 21, 211803

[2] R. Aaij et al. (LHCb collaboration), *Measurement of the time-integrated CP asymmetry in* $D0 \to K^-K+$ *decays*, Phys.Rev.Lett. 131 (2023) 9, 091802

[3] A. Khodjamirian and A.A. Petrov, *Direct CP asymmetry in* $D^0 \to \pi^-\pi^+$ *and* $D^0 \to K^-K^+$ *in QCD-based approach*, Phys. Lett. B 774, 235 (2017)

[4] E. Franco, S. Mishima, and L. Silvestrini, *The Standard Model confronts CP violation in* $D^0 \to \pi^+\pi^-$ *and* $D^0 \to K^+K^-$, JHEP, 05:140, 2012. arXiv:1203.3131

[5] R. Bause, H. Gisbert, G. Hiller, T. Höhne, D. F. Litim, and T. Steudtner, *U-spin-CP anomaly in charm*, Phys.Rev.D 108 (2023) 3, 035005

[6] A. Pich, E. Solomonidi, and L. Vale Silva, *final state interactions in the CP asymmetries of charm-meson two-body decays*, Phys.Rev.D 108 (2023) 3, 036026

[7] S. de Boer, B. Müller, and D. Seidel, *Higher-order Wilson coefficients for* $c \to u$ *transitions in the standard model*, JHEP 08 (2016) 091

[8] R. Kaminski, J. R. Pelaez, and F. J. Yndurain, *The Pion-pion scattering amplitude. III. Improving the analysis with forward dispersion relations and Roy equations*, Phys. Rev. D, 77:054015, 2008

[9] J.R. Peláez, A. Rodas, and J. Ruiz de Elvira, *Global parameterization of* $\pi\pi$ *scattering up to 2 GeV*, Eur. Phys. J. C 79, 1008 (2019)

[10] J.R. Peláez and A. Rodas, *Dispersive $\pi K \to \pi K$ and $\pi\pi \to K\overline{K}$ amplitudes from scattering data, threshold parameters, and the lightest strange resonance κ or $K_0^*(700)$*, Phys. Rep. 969, 1 (2022)

[11] B. Moussallam, *N_f dependence of the quark condensate from a chiral sum rule*, Eur. Phys. J. C 14, 111 (2000)

[12] A. Pich, E. Solomonidi, and L. Vale Silva, work in progress.