PROCEEDINGS OF SCIENCE

New bounds on magnetic monopoles from primordial magnetic fields

Daniele Perri *
dperri@sissa.it

SISSA, International School for Advanced Studies, via Bonomea 265, 34136 Trieste, Italy
INFN, Sezione di Trieste, via Valerio 2, 34127 Trieste, Italy
IFPU, Institute for Fundamental Physics of the Universe, via Beirut 2, 34014 Trieste, Italy

PoS(EPS-HEP2023)093

Abstract

Monopoles are inevitable predictions of GUT theories. They are produced during phase transitions in the early universe, but mechanisms like the Schwinger effect in strong magnetic fields could give relevant contributions to the monopole number density. I show that from the detection of intergalactic magnetic fields of primordial origin, we can infer additional bounds on the magnetic monopole flux. I also discuss the implications of these bounds for minicharged monopoles, magnetic black holes, and monopole pair production in primordial magnetic fields. The discussion on this paper is based on [1, 2].

1 Models of magnetic monopoles

The possible existence of magnetic monopoles was first theorized by Dirac in 1948 [3]. His model was based on the interpretation of monopoles as semi-infinite string solenoids. The magnetic field at the exit of the string is that of a point source:

$$\vec{B} = g\frac{\vec{r}}{r^3}. \tag{1}$$

In Dirac's theory the monopole is interpreted as an elementary particle and the string associated with it is not considered physical. This implies that when one goes around the Dirac string, the quantum wave function of the electron in the static magnetic field of a monopole should remain single-valued. This is possible once imposed the charge quantization condition:

$$g = \frac{2\pi n}{e} = n g_{\rm D}, \tag{2}$$

where n is an integer and $g_{\rm D}$ is called Dirac charge, i.e. the fundamental magnetic charge. As a consequence of the charge quantization condition, the existence of monopoles provides a strong theoretical motivation for the quantization of the electric charge.

An interpretation of magnetic monopoles in terms of modern gauge theories has been proposed by 'T Hooft and Poliakov in 1974 [4, 5]. They presented a model of magnetic monopoles as zero-dimensional topological defects of the vacuum manifold for gauge theories with spontaneous symmetry breaking (SSB). In particular, magnetic monopoles are interpreted as solitonic solutions of the vacuum manifold in the presence of non-triviality in the second group of homotopy:

$$\pi_2(G/H) \neq I, \tag{3}$$

where G is the symmetry group in the unbroken phase and H is the residual symmetry after the SSB, $G \to H$.

The European Physical Society Conference on High Energy Physics (EPS-HEP2023)
21-25 August 2023
Hamburg, Germany
*Speaker

The simplest model that contains all the ingredients for the monopole solitonic solution is the $SU(2)$ Georgi-Glashow model,

$$\mathcal{L}(t,\vec{x}) = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2}(D_\mu\phi^a)(D^\mu\phi^a) - \frac{1}{4}\lambda(\phi^a\phi^a - \eta^2)^2, \tag{4}$$

where $F^a_{\mu\nu}$ is the non-abelian field strength and ϕ^a is the vector triplet Higgs field responsible for the SSB. The monopole configuration is described by the so-called hedgehog solution for the Higgs field after the symmetry breaking:

$$\phi^a(\vec{x}) = \delta_{ia}\left(\frac{x^i}{r}\right)F(r), \tag{5}$$

where $F(r)$ is a shape function with $F(0) = 0$ and $F(r) \to \eta$ for $r \to \infty$. Within the monopole radius, the theory is still in the unbroken phase.

It can be shown that each time a simply connected gauge group is broken into a smaller group that contains a $U(1)$ the monopole solution exists. Consequently, monopoles are inevitable predictions of Grand Unified Theories. As an example, in the pattern of symmetry breaking of the $SU(5)$ Georgi-Glashow model,

$$SU(5) \to SU(3)\times SU(2)\times U(1) \to SU(3)\times U(1), \tag{6}$$

the monopole solution appears after the first symmetry breaking.

2 Monopole dynamics and primordial magnetic fields

By noting that a population of monopoles would short out the magnectic fields inside galaxies, Parker obtained upper bounds on the flux of monopoles [6]. In [1] we presented a comprehensive study of the Parker-type bounds based on the survival of primordial magnetic fields during reheating and the subsequent epoch of radiation domination. We then extended the results to arbitrary charged monopoles and magnetic black holes in [2]. Our analysis can be applied to both elementary and solitonic monopoles.

Primordial magnetic fields accelerate the monopoles and the process of monopole acceleration extracts energy from the fields. The energy that the monopoles extract from the primordial magnetic field is consequently transferred to the primordial plasma through scattering processes with the relativistic charged particles of the plasma. With a monopole number density large enough, this can cause the disappearance of the field. The evolution of the magnetic field energy density can be derived by solving the equation:

$$\frac{\dot{\rho}_{\rm B}}{\rho_{\rm B}} = -\Pi_{\rm red} - \Pi_{\rm acc}, \tag{7}$$

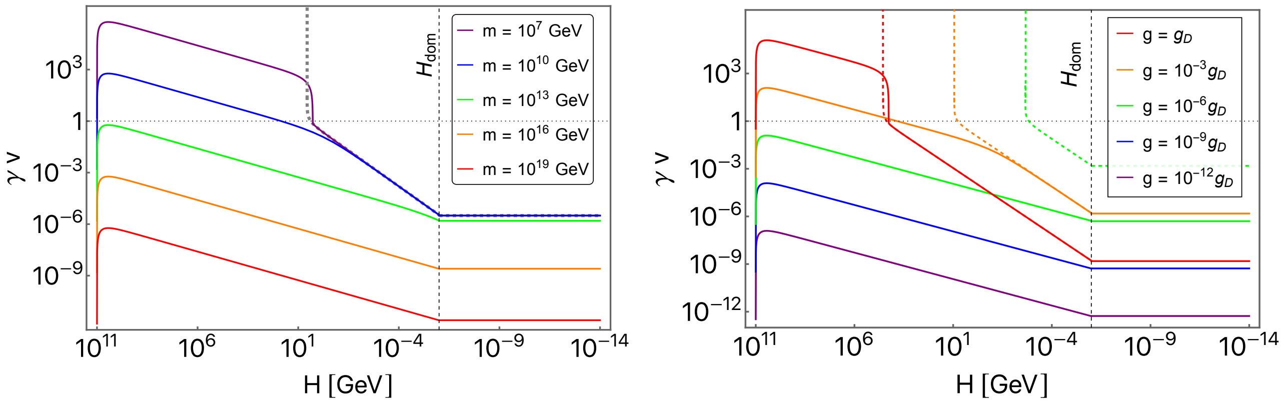
where we define the dissipation rates due to redshifting and monopole acceleration as:

$$\Pi_{\rm red} = 4H, \quad \Pi_{\rm acc} = \frac{4gvn}{B}, \tag{8}$$

with v the monopole velocity and n the monopole number density. If $\Pi_{\rm acc}/\Pi_{\rm red} \gg 1$, the magnetic fields completely lose their energy and eventually disappear. If $\Pi_{\rm acc}/\Pi_{\rm red} \ll 1$, the back-reaction of the monopoles on the magnetic fields is negligible and the fields simply redshift as $B \propto a^{-2}$, where a is the scale factor.

In order to rewrite the condition for the survival of the primordial magnetic fields in terms of the monopole abundance today, we study the monopole equation of motion in the early universe and substitute the results for the monopole velocity in $\Pi_{\rm acc}$. The motion of a monopole with charge g and mass m that moves at velocity v in a Friedmann-Robertson-Walker metric from the end of magnetogenesis to the epoch of e^+e^- annihilation can be described by the equation:

$$m\frac{d}{dt}(\gamma v) = gB - (f_{\rm p} + mH\gamma)\,v, \tag{9}$$



(a) (b)

Figure 1: Evolution in primordial magnetic fields of the velocity of monopoles with (a) fixed charge $g = 10^{-3} g_{\rm D}$ and different values of the mass and (b) fixed mass $m = 10^{11}$ GeV and different values of the magnetic charge. Here we assume a reheating temperature $T_{\rm dom} = 10^6$ GeV, an amplitude of the intergalactic magnetic fields today $B_0 = 10^{-15}$ G, $\mathcal{N}_c = 100$, and we use $H_i = 10^{11}$ GeV as the starting point of the evolution.

where $-f_{\rm p} v$ is the frictional force due to the interactions of the monopoles with the particles of the primordial plasma, with:

$$f_{\rm p} \sim \frac{e^2 g^2 \mathcal{N}_c}{16\pi^2} T^2, \tag{10}$$

and $\mathcal{N}_c$ the number of charged relativistic degrees of freedom. In Fig. 1 we show the time evolution of the monopole velocity for different values of the monopole mass and charge.

3 New bounds on the monopole abundance

From the condition $\Pi_{\rm acc}/\Pi_{\rm red} \ll 1$, we obtain two different upper bounds on the average monopole number density in the present universe: one during radiation domination and one during reheating.

When we apply the condition $\Pi_{\rm acc}/\Pi_{\rm red} \ll 1$ during radiation domination, we obtain an expression that generalizes the result of [7] to arbitrary masses and charges:

$$n_0 \lesssim \max\left\{10^{-20}\ {\rm cm}^{-3},\ 10^{-20}\ {\rm cm}^{-3} \left(\frac{m}{10^{19}\ {\rm GeV}}\right) \left(\frac{g_{\rm D}}{g}\right)^2\right\}. \tag{11}$$

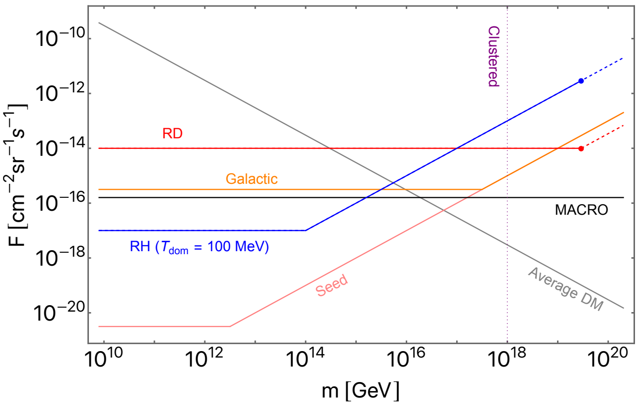
For the epoch of reheating we impose the condition $\Pi_{\rm acc}/\Pi_{\rm red} \ll 1$ assuming thermal equilibrium for the particles of the plasma, although stronger bounds can be obtained relaxing this assumption. For a process of magnetogenesis that ends sufficiently in the past, we obtain the bound:

$$\begin{aligned} n_0 \lesssim \max\Bigg\{ & 10^{-16}\ {\rm cm}^{-3} \left(\frac{B_0}{10^{-15}\ {\rm G}}\right)^{3/5} \left(\frac{T_{\rm dom}}{10^6\ {\rm GeV}}\right) \left(\frac{g_{\rm D}}{g}\right)^{3/5}, \\ & 10^{-13}\ {\rm cm}^{-3} \left(\frac{m}{10^{17}\ {\rm GeV}}\right) \left(\frac{T_{\rm dom}}{10^6\ {\rm GeV}}\right) \left(\frac{g_{\rm D}}{g}\right)^2 \Bigg\}. \end{aligned} \tag{12}$$

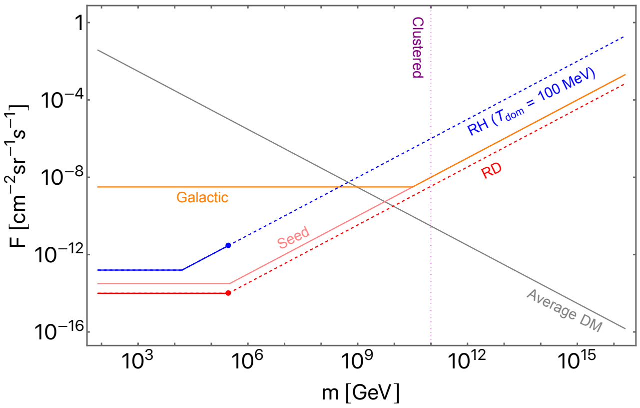
Here B_0 is the amplitude of the intergalactic magnetic field today. In this case, the bound depends on the reheating temperature, $T_{\rm dom}$.

For this analysis we assume that the monopoles are able to transfer the energy gained by the magnetic fields to the primordial plasma, although this is not always the case. In particular, the interaction between monopoles and primordial plasma is strong enough to dissipate the gained magnetic energy only under the condition:

$$m \lesssim 10^{19}\ {\rm GeV} \left(\frac{g}{g_{\rm D}}\right)^2. \tag{13}$$



(a)



(b)

Figure 2: Upper bounds on the magnetic monopole flux today for (a) $g = g_D$ and (b) $g = 10^{-7} g_D$. Here we assume $B_0 = 10^{-15}$ G and for the monopole velocity today $v_0 = 10^{-3}$. Grey: cosmological abundance bound, red: bound from primordial magnetic fields during radiation domination in Eq. (11), blue: bound from primordial magnetic fields during the reheating epoch in Eq. (12) for a reheating temperature $T_{dom} = 100$ MeV, orange: Galactic Parker bound [6], pink: seed Galactic Parker bound [10], black: direct search limit from the MACRO experiment [11] (only for Dirac charge). The lower mass limit for clustering with the Milky Way is shown in dashed purple lines.

Strictly speaking, for larger masses a violation of the bounds does not lead to the disappearing of the primordial magnetic fields, but only to a different redshift evolution.

4 Minicharged monopoles and magnetic black holes

Dark Matter candidates must meet two fundamental criteria in cosmological models: they should account for the required energy density, and they should cluster with observable galaxies, resulting in an overabundance of Dark Matter compared to the cosmic average. While magnetic monopoles have been considered as potential Dark Matter candidates, their efficient production remains a significant challenge. This is primarily due to the fact that traditional magnetic monopoles, in order to fulfill the dual requirements of covering the total Dark Matter content and clustering with galaxies, must be extraordinarily massive, with masses exceeding M_{Pl}.

To address this challenge, two distinct strategies are explored. First, there is the option of reducing the monopole charge, leading to the concept of "minicharged monopoles." The second strategy involves enlarging the monopole mass, considering the existence of "magnetic black holes."

Considering minicharged monopoles relaxes the mass requirements and allows lighter monopoles to potentially serve as Dark Matter constituents. One simple scenario where minicharged monopoles might emerge without violating the Dirac quantization condition has been proposed by [8]. This model involves a dark $SU(2)$ symmetry and two symmetry-breaking events, $SU(2) \to U(1) \to Z_2$. After the first SSB, there is production of dark monopoles, and after the second SSB the dark field confines into dark strings connecting the monopoles. In the presence of a mixing term with the visible sector, after the second simmetry breaking the dark monopoles are provided with a tiny electromagnetic charge and act as minicharged magnetic monopoles. However, this is just an example and we underline that our analysis does not assume any particular model of minicharged monopoles.

In Fig. 2 we show previous bounds on the monopole flux together with our new results for different values of the magnetic charge. The region of the primordial bounds where the condition is only on the redshift behaviour of the magnetic fields, and not on their disappearance, is shown in dashed lines. Notably, the primordial constraints on the monopole abundance are less dependent on the monopole charge and therefore become the strongest for small charges. Minicharged monopoles can cluster with the galaxies and be Dark Matter for masses much smaller than M_{Pl} and are less constrained by Parker-type bounds. As a result, minicharged monopoles are good candidates for Dark Matter.

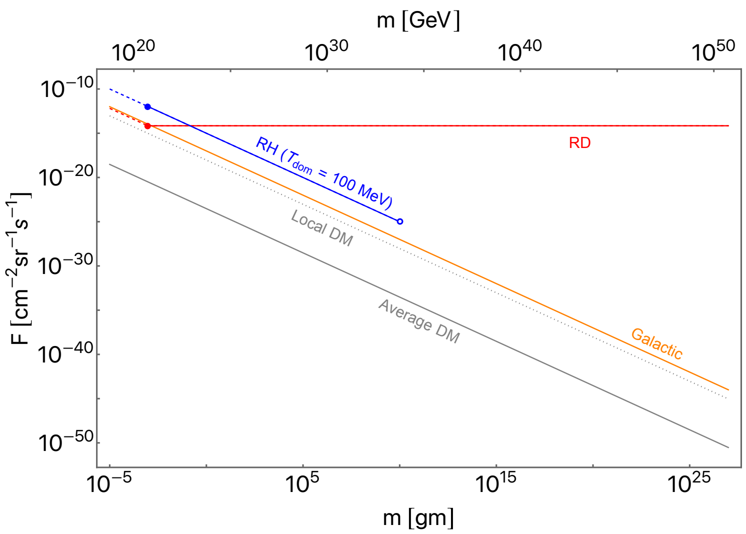


Figure 3: Upper bounds on the extremal magnetic black hole flux today. Here $B_0 = 10^{-15}$ G and $v_0 = 10^{-3}$. Grey: cosmological (solid) and local (dashed) abundance bound, red: bound from primordial magnetic fields during radiation domination in Eq. (11), blue: bound from primordial magnetic fields during the reheating epoch in Eq. (12) for a reheating temperature $T_{\rm dom} = 100$ MeV, orange: Galactic Parker bound [6], pink: seed Galactic Parker bound [10].

Because of the no-hair theorem, outside the event horizon magnetically-charged black holes act as extremely heavy magnetic monopoles. Therefore, they can be candidates of magnetically-charged Dark Matter. Because of the charge, such black holes cannot evaporate by Hawking radiation and stop the evaporation when they become extremal. This allows us to consider magnetic black holes with masses much smaller than the lightest possible mass for uncharged evaporating primordial black holes. Then, here we consider quasi-extremal magnetic black holes, which present a fixed mass-to-charge ratio:

$$g = \frac{m}{\sqrt{2}M_{\rm Pl}}. \tag{14}$$

In Fig. 3 we show how the bounds on monopole flux look like in the case of extremal magnetic black holes. Although cosmological bounds are stronger than galactic Parker bound for the Milky Way, this is not the case if we apply the galactic bound to the magnetic fields of M31. More than that, independently of their masses extremal magnetic black holes cluster with Milky Way but in general not with all the galaxies. Therefore, they are exluded as possible candidates of Dark Matter.

5 Schwinger effect and monopole pair production

Even in the absence of any initial monopole population, in the early universe the magnetic fields could have been strong enough to produce monopole pairs through the magnetic dual of the Schwinger effect [12, 13]. Hence we also apply our generic bounds to such pair-produced monopoles, in order to obtain the most conservative upper bound on the primordial magnetic field amplitude.

The rate of monopole-antimonopole pair production at arbitrary coupling in a static magnetic field has been derived through an instanton method:

$$\Gamma = \frac{(gB)^2}{(2\pi)^3} \exp\left[-\frac{\pi m^2}{gB} + \frac{g^2}{4}\right]. \tag{15}$$

In the case of strong coupling, $g \gg 1$, this result is valid under the following weak field condition:

$$B \lesssim \frac{4\pi m^2}{g^3}. \tag{16}$$

We found that the condition $\Pi_{\rm acc}/\Pi_{\rm red} \ll 1$ in the early universe applied to the pair-produced monopoles reduce to Eq. (16) on the initial strength of the primordial magnetic fields, with only a negligible log contribution.

The bounds we derive are comparable to those obtained in [14], which also examined the magnetic field screening through the Schwinger process and monopole overproduction. Therefore, once the weak field condition is verified the primordial magnetic fields survive both the processes of monopole pair production and the acceleration of the Schwinger-produced monopole pairs.

6 Conclusion

We carried out a comprehensive study of the monopole dynamics in the early universe and its back-reaction on primordial magnetic fields. We derived new bounds on the cosmic abundance of magnetic monopoles from the survival of primordial magnetic fields and generalized the results to arbitrary charged monopoles and extremal magnetic black holes. We found that, for a sufficiently small reheating temperature, primordial bounds become stronger than the galactic Parker bound and the limits from direct search even for a GUT scale monopole with Dirac charge. We also found that monopoles with different magnetic charges are best constrained by different astrophysical systems: Dirac charged monopoles by seed galactic magnetic fields, minicharged monopoles by primordial magnetic fields, and magnetic black holes by galactic magnetic fields.

We also studied under which conditions magnetic monopoles can be considered as possible Dark Matter. We found that although monopoles with Dirac charge can be Dark Matter only for masses comparable to or larger than $M_{\rm pl}$, minicharged magnetic monopoles are not constrained and cluster with galaxies for much smaller masses. Therefore, minicharged monopoles are good Dark Matter candidates. On the other hand, extremal magnetic black holes cannot cluster with all the galaxies and are strongly constrained by galactic Parker bounds. Then, they are excluded as Dark Matter candidates.

Finally, we obtained the most conservative bound on the primordial magnetic field amplitude from the Schwinger pair production of monopoles. We found that as long as the initial amplitude of the primordial magnetic field is sufficiently below the bound of the weak field condition in Eq. (16), any back-reaction from Schwinger-produced monopoles can be safely ignored.

References

[1] T. Kobayashi and D. Perri, Phys. Rev. D **106** (2022) no.6, 063016.

[2] T. Kobayashi and D. Perri, Phys. Rev. D **108** (2023) no.8, 083005.

[3] P. A. M. Dirac, Proc. Roy. Soc. Lond. A **133** (1931) no.821, 60-72

[4] G. 't Hooft, Nucl. Phys. B **79** (1974), 276-284.

[5] A. M. Polyakov, JETP Lett. **20** (1974), 194-195 PRINT-74-1566 (LANDAU-INST).

[6] M. S. Turner, E. N. Parker and T. Bogdan, Phys. Rev. D **26**, (1982) 1296.

[7] A. J. Long and T. Vachaspati, Phys. Rev. Lett. D **91** (2015) 103522.

[8] T. Hiramatsu, M. Ibe, M. Suzuki and S. Yamaguchi, JHEP **12** (2021), 122.

[9] J. Maldacena, JHEP **04**, 079 (2021).

[10] F. C. Adams, M. Fatuzzo, K. Freese, G. Tarle, R. Watkins and M. S. Turner, Phys. Rev. Lett. **70**, 2511 (1993).

[11] The MACRO collaboration, Eur. Phys. J. C **25** (2002) 511–522.

[12] J. S. Schwinger, Phys. Rev. **82** (1951) 664.

[13] I. K. Affleck and N. S. Manton, Nucl. Phys. B **194** (1982) 38.

[14] T. Kobayashi, Phys. Rev. D **104** (2021) no.4, 043501.