PROCEEDINGS OF SCIENCE

New avenues and observational constraints on functors of actions theories

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In this document, we introduce a novel formalism for any field theory and apply it to the effective field theories of large-scale structure. The new formalism is based on functors of actions composing those theories. This new formalism predicts the actionic fields. Furthermore, we discuss a generalised manifold with N-correlators of N_t- objects with or without contaminants. we discuss our findings in a cosmological gravitology framework. We present these results with a cosmological inference approach and give guidelines on how we can choose the best candidate between those models with some latest understanding of model selection using Bayesian inference.

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1. Introduction

This document is structured around two primary themes. Firstly, we embark on a journey into the realm of theoretical cosmology, where we delve into fundamental concepts crafted by cosmologists. Our exploration commences by introducing the intriguing notion of Functors of Actions (FA), a framework that holds significant promise. We will offer a concise guide on how to construct FA models, delve into the concept of actionic field fluctuations, and provide an overview of simulated constraints for these models. For those seeking a deeper understanding, a more detailed exposition is available in [1].

Secondly, we introduce a powerful tool—a (D_τ, D_x)-manifold coupled with N-point correlation functionals (NPCF) for N_t-objects. This formalism finds applications in the fascinating domains of large-scale structure (LSS) and quantum field theory (QFT) [2]. Our journey concludes with a brief summary and a glimpse into future directions.

2. Cosmological Gravitology

A concise overview of effective field theories (EFTs) is presented here to elucidate the Large Scale Structure (LSS) and introduce the Framework of Asymptotic (FA) behavior. A comprehensive review of EFTs and Modified Gravity (MG) in cosmology can be found in the literature [see, e.g., 3–5], along with further references. For a more detailed treatment, readers are encouraged to consult [1, 3] for notation specifics.

We illustrate most of these theories using an expanded schematic diagram as depicted in Fig. 1. In this diagram, distinct theory classes are represented with varying colors.

The canonical General Relativity (GR) is characterized by a local background metric $g_{\mu\nu}(x)$ describing a massless graviton [6]. In Fig. 1, GR is denoted by a black shape, and we also explore its extensions through various approaches.

In Fig. 1, we illustrate various models that challenge the assumptions of GR. These models encompass scenarios involving the compactification of dimensions, non-local models, ekpyrotic models (referred to as ΘCDM), extra dimensions, Lorentz violations, Einstein aether models, and Horava models. Notably, the DGP models belong to this category, exploring gravity in a 5D Minkowski space. It's worth mentioning that any model predicting extra dimensions falls into this category, including string theory, Anti-de Sitter space, and holography models.

Models in brown represent "massive gravity" models ($m_g > 0$), including the dRGT model, which is a resummation of massive gravity. Blue shapes denote models that introduce an additional field, grouped by the properties of the new field. Dark blue models incorporate a tensor field ($T_{\mu\nu}$), such as bigravity and multigravity, which are constrained by gravitational wave oscillations. Light blue models involve a vector field (V_μ), including Proca ($m_V > 0$), General Proca, and TeVeS (Modified Newtonian Dynamics - MOND) models. Green-blue shapes are for models introducing an additional scalar field (ϕ). Horndeski theories, are some of the simplest extensions in this category. These models encompass Love-Lock, quintessence, Brans-Dicke, $f(R)$, KGB, Gauss-Bonnet, Galileon, and unified dark energy and dark matter (UDEDM) models. The concept of Effective Field Theories of Large-Scale Structure (EFTofLSS) was introduced by [7], and mainly includes models from scalar theories.

Finally, we introduce novel ideas represented in orange. These ideas involve basic manipulations of the action, introducing a new set of functors of actions. These manipulations may generate new actions and theories, extending current models. We refer to these theories as "functors of actions" (FA). It's worth noting that the term "functors of actions" is a basic placeholder, and alternative names, such as "relations of actions," can be considered to better describe these novel theories. These theories are denoted by orange shapes. Two possible applications include actions of EFT (AofEFT), and AofEFTofLSS, which is detailed in section 3.

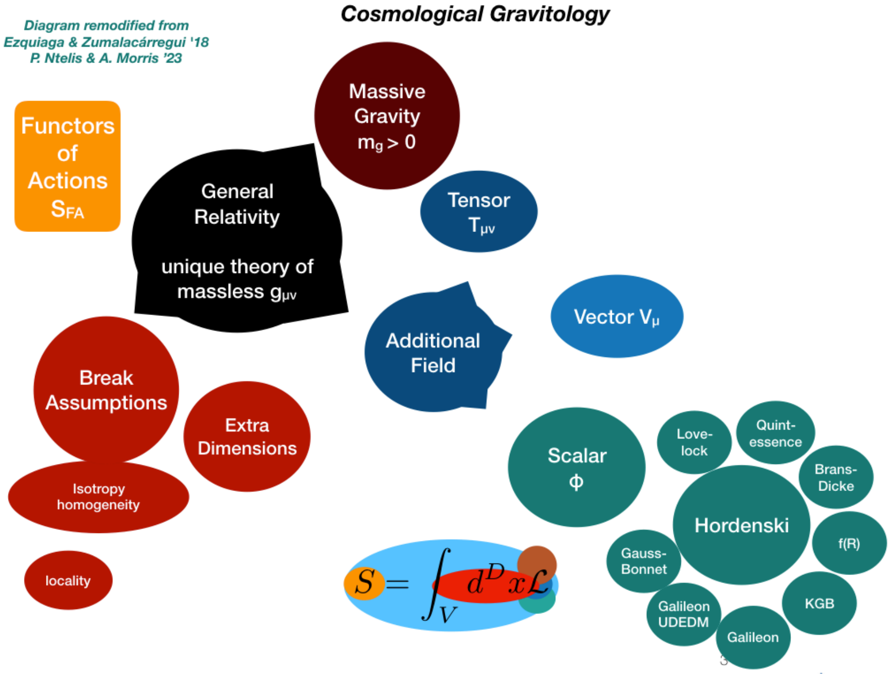

Figure 1: Cosmological gravitology is given by this diagram which describes the exploration of expansion of the general relativity theory, where each theory is color-coded. In this diagram, we add the novel functors of actions (FA) theories, which predict the actionic fluctuations.

3. Functors of actions

In mathematics, a **Functor (F)** is a generalization of the concept of a functional, which, in turn, is a generalization of the concept of a function.

In the realm of physics, an **action (S)** is a fundamental quantity that represents the product of energy and time, serving as a bridge between topology and energy. It provides insight into the diverse ways a particle can traverse from one point to another within a specific spacetime region.

The **functor of actions (FA)** is the application of the mathematical quantity functors on the physical quantity action. FA goes further by predicting the existence of actionic fluctuations and field-particles, analogous to energetic and topological fluctuations and field-particles.

We denote the category of FA theories as $\mathcal{S}_{\rm FA}$. Let a possible action, S, defined as an integral of a hyper-manifold, $\mathcal{M}$, with infiinitesimal element of D-dimensions, $d^D x$, and a determinant of a metric, g. The kernel of this integral is a lagrangian density $\mathcal{L}$. Then the action is given by the following expression,

$$\mathcal{S}_{\rm FA} \supset S = \int_{\mathcal{M}} d^D x \, \sqrt{-g}\mathcal{L} \ . \tag{1}$$

We can guild new FA theories by using a functional form of an action, such as

- considering a functional of an action S, i.e. $S_{\rm FA} = F[S]$,
- considering an integral of S, i.e. $S_{\rm FA} = \int_0^{S'} dS$,
- considering a contraction of a tensor of S, i.e. $S_{\rm FA} = S^{\mu_1\cdots\mu_\nu} S_{\mu_1\ldots\mu_\nu}$,
- considering a combination of those, $S_{\rm FA} = \int_0^{S'} dS \ F[S^{\mu_1\cdots\mu_\nu} S_{\mu_1\ldots\mu_\nu}]$.

Note that FA theories have as a limit all known and studied actions, such as GR, strings, Hordenski, EFTofLSS, QFT, and others. In particular, we have the following.

- The FA theories have a limit the GR-action, S_{GR}, an AofEFTofLSS. This can be shown by the following expression

$$\mathcal{S}_{\rm FA} \supset S_{FA} = \int_0^{S_{GR}} dS = S_{GR} = \int d^D x\sqrt{-g}\left[\frac{R}{16\pi G_{\rm N}} + \mathcal{L}_m\right] \ , \tag{2}$$

  where R is the Ricci scalar, G_N is the Newton gravitational constant, and $\mathcal{L}_m$ is the matter lagrangian density.

- The FA theories have a limit the string-action, S_{string}, an AofEFT or AofEFTofLSS. This can be shown by the following expression, $\mathcal{S}_{\rm FA} \supset S_{FA} = \int_0^{S_{string}} dS = S_{string}$.
- The FA theories have a limit the universe-action, $S_{universe}$, an AofEFTofLSS. This can be shown by the following expression, $\mathcal{S}_{\rm FA} \supset S_{FA} = \int_0^{S_{Universe}} dS = S_{Universe}$.

A particular model that we can build with some observational constrains is the following,

$$S_{\rm FA} = \beta S_{\rm R} + S_m + \delta S_3 \tag{3}$$

where β is the modulation parameter which shows the amount of actionic curvature and δS_3 is an actionic fluctuation. Applying the least action principle, $\delta S = 0$, we get

$$R_{\mu\nu} + \frac{1}{2}R g_{\mu\nu} = \frac{1}{\beta}\frac{8\pi G_{\rm N}}{c^3}\left[T_{\mu\nu} + \delta(\mathcal{L}_3)_{\mu\nu}\right] \tag{4}$$

where $\delta(\mathcal{L}_3)_{\mu\nu}$ is the lagrangian fluctuation tensor. Modelling the lagrangian fluctation using a simple parameter $\delta\mathcal{L}_3$, i.e. $\delta(\mathcal{L}_3)_{\mu\nu} = \delta\mathcal{L}_3\rho$, for $\mu = \nu = 1$ while $\delta(\mathcal{L}_3)_{\mu\nu} = 0$ otherwise, where ρ is the energy density of the universe, we have that the equation of state is

$$w = [1 + \delta\mathcal{L}_3]^{-1} \ . \tag{5}$$

By constraining the standard equation of state w, then we can constrain this model since we constrain $\delta\mathcal{L}_3$. Therefore, a measurement of a constant (non-redshift-dependent) equation of state, *i.e.* $w \simeq -0.9$, corresponds to an exotic AofEFT model with $\delta\mathcal{L}_3 \simeq 0.\dot{1}$. For more details see [1].

4. A (D_τ, D_x)-manifold with NPCF of N_t-objects

Consider the following experimental set up shown in Fig. 2, where with green we define the (D_τ, D_x)-dimensional target manifold, $\mathcal{M}^T$. We define the (D_τ, D_x)-dimensional contaminant manifold, $\mathcal{M}^C$, with red and green. Red corresponds to the case where the contaminants live in higher redshifts than the target sample one, while green corresponds to the case where the contaminants live in lower redshifts.

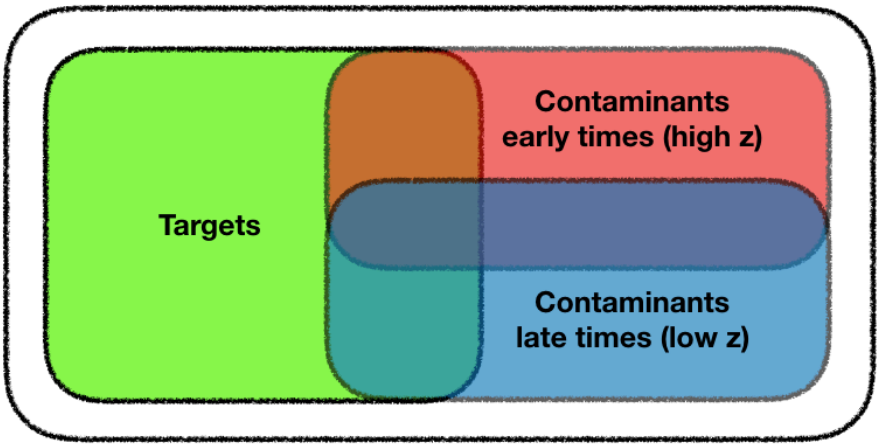

Figure 2: Experimental set up, where green shows the (D_τ, D_x)-dimensional target manifold, $\mathcal{M}^T$. We colocode the (D_τ, D_x)-dimensional contaminant manifold, $\mathcal{M}^C$, with red and green. Red corresponds to the case where the contaminants live in higher redshifts than the target sample one, while green corresponds to the case where the contaminants live in lower redshifts.

Under reasonable assumptions detailed in [2], in LSS, we can define the functional of bias and growth of structure, $\mathcal{FBD}(\vec{\tau})$ as a function of the observed NPCF, $\xi_O^{(N)}(\vec{\tau}, \vec{x})$, over the matter one, $\xi_m^{(N)}(\vec{\tau}, \vec{x})$. This is the same using their Fourier space respectively. In a (1,3) dimensional space, i.e. the spacetime continuum, this functional is only a function of redshift z, i.e.

$$\mathcal{FBD}(\vec{\tau}) \rightarrow \mathcal{FBD}(z) = \left[\frac{P_O^{(N)}(z, \vec{k})}{P_m^{(N)}(z, \vec{k})} \right]^{1/N} \tag{6}$$

where P is the power spectrum of ξ.

According to the model from Eq. 6 built in [2], As anticipated, our analysis reveals the following trends in the behavior of the functional $\mathcal{FBD}(z)$:

1. increasing (or decreasing) the number of tracers.
2. increasing (or decreasing) the contaminant factor.

lead to corresponding changes in the functional $\mathcal{FBD}(z)$. Specifically, doubling the number of tracers within the same redshift range while employing the same bias model results in a proportional doubling of $\mathcal{FBD}(z)$. Additionally, a 10% increase in the contaminant factor corresponds to a

2-1% (or 4-2.4%) increase in the functional $\mathcal{FBD}(z)$, depending on whether one (or two) tracers are within the targeted redshift range of interest. Note that the behaviour changes when the contaminants live in lower redshift regions than the target ones.

It is well-established that natural physical quantum systems can be effectively described by Minkowski spacetime at quantum scales. In the context of this study, we extend this description to encompass a generalized Minkowski spacetime within the framework of NPCF for quantum mechanical systems. Much like the concept of LSS, we anticipate a similar requirement in quantum systems: the distinction between a targeted quantum system and a contaminant one, possibly arising from elements we prefer not to observe or target. Consequently, we can formulate an NPCF framework for quantum systems akin to what has been achieved for LSS. For further details please read [2].

5. Summary and conclusion

In summary, the functors of actions (FA) theories represent innovative concepts stemming from the application of functors to actions. These FA theories find application across both large-scale and quantum-scale phenomena, where they offer predictions related to actionic fluctuations and field-particles.

Furthermore, we establish a (D_τ, D_x)-dimensional manifold, featuring N-point correlation functionals (NPCF) applied to N_t-objects, both with and without contaminated samples. This framework proves relevant in both the study of large-scale structures and quantum-scale structures.

Ongoing research persists in testing these theories using various observables, encompassing data from telescopes and particle colliders. Efforts are directed towards expanding and incorporating specific models pertaining to the Hubble expansion rate, as related to Dark Energy and Dark Matter, within the overarching framework of functors of actions. These endeavors are aimed at addressing and mitigating existing tensions and discrepancies in the field.

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